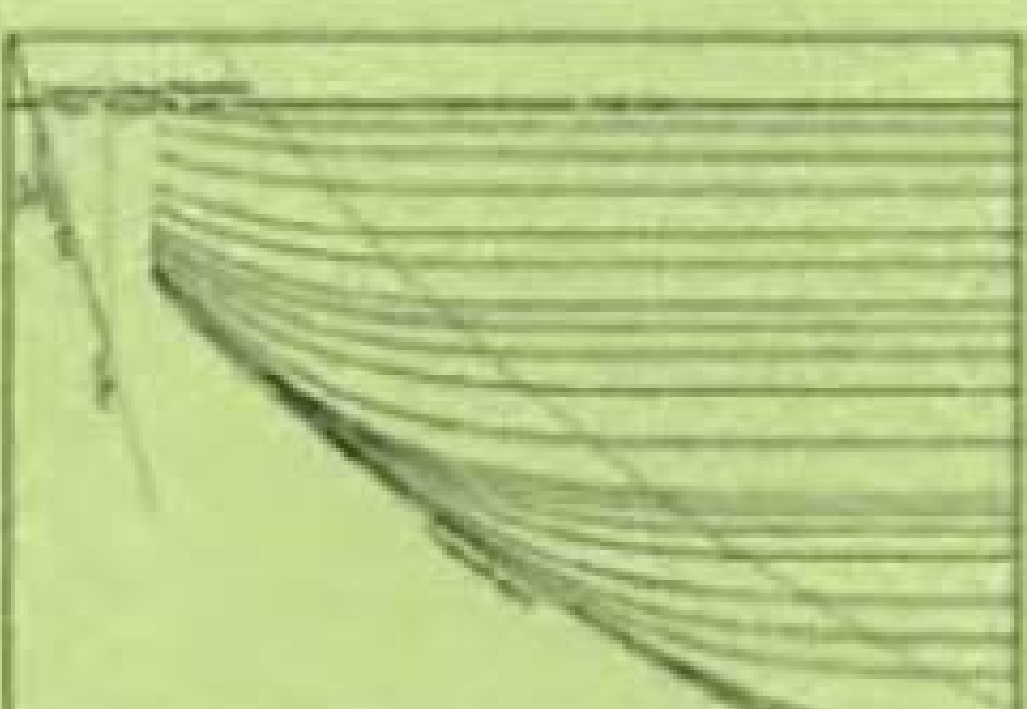
PRACTICAL FLUID MECHANICS FOR ENGINEERING APPLICATIONS



JOHN J. BLOOMER

PRACTICAL FLUID MECHANICS FOR ENGINEERING APPLICATIONS

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To the memory of James Murdock

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Preface

When Jim Murdock first approached me about the idea for this book, I was impressed. It seemed to me that a book covering the practical side of fluid mechanics would be ideal for the practicing engineer. Jim asked me to become a coauthor of the project, but unfortunately he passed away before we began work. I inherited a partially completed manuscript that was based on his earlier introductory book on the subject, *Fundamental Fluid Mechanics for the Practicing Engineer*.

I completed this "stand-alone" book, trying as much as possible to keep Jim's original ideas alive. What has resulted is a book that can be used by engineers and technicians who have no memory of or no formal training in the theoretical mathematics that provides a foundation for most of today's undergraduate fluid mechanics texts. The book is designed as both a reference and a primer. Although intended for the practicing engineer, this book can also be used to augment the material covered in an undergraduate fluid mechanics class because it presents useful design procedures and many hard-to-find fluid properties.

The book is organized into eight chapters. Chapters 1–3 present introductory definitions, equations, and derivations that are useful for the material that follows. The material can be used to refresh the reader's knowledge in a particular area or it may be skipped and referred to later as questions arise. Chapters 4–8, the heart of the book, cover subjects such as pipe flow and system design, flow in open channels, flow measurement methods, forces on immersed objects, and unsteady flow. Over 50 fully solved examples are given to illustrate each concept. The examples are solved in the U.S. Customary System (USCS) of units, with conversions provided for use with the System International (SI) units.

Appendixes are provided that give useful supplementary information for this text and many other applications. Appendix A provides engineering conversion factors. Appendix B supplies information on pipe schedules. Appendix C is a compilation of properties of areas, pipes, and tubing. Finally, Appendix D contains the saturated, critical, and gas properties of 49 fluids, and the viscosity and density of compressed water and saturated steam.

Writing a book is an enormous undertaking and it requires the efforts, cooperation, and understanding of many. I wish to thank the editorial staff of Marcel Dekker, Inc., for their belief in this project and the encouragement to finish. I also need to thank my former mentors at Drexel University, my current colleagues at Selas Corporation of America, and the estate of James Murdock for their help at various points during this process. I am most grateful for the support of my Mom and Dad, brothers and sisters, Keenans, and friends (in fact, my Dad and brothers-in-law Ed and Steve are my inspiration for all things practical involving pipe flow). Finally, my heartfelt thanks to my wife, Regina, and my daughter, Mary Beth, for their patience, love, and motivation through this and all my endeavors.

John J. Bloomer

Contents

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1

Basic Concepts of Fluid Mechanics

1.1 INTRODUCTION

The purpose of this chapter is to review the fundamental concepts of fluid mechanics and to provide a sound foundation for the development of equations throughout this book. It is assumed that the reader has an understanding of basic undergraduate mathematics and engineering principles. If further development of the principles outlined in this chapter is desired, the reader is directed to any introductory fluid mechanics text (suggested references are provided at the conclusion of this chapter).

This chapter outlines basic fluid characteristics, classifications, and properties. A brief outline of the fundamental thermodynamic concepts required in fluid mechanics is also provided. Chapter 2 follows with the development of the equations that describe fluids at rest, and Chapter 3 completes the introductory material of the text with a description of the equations of fluids in motion. Chapters 4 through 8 describe the practical applications of this introductory material to engineering problems.

1.2 CHARACTERISTICS OF A FLUID

Substances may be classified by their response when at rest to the imposition of a shear force. Consider the two very large plates, one moving, the other stationary, separated by a small distance y shown in Figure 1.1. The space between these plates is filled with a substance whose surfaces adhere in such a manner that the upper surface of the substance moves at the same velocity as the upper plate while the bottom surface is stationary. As a result of the imposition of the shear force F_s when the upper plate moves, the upper surface of the substance attains a velocity U. As y approaches dy, U approaches dU and the rate of deformation of the substance becomes dU/dy. The unit shear stress $\tau = F_s/A_s$, where A_s is the shear area.

A plot of shear stress as a function of deformation rate (dU/dy) is illustrated in Figure 1.2 for several substances. An ideal solid, or *elastic solid*, will resist an imposed shear force, and its rate of deformation will be zero, regardless of loading. Shear stress for an elastic solid is therefore coincident with the y-axis of Figure 1.2. A *plastic* will resist shear until its yield stress is attained, and then the application of additional loading will cause it to deform continuously or flow. If the deformation rate is directly proportional to the applied shear stress less that required to start flow, then it is called an ideal plastic or *Bingham plastic*. If the substance is unable to resist even the slightest amount of shear force without *flowing*, then it is called a *fluid*. An *ideal fluid* has no internal friction, and hence its deformation rate coincides with the x-axis of Figure 1.2. All real fluids have internal friction, so their rate of deformation is a function of the applied shear stress. If the rate of deformation is directly proportional to the applied shear stress, the fluid is called a *Newtonian fluid*. Under normal conditions, fluids such as air and water are Newtonian fluids. The deformation rate of a *non-Newtonian fluid* is not directly proportional to an applied shear stress. Examples of non-Newtonian fluids are polymer solutions and blood.

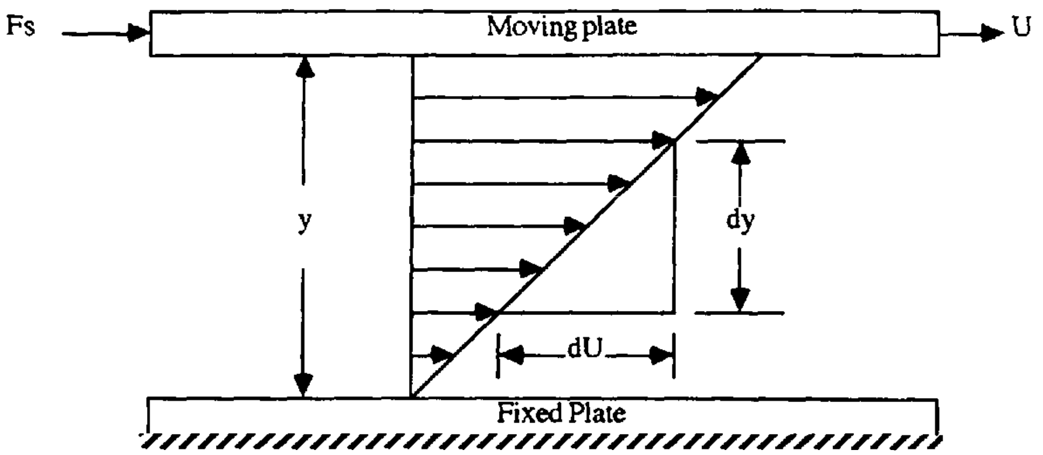

Figure 1.1 Example of substance flow between parallel plates.

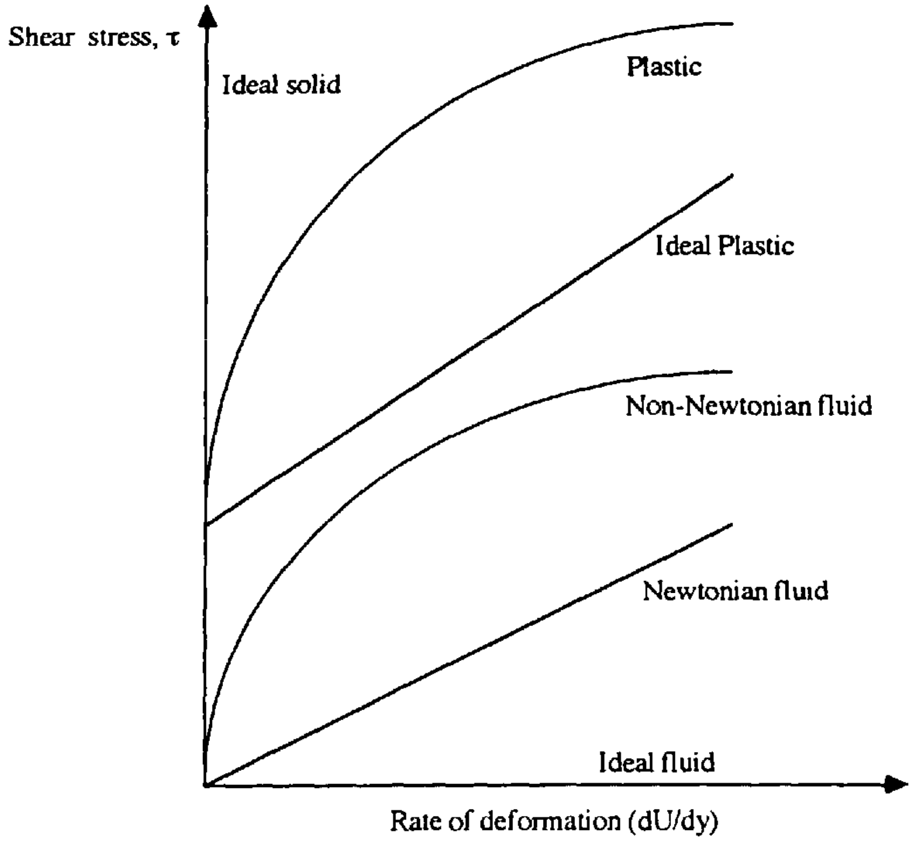

Figure 1.2 Deformation rate characteristics of several substances.

The behavior of Newtonian fluids is of interest for most common engineering problems and, thus, is the main focus of this book. The fact that the rate of deformation is directly proportional to the applied shear stress for Newtonian fluids is mathematically expressed as equation (1.1):

$$\tau \propto \frac{dU}{dy} \tag{1.1}$$

The constant of proportionality required for equality of equation (1.1) is known as the *absolute* or *dynamic viscosity* (μ). Equation (1.1) can therefore be rewritten as equation (1.2):

$$\tau = \mu \frac{dU}{dy} \tag{1.2}$$

Equation 1.2 is known as Newton's law of viscosity.

Absolute viscosity will be discussed further in Section 1.4; for now it can be said that it is a measure of the internal friction. As will be seen throughout this book, it is sometimes convenient to assume that a fluid behaves ideally (i.e., no internal friction); such fluids are commonly referred to as *inviscid fluids*.

1.3 KINDS OF FLUIDS

For the purposes of the application of fluid mechanics to design it is convenient to consider two kinds of fluids: compressible and incompressible. These characteristics are determined by the molecular spacing and arrangement, or *phase*, of the substance. In general, changes in density are negligible for incompressible flows but are not negligible for compressible flows. Several statements can be made about the compressible/incompressible nature of fluids that depend on the phase:

Liquids are considered to be incompressible except at very high pressures. Unless otherwise specified in this book, liquids are assumed to be incompressible.

Vapors are gases at temperatures below their critical temperatures and are compressible, but their temperature–pressure–volume relationships cannot be expressed by simple mathematical equations like the ideal gas law. Vapor properties are usually tabulated, as, for example, in steam and refrigeration tables.

Gases are compressible fluids. Gases behave ideally (i.e., the pressure–volume–temperature relation can be expressed by the ideal gas law, as discussed in Section 1.5) if the ratio of the temperature of the substance to the critical temperature approaches infinity and the ratio of the pressure to the critical pressure approaches zero. No real gas follows this law exactly, and a simple nonideal gas equation of state is also presented in Section 1.5.

1.4 FLUID PROPERTY DEFINITIONS

Basic fluid properties used throughout this book are defined in this section. The units commonly used for each property are provided in both the U.S. Customary System (USCS) and the Système Internationale (SI) units. Dimensions for each property are also provided using the following nomenclature: M = mass, L = length, T = time, and F = force.

Temperature

Symbol: T
Units: USCS: °F, °R; SI: °C, K

Temperature is a macroscopic property that is a measure of the average molecular (microscopic) energy within a substance. The thermodynamic temperature is called an *absolute temperature* because its datum is absolute zero (i.e., zero molecular energy). The *thermodynamic temperature scale* has little practical value unless numbers can be assigned to the temperatures at which real substances freeze or boil so that temperature-sensing devices may be calibrated. The following are the commonly used temperature scales in engineering applications:

Fahrenheit temperature scale: used in the United States for ordinary temperature measurements. On this scale, water freezes (ice point) at 32°F and boils at 212°F (steam point) at standard atmospheric pressure.

Celsius temperature scale (formerly centigrade): the S.I. unit of temperature measurement. On the Celsius scale, water's ice point is 0°C and the steam point is 100°C.

Kelvin temperature scale: the absolute Celsius scale. The relationship of the kelvin (K with no degrees sign) to the degree Celsius is shown in equation 1.3:

$$T_K = T_C + 273.15 \tag{1.3}$$

For most engineering calculations, this can be simplified as follows:

$$T_K = T_C + 273 \tag{1.4}$$

Rankine temperature scale: the absolute Fahrenheit scale. The degree Rankine is related to the degree Fahrenheit as follows:

$$T_R = T_F + 459.67 \tag{1.5}$$

For most engineering calculations, this can be simplified as follows:

$$T_R = T_F + 460 \tag{1.6}$$

Temperature scale relations are shown in Figure 1.3. On the Celsius scale, there is 100° difference between the water ice and steam points, while on the Fahrenheit scale the difference is 180°. The conversions between the two scales are expressed in equations (1.7) and (1.8):

$$T_F = 1.8T_C + 32 \tag{1.7}$$

$$T_C = \frac{T_F - 32}{1.8} \tag{1.8}$$

The conversion between the two absolute temperature scales is shown as equation (1.9):

$$T_R = 1.8T_K \tag{1.9}$$

Unless otherwise specified in this text, all temperatures must be converted to absolute units for use in calculations.

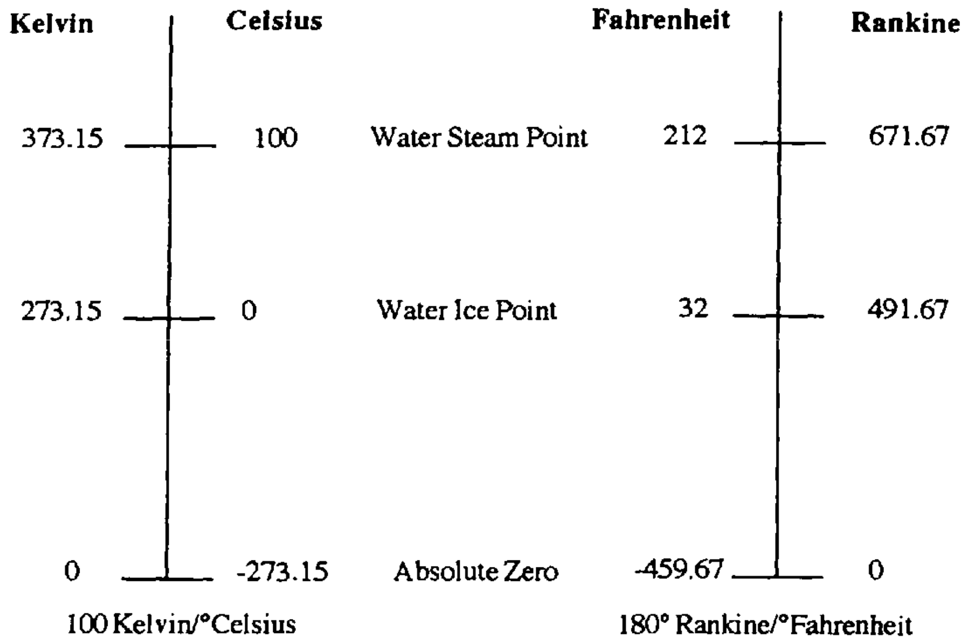

Figure 1.3 Temperature scales.

Pressure

Definition: Force per unit area
Symbol: p
Dimensions: FL^{-2} or ML^{-1}/T^{-2}
Units: USCS: lbf/in.^{2} (psi), lbf/ft^2; SI: N/m^2 or Pa

Shear forces acting on a fluid have been previously discussed in the context of viscosity. Fluids can also exert compressive (normal) forces on a boundary or surface. *Fluid pressure* is defined as the normal force per unit area exerted by the fluid on a boundary.

Atmospheric Pressure

The *actual* atmospheric pressure is the weight per unit area of the air above a datum, and it varies with weather conditions. Since this pressure is usually measured with a barometer, it is commonly called *barometric pressure*.

Standard Atmospheric Pressure

By international agreement the standard atmospheric pressure is defined as 101.325 kN/m^2 (kPa). In USCS units this value is 14.696 lbf/in.^{2} (psi), or

29.92 inches of mercury at 32° F. For most practical purposes 14.70 psi may be used for atmospheric pressure.

Observed Pressures

Most pressure-sensing devices (the barometer is an exception) indicate the difference between the pressure measured and atmospheric pressure. As shown in Figure 1.4, pressures measured above atmospheric pressure are called *gage pressures*, while pressures measured lower than atmosphere are called *vacuum (negative gage) pressures*. The algebraic sum of the instrument reading and the actual atmospheric pressure is the true, or absolute, pressure, as shown in equation (1.10):

$$p = p_b + p_i \tag{1.10}$$

In equation (1.10), p is the absolute pressure, p_b is the atmospheric (barometric) pressure, and p_i is the instrument reading (positive for gage pressure, negative for vacuum). *All instrument readings must be converted to absolute pressure before they are used in calculations.*

Conventional American engineering practice is to use the psi unit for pressure. Gage pressures are indicated by psig and absolute pressures by psia. Vacuums are almost always reported in inches of mercury at 32°F. There is no equivalent of gage pressure in SI, so all pressures are absolute unless gage is specified.

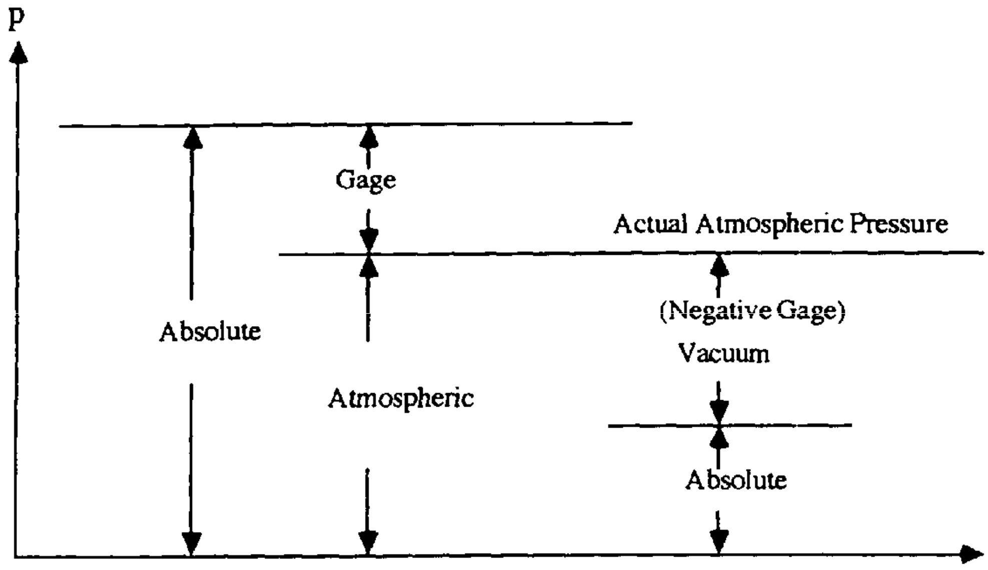

Figure 1.4 Pressure relations.

Mass, Force, and Weight

Mass is a quantity of matter. Its value is the same any place in the universe. *Force* and *mass* are related by Newton's second law of motion. *Weight* is the force exerted by a *mass* due to the acceleration of gravity.

Newton's second law of motion states that an unbalanced force acting on a body causes the body to accelerate in the direction of the force. The acceleration is directly proportional to the unbalanced force and inversely proportional to the mass of the body. This law may be expressed mathematically as equation (1.11):

$$F = \frac{m\alpha}{g_c} \tag{1.11}$$

In this equation, F is the unbalanced force, m is the mass of the body, α is the acceleration, and g_c is a *proportionality constant.*

The numerical value of g_c depends upon the units used in equation (1.10). For SI, the *newton* is defined as the force produced by the acceleration of the mass of 1 kg at a rate of 1 m/s^2. Substituting these values into equation (1.11) and solving for g_c yields the following:

$$g_c = \frac{m\alpha}{F} = 1\ \frac{\text{kg·m}}{\text{N·s}^2} \tag{1.12}$$

In USCS units the *pound-mass* (lbm) is defined by international agreement to be equal to 0.45359237 kg. The *pound-force* (lbf) is defined as the weight of 1 lbm when subjected to the standard acceleration of gravity (32.174 ft/s^2). Again, solving equation (1.11) for units of g_c yields the following:

$$g_c = \frac{1\ \text{lbm} \cdot 32.174\ \text{ft/s}^2}{1\ \text{lbf}} = 32.174\ \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.13}$$

Density

Definition:	mass per unit volume
Symbol:	ρ
Dimensions:	ML^{-3} or FT^2L^{-4}
Units:	USCS: lbm/ft^3; SI: kg/m^3

Density is the mass per unit volume of a substance and its numerical value is the same any place in the universe because it represents a quantity of matter.

Specific Weight

Definition: weight (force) per unit volume
Symbol: γ
Dimensions: FL^{-3} or $ML^{-2}T^{-2}$
Units: USCS: lbf/ft^3; SI: N/m^3

Specific weight is the weight or force exerted by mass of a substance per unit volume due to the local acceleration of gravity. Unlike density, the numerical value of specific weight varies with local gravity.

Using Newton's second law of motion [equation (1.11)], the specific weight is related to the density as follows:

$$\gamma = \frac{F}{V} = \frac{mg}{Vg_c} = \frac{\rho g}{g_c} \tag{1.14}$$

In equations of fluid statics and dynamics, it is often convenient to represent specific weight in terms of density.

Specific Volume

Definition: volume per unit mass
Symbol: ν
Dimensions: $L^{-3}M^{-1}$ or $F^{-1}L^{-4}T^{-2}$
Units: USCS: ft^3/lbm; SI: m^3/kg

Specific volume is defined as the volume of a substance divided by its mass. Like density, specific volume has the same numerical value any place in the universe. Since specific volume is the inverse of density, it follows that:

$$\nu = \frac{1}{\rho} \tag{1.15}$$

Substitution of equation (1.15) into equation (1.14) yields the following expression for specific volume in terms of specific weight:

$$\nu = \frac{g}{\gamma g_c} \tag{1.16}$$

Specific Gravity

Definition:	fluid density/reference fluid density
Symbol:	S
Dimensions:	dimensionless ratio
Units:	none
Reference fluids	solids and liquids, water; gases, air

Specific Gravity of Liquids

It is common practice to tabulate liquid densities in terms of specific gravity, the liquid density normalized by a reference density (usually water). Since the density of liquids varies with pressure (especially at high pressures) and temperature, the temperature and pressure of the liquid and water should be stated when defining the specific gravity. In actual practice (at low to moderate pressures), two temperatures are stated, for example, 60/60°F (15.56/15.56°C), where the upper temperature refers to the fluid and the lower to water. The density of water at 60°F (15.56°C) is 62.37 lbm/ft^3 (999.1 kg/m^3). If no temperatures are stated it should be assumed that reference is made to water at its maximum density. The maximum density of water at atmospheric pressure is at 39.16°F (3.98°C) and has a value of 62.43 lbm/ft^3 (1000.0 kg/m^3). The specific gravity is calculated using the expression shown in equation (1.17):

$$S_{T_f/T_w} = \frac{\rho_f}{\rho_w} \tag{1.17}$$

where ρ_f is the density of the fluid at temperature T_f and ρ_w is the density of water at temperature T_w.

Specific Gravity of Gases

For gases it is common practice to use the ratio of molecular weight (molar mass) of the gas to that of air (28.9644), thus eliminating the necessity of stating the pressures and temperatures for ideal gases.

Hydrometer Scale Conversions

In certain fields of industry, hydrometer scales are used that have arbitrary graduations. In the petroleum and chemical industries, the Baumé (°Be) and the American Petroleum Institute (°API) scales are used. Conversions are as follows:

Baumé Scale:

For liquids heavier than water:

$$S_{60/60^\circ F(15.56/15.56^\circ C)} = \frac{145}{145 - {}^\circ Be} \tag{1.18}$$

For liquids lighter than water:

$$S_{60/60^\circ F(15.56/15.56^\circ C)} = \frac{140}{130 + {}^\circ Be} \tag{1.19}$$

American Petroleum Institute (API) Scale:

$$S_{60/60^\circ F(15.56/15.56^\circ C)} = \frac{141.5}{131.5 + {}^\circ API} \tag{1.20}$$

The Baumé scale for liquids lighter than water is nearly the same as the American Petroleum Institute (API) scale. The use of the API scale is recommended by the American National Standards Institute (ANSI). Standardized hydrometers are available in various ranges, from -1°API to 101°API, for specific gravity ranges of 1.0843 to 0.6068 at 60/60°F (15.56/15.56°C).

Viscosity

Dynamic Viscosity

Definition:	shearing stress/rate of shearing strain
Symbol:	μ
Dimensions:	$FL^{-2}T$ or $ML^{-1}T^{1}$
Units:	USCS: lbf s/ft^2; SI: N · s/m^2 or Pa · s

The concept of viscosity was introduced in Section 1.2. *Viscosity* is the resistance of a fluid to motion, a measure of its internal friction. As discussed in Section 1.2, a fluid in a static state is by definition unable to resist even the slightest amount of shear stress. Application of shear force results in the continual and permanent distortion known as *flow*.

In Section 1.2, consideration of Figure 1.1 led to the development of the unit shear stress $\tau = F_s/A_s$, where F_s is the shear force and A_s is the shear area. Also developed from consideration of Figure 1.1 was the relationship between shear stress, rate of deformation (shearing strain, dU/dy), and a proportionality constant known as the *absolute viscosity* (μ). This relationship was mathematically expressed as equation (1.2), Newton's law of viscosity:

$$\tau = \mu \frac{dU}{dy} \tag{1.2}$$

Rearrangement of equation 1.2 leads to the following definition of viscosity:

$$\mu = \frac{\text{shearing stress}}{\text{rate of shearing strain}} = \frac{F_s/A_s}{dU/dy} = \frac{\tau\, dy}{dU} \tag{1.21}$$

In various publications, μ is known as (a) coefficient of viscosity, (b) absolute viscosity, and (c) dynamic viscosity (used in this book).

Kinematic Viscosity

Definition: dynamic viscosity/density
Symbol: ν
Dimensions: L^2T^{-1}
Units: USCS: ft^2/s; SI: m^2/s

The ratio of dynamic viscosity to density, defined as the *kinematic viscosity*, often arises in fluid mechanics equations. Because the dynamic viscosity is in force units and the density in mass units in both USCS and SI, it is necessary to introduce the proportionality constant (g_c) to relate dynamic and absolute viscosities. Therefore, kinematic viscosity is written as follows:

$$\nu = \frac{g_c\mu}{\rho} \tag{1.22}$$

In USCS units:

$$\nu = \frac{g_c\mu}{\rho} = \frac{(\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2)(\text{lbf}\cdot\text{s/ft}^2)}{(\text{lbm/ft}^3)} = \frac{\text{ft}^2}{\text{s}} \tag{1.23}$$

In SI Units:

$$\nu = \frac{g_c\mu}{\rho} = \frac{\text{kg}\cdot\text{m/N}\cdot\text{s/m}^2)}{(\text{kg/m}^3)} = \frac{\text{m}^2}{\text{s}} \tag{1.24}$$

Characteristics

In a flowing fluid, tangential (shear) stresses arise from two different molecular phenomena. The first is the cohesive (attractive) forces of the molecules, which resist motion. The second is the molecular activity, which causes resistance to flow due to molecular momentum transfer. These phenomena help explain the variation of viscosity in liquids and gases.

Liquid Viscosities

In liquids, cohesive forces are dominant. Since cohesive forces decrease with increasing temperatures, so do the liquid viscosities. An empirical equation that relates liquid viscosity to temperature is given in equation (1.25).

$$\mu = Ae^{B/T} \tag{1.25}$$

The constants A and B depend upon the individual fluid and can be found in many references.

At moderate pressures, liquid viscosities change only a small amount as pressure varies. However, at high pressures, cohesive forces and, therefore, viscosity increase with pressure. The change in liquid viscosity with pressure is complicated, and a simple empirical expression, such as equation (1.25), has not been developed.

Gas Viscosities

Cohesive forces are absent in ideal gases. Molecular activity and viscosity increase with temperature. The variation of gas viscosity with temperature is well represented by the empirical Sutherland equation, as follows:

$$\mu = \frac{bT^{3/2}}{S + T} \tag{1.26}$$

The following values of the constants b and S have been determined for air:

$$b = 1.458 \times 10^{-6} \text{kg/m·s·K}^{1/2}$$
$$S = 110.4\text{K}$$

Values of the Sutherland constants for other gases can be found in most standard handbooks.

At moderate pressures, gas viscosities change only a small amount as pressure varies. However, at high pressures, molecular activity and, therefore, viscosity increase with pressure. As with liquids, the change in gas viscosity with pressure is complicated, and a simple empirical expression, such as equation (1.26), has not been developed.

Other Units of Viscosity

The *poise*, a unit of dynamic viscosity named after French scientist Jean Louis Poiseuille, is defined as one dyne-second per square centimeter. In SI this is equal to 0.1 N·s/m^2 or 0.1 Pa·s. The viscosity of water at 20 C is 1 002 μPa·s or 1.002×10^{-2} poises. Because of the magnitude of the poise, the *centipoise* (1/00 poise) is often used. The viscosity of water at 20°C is thus approximately 1 centipoise. The conversion factor for USCS units is 2.089×10^{-5} lbf·s/ft^2/centipoise.

The *stoke*, a unit of kinematic viscosity named after English scientist George Gabriel Stokes, is defined as 1 square centimeter per second. In SI units, the stoke is equal to 1×10^{-4} m^2/s. In USCS units, the stoke is equal to 1.076×10^{-3} ft^2/s. Like the poise, the *centistoke* is often used because of the magnitude.

The standard viscometer for industrial work in the United States is the Saybolt universal viscometer. Essentially it consists of a metal tube and an orifice built to rigid specifications and calibrated with fluids of known viscosity. The time required for a gravity flow of 60 cm^3 is a measure of the kinematic viscosity of the fluid and is called SSU (Saybolt seconds universal). Approximate conversions of SSU to centistokes may be made using the following equations:

$$\text{centistokes} = 0.226(\text{SSU}) - \frac{195}{\text{SSU}}; \qquad 32 < \text{SSU} < 100 \tag{1.27}$$

$$\text{centistokes} = 0.220(\text{SSU}) - \frac{135}{\text{SSU}}; \qquad \text{SSU} > 100 \tag{1.28}$$

For very viscous oils a larger orifice is used in the Saybolt viscometer, and the time, in seconds, is called SSF (Saybolt seconds furol). The term *furol* is a contraction of fuel and road oils. Approximate conversions of SSF to centistokes may be made using the following equations:

$$\text{centistokes} = 2.24(\text{SSF}) - \frac{184}{\text{SSF}}; \qquad 25 < \text{SSF} < 40 \tag{1.29}$$

$$\text{centistokes} = 2.16(\text{SSF}) - \frac{60}{\text{SSF}}; \qquad \text{SSF} > 40 \tag{1.30}$$

Exact SSU and SSF conversions depend upon specific viscometer calibration data.

1.5 THERMODYNAMIC CONCEPTS

Thermodynamic Processes

The *state* of a substance is the condition of its existence, and it is determined by any two independent properties. Consider the *pv* diagram shown in Figure 1.5. In this case, state point 1 is determined by p_1 and v_1. If one or more properties are changed, the fluid is said to have undergone a *process*. For example, if the pressure in Figure 1.5 is changed from p_1 to p_2, the

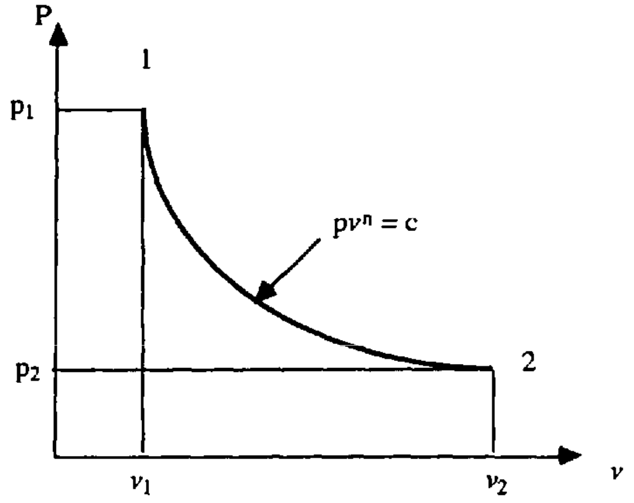

Figure 1.5 Generalized thermodynamic process diagram.

resulting specific volume is v_2. The manner in which this change takes place determines the *path* of the process. If the fluid can be made to return to its original state by retracing exactly the original path, the process is said to be *reversible*. A reversible process is frictionless and in reality does not occur in nature. However, a reversible process is a useful concept, since it serves as an ideal case.

Polytropic Process

All ideal gas processes are *polytropic* processes, and the processes discussed next (isentropic, isothermal, isobaric, and isometric) are all special cases of the polytropic process. For an ideal gas the relation between pressure and specific volume is given by equation (1.31):

$$pv^n = c \tag{1.31}$$

The value n can be expressed if equation (1.31) is written in logarithmic form ($\log_e p + n \log_e v = \log_e c$) and differentiated, yielding the following:

$$\frac{dp}{p} + \frac{n\,dv}{v} = 0 \qquad \text{or} \qquad n = -\frac{v\,dp}{p\,dv} \tag{1.32}$$

Equation (1.32) indicates that n is the slope of the p–v curve and establishes the pressure-specific volume relationship for the process. The value of n for a polytropic process ranges from $+\infty$ to $-\infty$, depending upon the nature of the process.

Isentropic Process

If a process takes place without heat transfer and is reversible (frictionless), it follows a path of constant entropy (s), and, hence, is called *isentropic*. This same process is also called a *reversible adiabatic* process. The path of this process is given by:

$$(s = c) \qquad pv^n = pv^k = c \tag{1.33}$$

where k is the isentropic exponent ($k = c_p/c_v$), c_p is the specific heat at constant pressure, and c_v is the specific heat at constant volume.

Isothermal Process

If a process takes place at constant temperature it is called an *isothermal* process. From the equation of state for an ideal gas, $pv = RT$ [equation (1.37)]. Differentiating equation (1.37) for $T = c$, yields $d(pv) = 0$, or $v\,dp = -p\,dv$. Substituting this relation into equation (1.32) gives the following for n:

$$(T = c) \qquad n = -\frac{v\,dp}{p\,dv} = -\frac{v\,dp}{-v\,dp} = 1 \tag{1.34}$$

Isobaric Process

If a process takes place at constant pressure it is called an *isobaric* process. For a constant-pressure process, $dp = 0$. Substituting into equation (1.32) yields the following:

$$(p = c) \qquad n = -\frac{v\,dp}{p\,dv} = -\frac{v(0)}{p\,dp} = 0 \tag{1.35}$$

Isometric Process

If a process takes place at constant volume (or constant specific volume) it is called an *isometric* process. The value of n for this process is as follows:

$$(v = c) \qquad n = -\frac{v\,dp}{p\,dv} = -\frac{v\,dp}{p\,(0)} = \infty \tag{1.36}$$

Equations of State

An equation of state is one that defines the relationships of pressure-temperature and volume. The ideal gas law is the equation of state that is valid for ideal gases. Real gas behavior can be described by modifying the ideal gas law.

Equation of State for an Ideal Gas

An ideal gas is one that obeys the equation of state (1.37) and whose internal energy is a function of temperature. The equation of state for an ideal gas is shown as follows:

$$pv = RT \tag{1.37}$$

where

p = pressure, lbf/ft^2(Pa)

v = specific volume, ft^3/lbm (m^3/kg)

R = gas constant, ft·lbf/lbm · °R (J/kg K)

T = temperature, °R (K)

The gas constant R is computed using the molecular weight (molar mass) from the following:

$$R = \frac{R_u}{M} \tag{1.38}$$

where

R_u = universal gas constant, 1545 ft lbf/(lb · mol)(°R) = 8314 J/(kg·mol)(K)

M = molecular weight (molar mass), lbm/lb · mol (kg/kg·mol)

For computation of density, substitution of equation (1.15) for v in equation (1.37) yields the following after some rearrangement:

$$\rho = \frac{p}{RT} \tag{1.39}$$

Other p–v–T relations for ideal gases may be obtained by combining the equation of state $pv = RT$ (1.37) with the polytropic relation $pv^n = c$ (1.31). These equations are as follows:

For pressure:

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n = \left(\frac{T_2}{T_1}\right)^{n/(n-1)} \tag{1.40}$$

For specific volume:

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/n} = \left(\frac{T_1}{T_2}\right)^{1/(n-1)} \tag{1.41}$$

For temperature:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \tag{1.42}$$

Equation of State for a Real Gas

Generally, the ideal gas law is an adequate model of gas behavior for pressures below the critical state pressure and temperatures above the critical state temperature. The ideal gas law may be modified for a real gas as follows:

$$pv = ZRT \qquad \text{or} \qquad Z = \frac{pv}{RT} \tag{1.43}$$

In this equation, Z is the *compressibility factor*. Note that Z is unity when the substance is in the ideal gas state. Thus the deviation of the compressibility factor from unity is a measure of nonidealness of the state of the substance. The value of Z can be determined using the critical properties of a substance and the compressibility charts found in any thermodynamics text.

1.6 OTHER FLUID PROPERTIES

Bulk Modulus of Elasticity

Definition:	stress/volumetric strain
Symbol:	E
Dimensions:	FL^{-2} or $ML^{-1}T^{-2}$
Units:	USCS: lbf/in.^{2}, lbf/ft^2; SI: kN/m^2 or Pa

Derivation of Basic Equations

Consider the piston and cylinder of Figure 1.6. A fluid originally under a pressure p had a volume V. When an additional pressure dp is imposed, the result is a decrease of volume, dV. The normal stress acting on the fluid is the imposed pressure, dp. The definition of strain from fundamental engineering mechanics is the ratio of the deformation length to the original length, which in terms of volume can be expressed as dV/V. Therefore, the following equation can be developed from the basic definition of bulk modulus of elasticity:

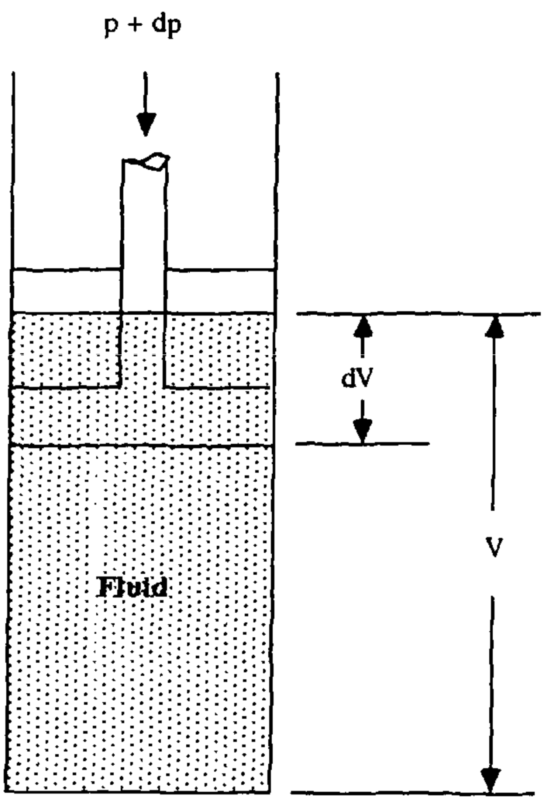

Figure 1.6 Notation for bulk modulus of elasticity.

$$E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{-dV/V} = -V\left(\frac{dp}{dV}\right) \tag{1.44}$$

The negative sign is used to represent the fact that the volume is decreased by dV. Substitution of the definition of specific volume ($v = V/m$) in equation (1.44) yields the following:

$$E = -vm\left(\frac{dp}{m\,dv}\right) = -v\left(\frac{dp}{dv}\right) \tag{1.45}$$

Equation (1.45) cannot be evaluated unless the process is known so that the pressure-specific volume relationship can be established. Equation (1.45) is therefore more correctly written as equation (1.46):

$$E_n = -v\left(\frac{\partial p}{\partial v}\right)_n \tag{1.46}$$

E_n in this equation is the bulk modulus of elasticity for process n, and $(\partial p/\partial v)_n$ indicates the differential pressure-specific volume ratio for that process (the partial derivative of p with respect to v with n constant). Although any number of processes are possible, conventional practice is to use only the *isothermal bulk modulus* (E_T) and the *isentropic bulk modulus* (E_s).

Ideal Gases

If equation (1.32) is written as $np = -v(\partial p/\partial v)_n$, and substituted into equation (1.46), the following expression is valid for the bulk modulus of ideal gases:

$$E_n = -v\left(\frac{\partial p}{\partial v}\right)_n = np \tag{1.47}$$

For an isothermal process, $n = 1$ and equation (1.47) reduces to the following:

$$E_T = np = p \tag{1.48}$$

For an isentropic process, $n = k$ and equation (1.47) reduces to the following:

$$E_s = np = kp \tag{1.49}$$

The relationship between E_s and E_T is established by dividing equation (1.49) by equation (1.48), resulting in the following:

$$\frac{E_s}{E_T} = \frac{kp}{p} = k = \frac{c_p}{c_v} \tag{1.50}$$

It may be demonstrated that the relationship expressed by equation (1.50) can be applied to all fluids, not just ideal gases.

Liquids

At constant pressure, the bulk modulus of most liquids decreases with temperature. Water is one exception; the bulk modulus of water increases to a maximum value at 120°F (49°C) and decreases in value above that temperature at atmospheric pressure. At constant temperature, the bulk modulus increases with pressure for all liquids. No simple relationship similar to $pv^n = c$ for ideal gases exists for liquids. For liquids, equation (1.46) may be approximated over small intervals as follows:

$$E_n = -v\left(\frac{\partial p}{\partial v}\right)_n = -v_1\left(\frac{\Delta p}{\Delta v}\right)_n = v_1\left(\frac{p_2 - p_1}{v_1 - v_2}\right)_n \tag{1.51}$$

Some handbooks and other sources use equation (1.51) as a definition of liquid bulk modulus.

Acoustic Velocity

Definition: speed of a sound wave in a fluid
Symbol: c
Dimensions: LT^{-1}
Units: USCS: ft/s; SI: m/s

Derivation of Basic Equations

Consider an elastic fluid in a rigid pipe fitted with a piston, as shown in Figure 1.7. The pipe has a uniform cross-sectional area, A. As the result of the application of a differential force, dF, the piston is suddenly advanced with a velocity v for a time dt. The fluid pressure p is increased by the amount of dp, which travels as a wave front with a velocity of c. During the application time dt, the piston moves a distance of $v\,dt$ and the wave front advances a distance of $c\,dt$. The result of this piston movement decreases the volume $c\,dt\,A$ by the amount of the volume $v\,dt\,A$.

From equation (1.44), the bulk modulus of the fluid is the following:

$$E = -\frac{dp}{-dV/V} = -\frac{dp}{(v\,dt\,A)/c\,dt\,A)} = -\frac{c\,dp}{v} \tag{1.52}$$

or

$$c = -\frac{vE}{dp} \tag{1.53}$$

From a force balance, dF can be written as $(p + dp)A - pA = dp\,A$. The mass of fluid accelerated in time dt is $\rho c\,dt\,A$, so the mass flow rate is

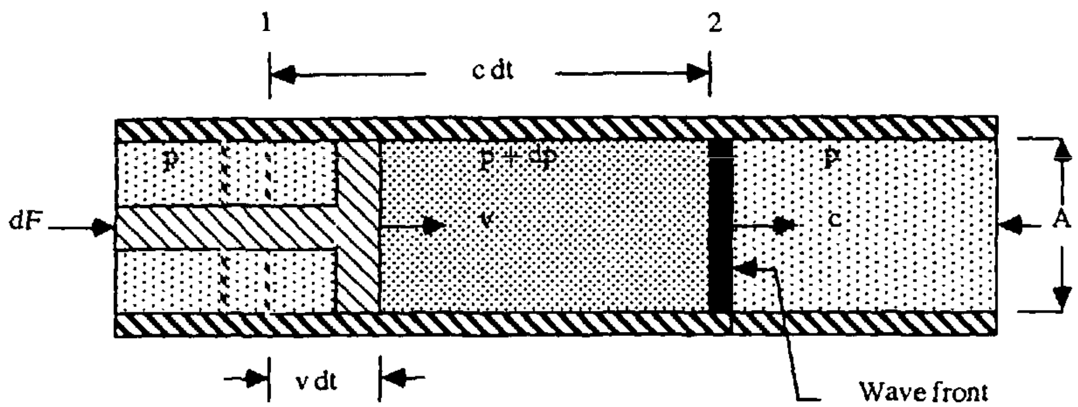

Figure 1.7 Notation for acoustic velocity.

$\dot{m} = m/dt = (\rho c\ dt\ A)/dt = \rho c A$. Newton's second law of motion (equation (1.11) can be rewritten as follows:

$$F = \frac{m\alpha}{g_c} = \frac{\dot{m}(v_2 - v_1)}{g_c} \tag{1.11}$$

For the example shown in Figure 1.7, the velocity change is from v to 0. Therefore, the imposed force can be expressed as follows:

$$dF = dpA = \frac{\rho c A(0 - v)}{g_c} \tag{1.54}$$

Solving for c in equation (1.54) yields the following:

$$c = -\frac{dp\ g_c}{\rho v} \tag{1.55}$$

A simplified expression for c can be obtained by multiplying equations (1.53) and (1.55) as follows:

$$c^2 = \left(-\frac{dp\ g_c}{\rho\ v}\right)\left(-\frac{vE}{dp}\right) = \frac{Eg_c}{\rho} \tag{1.56}$$

or

$$c = \sqrt{\frac{Eg_c}{\rho}} \tag{1.57}$$

The numerical value of E depends upon the process, as discussed earlier in Section 1.6. It is assumed that a small pressure (sound) wave will travel through the fluid without either heat transfer or friction. With these assumptions, the process becomes isentropic and equation (1.57) is written as follows:

$$c = \sqrt{\frac{E_s g_c}{\rho}} \tag{1.58}$$

Equation (1.58) may be used for any fluid, provided E_s and ρ are known.

Ideal Gases

From equation (1.49) $Es = kp$, and from equation (1.39) $= p/RT$. Substituting these expressions into equation (1.58) yields the following:

$$c = \sqrt{\frac{E_s g_c}{\rho}} = \sqrt{\frac{(kp)g_c}{p/RT}} = \sqrt{k}RTg_c \tag{1.59}$$

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2
Fluid Statics

2.1 INTRODUCTION

This chapter is concerned with establishing the basic relations of fluid statics. Included are the basic equation of fluid statics, pressure height relations for incompressible fluids and ideal gases, pressure-sensing devices, and buoyancy and flotation.

This chapter may be used as a text for tutorial or for refresher purposes. Each concept is explained and derived mathematically as needed. The basic purpose of this introductory chapter is to provide a foundation for the equations developed in the remainder of the book. If the reader is already familiar with this subject matter, this chapter can be skipped.

2.2 FLUID STATICS

Fluid statics is the branch of fluid mechanics that deals with fluids that are at rest with respect to the surfaces that bound them. The entire fluid mass may be in motion, but all fluid particles are at rest relative to one another.

There are two kinds of forces to consider: (1) *surface forces*, forces due to direct contact with other fluid particles or solid walls (forces due to pressure and tangential or shear stress), and (2) *body forces*, forces acting on the fluid particles at a distance (e.g., gravity, magnetic field). Since there is no motion of a fluid layer relative to another fluid layer, the shear stress everywhere in the fluid must be zero and the only surface force that can act on a fluid particle is normal pressure force. Because the entire fluid mass may be accelerated, body forces other than gravity may act in any direction on a fluid particle.

2.3 BASIC EQUATION OF FLUID STATICS

Body Forces

The infinitesimal fluid cube shown in Figure 2.1 has a mass of $\rho\, dx\, dy\, dz$. This cube is a particle in a large container of fluid where all the particles are at rest with respect to one another. The entire fluid mass is subject to body force accelerations of α_x, α_y, and α_z, opposite the directions of x, y, and z, respectively. In addition, the acceleration due to gravity, g, acts opposite to the direction of z. For clarity, only the z-direction forces are shown in Figure 2.1, but forces also act in the x- and y-directions. From equation (1.11), $F = m\alpha/g_c$. The body forces are: $F_{bx} = (\rho\, dx\, dy\, dz)\alpha_x/g_c$, $F_{by} = (\rho\, dx\, dy\, dz)\alpha_y/g_c$, $F_{bz} = (\rho\, dx\, dy\, dz)\alpha_z/g_c$. The gravity force is $F_g = (\rho\, dx\, dy\, dz)g/g_c$.

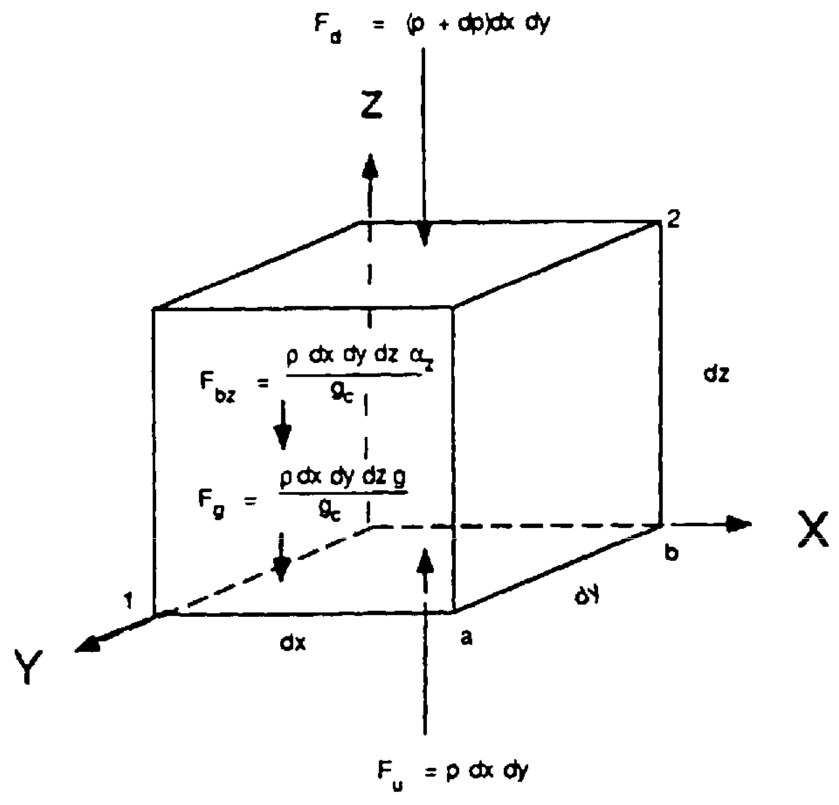


Figure 2.1 Notation for basic equation of fluid statics.

Vertical Forces

By the definition of pressure, $F = pA$, the upward pressure force is $F_u = p\,dx\,dy$ and the downward pressure force is $F_d = (p + dp)dx\,dy$. Considering the cube of Figure 2.1 to be a free body and only vertical components acting, the following force balance can be written:

$$\sum F_z = F_u - F_d - F_{bz} - F_g = 0$$

or

$$p\,dx\,dy - (p + dp)dx\,dy - \frac{(\rho\,dx\,dy\,dz)\alpha_z)}{g_c} - \frac{\rho\,dx\,dy\,dz)g}{g_c} = 0$$

which reduces to the following:

$$dp = \frac{\rho(\alpha_z + g)dz}{g_c} \qquad (x, y \text{ constant}) \tag{2.1}$$

Combined Forces

In a like manner, it may be shown that with only y-direction forces acting, dp equals the following:

$$dp = -\frac{\rho\alpha_y\,dy}{g_c} \qquad (x, z \text{ constant}) \tag{2.2}$$

With only x-direction forces acting, dp equals the following:

$$dp = -\frac{\rho\alpha_x\,dx}{g_c} \qquad (y, z \text{ constant}) \tag{2.3}$$

Forces may be combined by considering the pressure difference between points 2 and 1 of Figure 2.1. In path $1 \to a$, x is varied and y and z are held constant, so equation (2.3) applies to the difference between a and 1. In a like manner, equation (2.2) may be used for path $a \to b$ and equation (2.1) for path $b \to 2$. The total difference is the sum of each component, or the following:

$$dp = -\frac{\rho\alpha_x}{g_c} - \frac{\rho\alpha_y\,dy}{g_c} - \frac{\rho(\alpha_z + g)dz}{g_c}$$

or

$$dp = -\frac{\rho[\alpha_x\,dx + \alpha_y\,dy + (\alpha_z + g)dz]}{g_c} \tag{2.4}$$

Equation (2.4) is the basic equation of fluid statics.

2.4 PRESSURE–HEIGHT RELATIONS FOR INCOMPRESSIBLE FLUIDS

For a fluid at rest and subject only to gravitational force, α_x, α_y, and α_z are zero, so equation (2.4) reduces to the following:

$$dp = -\frac{\rho g\, dz}{g_c} \tag{2.5}$$

Integrating equation (2.5) for an incompressible fluid in a field of constant gravity and substituting $\gamma = \rho g/g_c$ from equation (1.14), yields the following:

$$\int_1^2 dp = -\frac{\rho g}{g_c}\int_1^2 dz = p_2 - p_1 = -\frac{\rho g(z_2 - z_1)}{g_c} = -\gamma(z_2 - z_1) = -\gamma h$$

which reduces to

$$(p_1 - p_2) = \Delta p = \gamma h \tag{2.6}$$

where $h = z_2 - z_1$, or the height of a liquid column. The relationships of equation (2.6) are shown in Figure 2.2.

2.5 PRESSURE-SENSING DEVICES

Bourdon tube gages are used for measuring pressure differences. The essential features are shown in Figure 2.3. The Bourdon tube is made of metal and has an elliptical cross section. The tube is fixed at B and free to move at C. As the difference between the internal and the external pressures increases, the elliptical cross section tends to become circular, and the free end of the tube (point C) moves outward, moving the pointer D through a suitable linkage. It should be noted that when the outside pressure is atmospheric, the Bourdon tube indicates *gage* pressure, and when the internal pressure is less than the atmospheric, a negative gage pressure, or *vacuum*, is sensed. Refer to Figure 1.4 for these relationships.

Credit for the discovery of the *barometer* is given to Evangelista Torricelli (1608–1647), an Italian mathematician and scientist who related barometric height to the weight of the atmosphere. Figure 2.4 shows the essential features of an elementary barometer. In its most primitive form, the barometer is made by filling a long glass tube with mercury and inverting it in a pan of mercury. If the height of the mercury column is less than that of the tube, mercury vapor will form at the top of the tube. Application of equation (2.6) yields the following relationship:

$$p_b = \gamma h + p_v \tag{2.7}$$

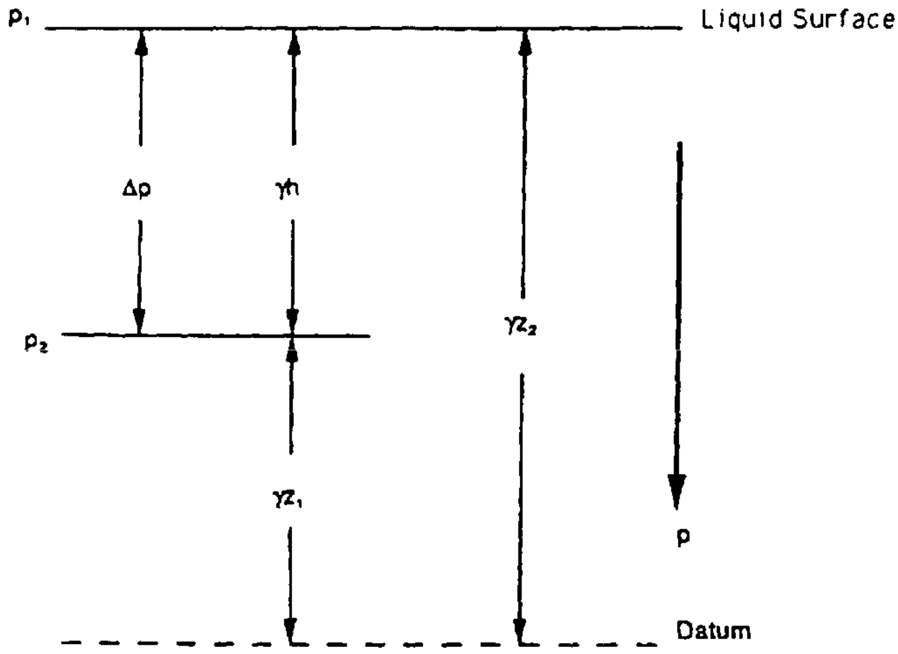


Figure 2.2 Pressure equivalence.

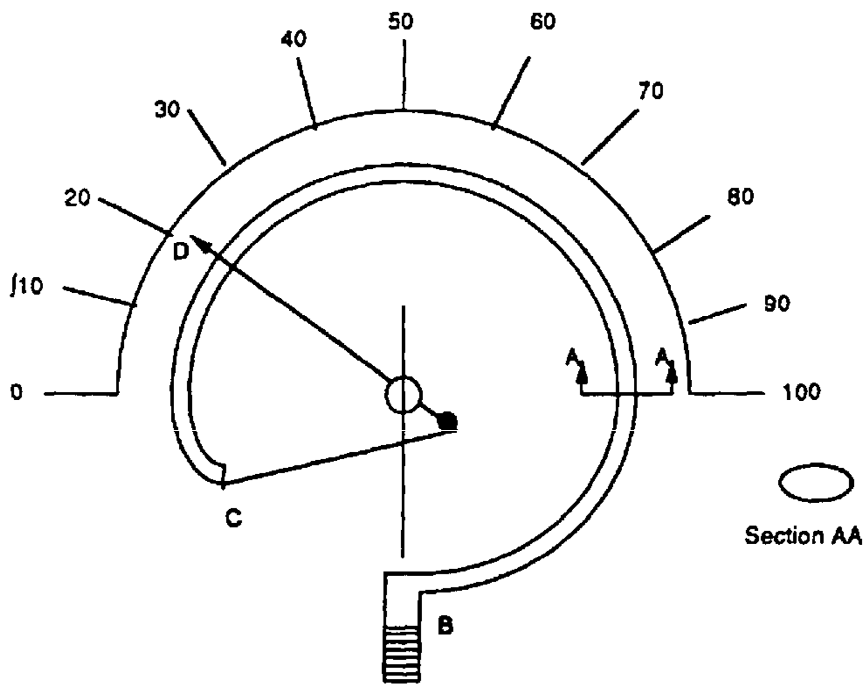


Figure 2.3 Bourdon tube gage.

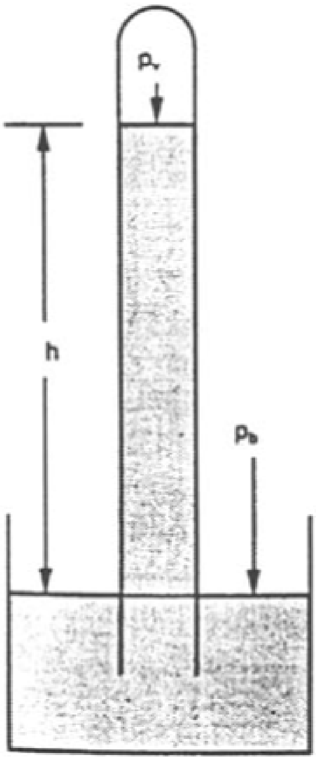


Figure 2.4 Barometer.

The vapor pressure of mercury is very small; from Table D.1 (see Appendix D), the vapor pressure of mercury at 32 F (0 C) is 3.957×10^{-6} psia (2.728×10^{-2} Pa). For all practical purposes, $p_b = \gamma h$ (for a mercury-filled barometer). However, when a barometer-type arrangement is filled with a fluid other than mercury, the vapor pressure must be taken into account in equation (2.7).

Manometers are one of the oldest means of measuring pressure. They were used as early as 1662 by Robert Boyle to make precise measurements of steady fluid pressures. Because of its inherent simplicity, the manometer serves as a pressure standard in the range of 1–10 in. of water to 100 psig (2.5 Pa to 790 kPa).

The arrangement of the *U-tube manometer* is shown in Figure 2.5. The manometer is acted upon by a pressure p_1 on the left and p_2 on the right. If $p_1 > p_2$, the fluid in the left leg of the manometer will be displaced to the right by a volume of $z_1 A_1$, resulting in an increase of volume of $z_2 A_2$ in the right leg. Application of equation (2.6) for equilibrium in the U-tube manometer results in the following:

$$p_1 + \gamma_f(z_1 + z_2) = p_2 + \gamma_m(z_1 + z_2)$$

which reduces to

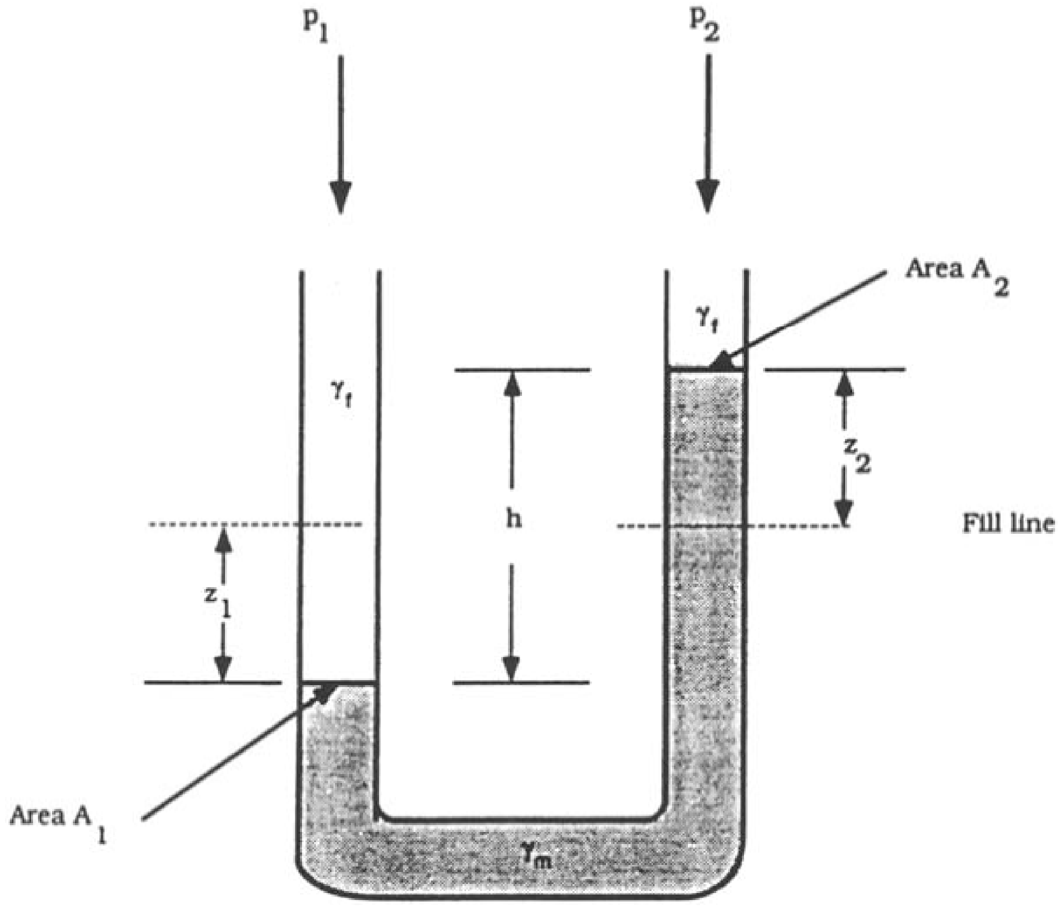


Figure 2.5 U-tube manometer.

$$(p_1 - p_2) = (\gamma_m = \gamma_f)(z_1 + z_2) = (\gamma_m - \gamma_f)h \tag{2.8}$$

In these equations, γ_m is the specific weight of the manometer fluid and γ_f is the specific weight of the fluid whose differential is being sensed.

One of the disadvantages of the U-tube manometer is that unless $A_1 = A_2$ exactly, both legs must be observed simultaneously. For this reason, the *well*, or *cistern*, type shown in Figure 2.6 is sometimes used. In the well or cistern type of manometer, the areas A_1 and A_2 are controlled to give a maximum deflection for z_2 and a minimum for z_1. From consideration of volumetric displacement of the liquid from one leg to the other, the following can be written:

$$z_1 A_1 = z_2 A_2$$

or

$$z_1 = \frac{z_2 A_2}{A_1}$$

Substitution in equation (2.8) yields the following:

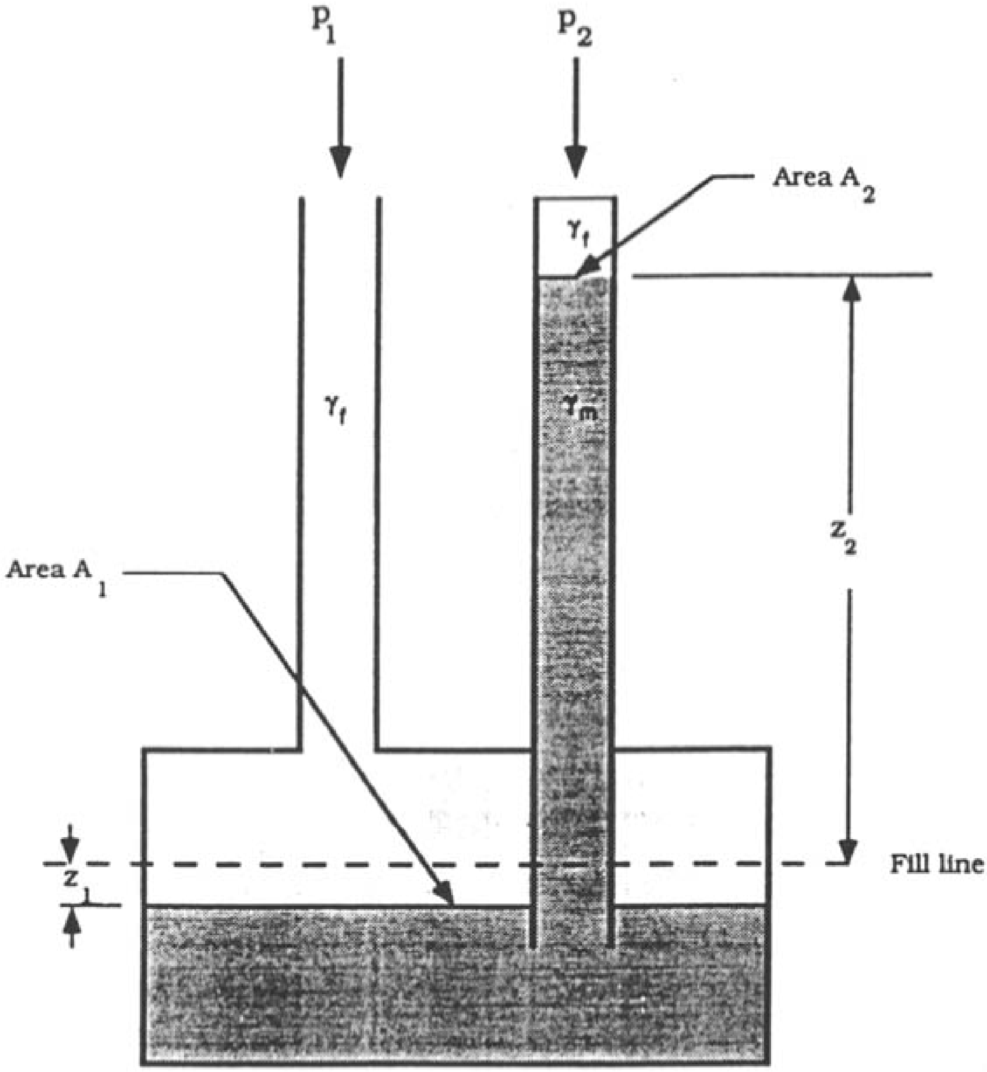


Figure 2.6 Well or cistern type of manometer.

$$p_1 - p_2 = (\gamma_m - \gamma_f)\left(\frac{z_2 A_2}{A_1} + z_2\right) = (\gamma_m - \gamma_f)\left(1 + \frac{A_2}{A_1}\right)z_2 \tag{2.9}$$

Note that as $A_1 \to \infty$, $A_2/A_1 \to 0$. By making the area A_1 very large, the designer of a well type of manometer can create a condition where $z_2 \to h$. The difference in area ratios is usually accounted for with scale graduations.

Commercial manufacturers of the well type of manometer correct for the area ratios so that $p_1 - p_2 = (\gamma_m - \gamma_f)S_u$, where S_u is the scale reading, which is equal to $z_2(1 + A_2/A_1)$. For this reason, scales should not be interchanged between U-tube and well type or among well types without consulting the manufacturer.

The *inclined manometer*, as shown in Figure 2.7, is a special form of the well type. It is designed to enhance the readability of small pressure differ-

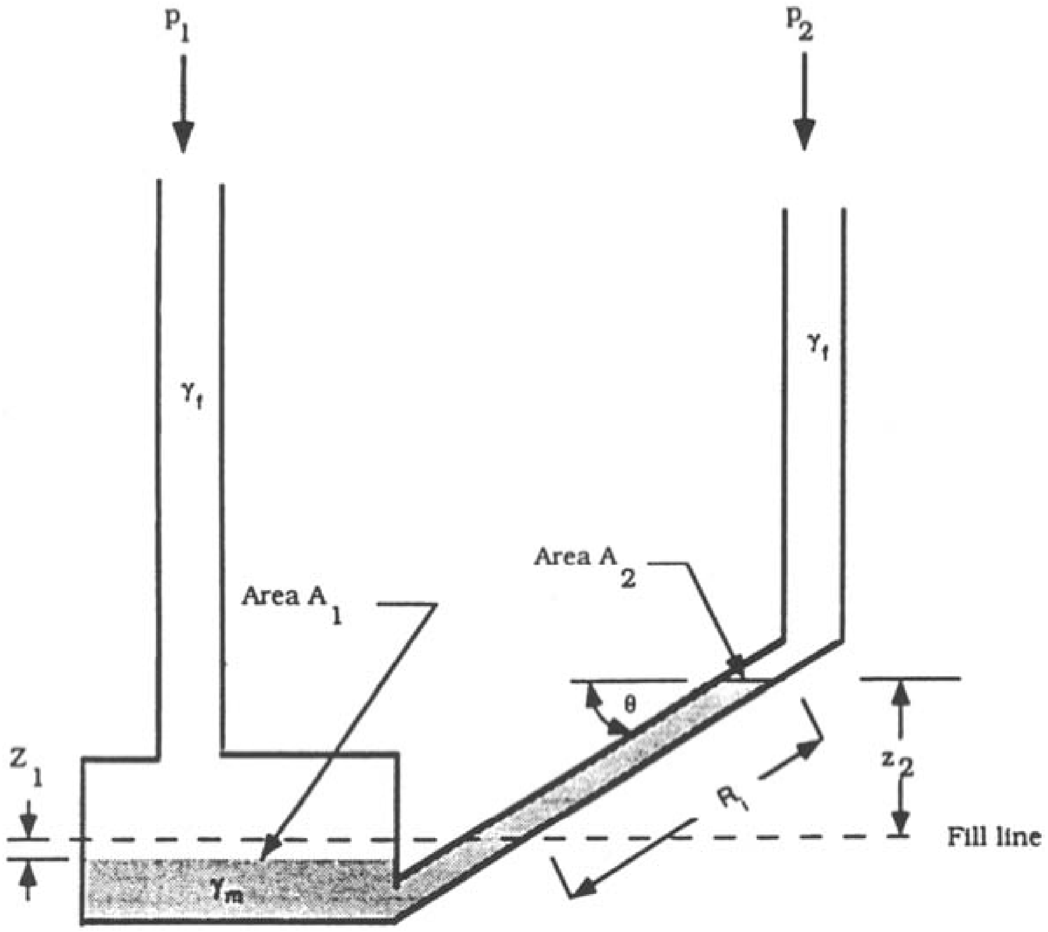


Figure 2.7 Inclined manometer.

entials. From consideration of the geometry of this device for displacement, the following can be written:

$$z_1 A_1 = R_t A_2$$

or

$$z_1 = \frac{R_t A_2}{A_1}$$

and

$$z_2 = R_t \sin\theta$$

Substitution in equation (2.8) yields the following:

$$p_1 - p_2 = (\gamma_m - \gamma_f)\left(\frac{A_2}{A_1} + \sin\ \theta\right)R_t \tag{2.10}$$

In this equation, R_i is the distance along the inclined tube. Commercial inclined manometers also have special scales so that the following can be used to determine pressure differential:

$$p_1 - p_2 = (\gamma_m - \gamma_f)S_i$$

Here, S_i is the inclined manometer scale, equal to $(A_2/A_1 + \sin\theta)R_i$.

In actual practice, inclined manometers are used for measurement of small air pressures. Their scales are usually graduated to read in inches of water, but many other fluids can be used. Care must be taken to "level" these instruments and to ensure that the correct liquid is used as specified by the manufacturer. Scales should never be interchanged.

Application

The equations just derived are simple, but actual installations may require more complex ones. Since there is almost an infinite number of combinations and arrangements that can be used, equations for each actual case must sometimes be derived individually.

2.6 PRESSURE–HEIGHT RELATIONS FOR IDEAL GASES

The equation for a static fluid in a gravitational field may be written as equation (2.5):

$$dp = -\frac{\rho g\, dz}{g_c} \tag{2.5}$$

To integrate the left-hand term of this equation, the functional relationship between pressure and density must be established for a compressible fluid. The right-hand term requires that the relationship between the acceleration due to gravity and altitude be established.

To establish these relationships, first note the following from equation (1.15):

$$\rho = \frac{1}{\nu} \tag{1.15}$$

Specific volume can be expressed in the form shown in equation (1.41):

$$\nu = \nu_1 \left(\frac{p_1}{p}\right)^{1/n} \tag{1.41}$$

Specific volume is also a function of temperature and pressure, from the equation of state of an ideal gas:

$$v_1 = \frac{RT_1}{p_1} \tag{1.39}$$

Substituting this in the left-hand side of equation (2.5) yields the following:

$$-\frac{dp}{\rho} = -v\,dp = -v_1 p_1^{1/n}\frac{dp}{p^{1/n}} = -RT_1 p_1^{(1/n)-1}\frac{dp}{p^{1/n}} = \frac{g}{g_c}\,dz$$

or

$$-RT_1 p_1^{(1/n)-1}\frac{dp}{p^{1/n}} = \frac{g}{g_c}\,dz \tag{2.11}$$

Gravity at elevations other than sea level can be estimated as follows:

$$g = \frac{g_0 r_e^2}{(r_e + z)^2} = \frac{g_0}{(1 + z/r_e)^2}$$

In the above equation, g_0 is the gravity at sea level, z is the elevation above sea level, and r_e is the mean radius of the Earth = 20.86×10^6 ft (6,357 km). Substituting this expression in equation (2.11) yields the following:

$$-RT_1 p_1^{(1/n)-1}\frac{dp}{p^{1/n}} = \frac{g_0}{g_c}\frac{dz}{(1 + z/r_e)^2} \tag{2.12}$$

Integration of the right-hand term of equation (2.12) leads to the following:

$$-RT_1 p_1^{(1/n)-1}\int_{p_1}^{p_1}\frac{dp}{p^{1/n}} - \frac{g_0}{g_c}\int_{z_1}^{z_2}\frac{dz}{(1 + z/r_e)^2} = \frac{g_0(z_2 - z_1)}{g_c(1 + z_2 r_e)(1 + z_1/r_e)} \tag{2.13}$$

It is apparent from equation (2.13) that mathematically there are only two values of $1/n$ that need be considered, when $n = 1$ and when $n \neq 1$. Since the value of n for an isothermal process of an ideal gas = 1 (Section 1.5), there are two equations to consider, one for isothermal processes and another for nonisothermal processes.

Isothermal Processes

Integrating the left-hand term of equation (2.13) for $n = 1$ yields the following:

$$-RT\,\log_e\left(\frac{p_1}{p_2}\right) = \frac{g_0(z_2 - z_1)}{g_c(1 + z_2/r_e)(1 + z_1/r_e)} \qquad (T = c) \tag{2.14}$$

Nonisothermal Processes

For all other processes, the left-hand term of equation (2.13) integrates as follows:

$$\left(\frac{nRT_1}{m-1}\right)\left[1-\left(\frac{p_2}{p_1}\right)^{(n-1)/n}\right]=\frac{g_0(z_2-z_1)}{gc(1+z_2/r_e)(1+z_1/r_e)} \quad (T \neq c) \qquad (2.15)$$

Temperature relations may be established from equation (1.42).

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{(n-1)/n} \qquad (1.42)$$

Substituting equation (1.42) in equation (2.15) yields the following:

$$\left(\frac{nRT_1}{n-1}\right)\left[1-\left(\frac{T_2}{T_1}\right)\right]=\frac{g_0(z_2-z_1)}{g_c(1+z_2/r_e)(1+z_1/r_e)}$$

Solving for temperature gradient leads to equation (2.16):

$$\frac{\Delta T}{\Delta z}=\frac{T_2-T_1}{z_2-z_1}=\frac{g_0(z_2-z_1)}{g_c nR(1+z_2/r_e)(1+z_1/r_e)} \qquad (2.16)$$

2.7 ATMOSPHERE

The atmosphere is a gaseous envelope that surrounds the Earth, extending from sea level to an altitude of several hundred miles. The altitude for near space has been set arbitrarily at 50 miles (80 km).

The Earth's atmosphere is divided into five levels based on temperature variation. The *troposphere* extends from sea level to 54,000 ft (16.5 km) at the equator, decreasing to 28,000 ft (8.5 km) at the poles; it is composed of approximately 79% nitrogen and 21% oxygen. With increasing altitude from sea level, the temperature decreases from 59°F (15°C) to −69.7°F (−56.5°C). Above the troposphere is the *stratosphere*, which extends to approximately 65,000 ft (19.8 km) and exists at a relatively constant temperature of −69.7°F (−59.5°C). The *mesosphere* extends from nearly 65,000 ft (10.8 km) to 300,000 ft (91.4 km), its temperature increases from −69.7°F (−56.5°C) to +28.67°F (1.85°C) and then decreases to −134°F (−92°C). The mesosphere is characterized by an ozone layer that absorbs ultraviolet radiation from the sun. Above the mesosphere is the *thermosphere*, also called the *ionosphere*, which extends from approximately 300,000 ft (91.4 km) to 1,000,000 ft (305 km). The temperature in this layer increases from −134°F (−92°C) to nearly 2,200°F (1,200°C). The composition is primarily ionized atoms of the lighter gases. The last level is the *exosphere*, which extends to the space environment.

U.S. Standard Atmosphere

Because of wide variations in the atmosphere, a standard atmosphere is used for design purposes. The United States standard atmosphere, formulated in 1976, extends from sea level to 3,280,840 ft (1,000 km). For altitudes above 282,152 ft (86 km), however, the hydrostatic equilibrium of the atmosphere gradually breaks down due to diffusion and vertical transport of the individual gas species. For this reason the pressure-height relations given in this section are valid only for altitudes below 282,152 ft (86 km).

The standard assumes that gravity is constant at all sea level locations, or $g_0 = 32.1740$ ft/s (9.80665 m/s). Using relationships previously established for the variation in temperature, gravity, and pressure with height, the U.S. standard atmosphere can be calculated. The results of such a calculation are shown in Tables 2.1 and 2.2.

2.8 BUOYANCY AND FLOTATION

Principles

The elementary principles of buoyancy and flotation were established by Archimedes (287–212 B.C.). These principles are usually stated as follows: (1) a body immersed in a fluid is buoyed up for a force equal to the weight of fluid displaced by the body, and (2) a floating body displaces its own weight of the fluid in which it floats.

Buoyant Force

Consider the body $ABCD$ shown in Figure 2.8. Dashed lines AE and BF are vertical projections. The force F_{zd} exerted by the fluid vertically on the body is the weight of the fluid above ABC. This weight is expressed as follows:

$$F_{zd} = \gamma V_{EACBF} \tag{2.17}$$

In a like manner, the upward vertical force is the weight above ABD, or the following:

$$F_{zu} = \gamma V_{EADBF} \tag{2.18}$$

The net upward force is the buoyant force F_B, defined as follows:

$$F_B = F_{zu} - F_{zd} = \gamma V_{EADBF} - \gamma V_{EACBF} = \gamma V_{ABCD} \tag{2.19}$$

Table 2.1. U.S. Standard Atmosphere – SI Units

Altitude (m)	Temperature (K)	Pressure (Pa)	Density (kg m^{-3})	Gravity (m s^{-2})	Sonic velocity (m·sec^{-1})	Dynamic viscosity (Pa·sec × 10^6)
0	288.15	1.013E + 05	1.225E + 00	9.807	340.29	17.89
1,000	281.66	8.989E + 04	1.112E + 00	9.804	336.44	17.58
2,000	275.17	7.953E + 04	1.007E + 00	9.800	332.54	17.26
3,000	268.68	7.015E + 04	9.096E − 01	9.797	328.60	16.94
4,000	262.19	6.170E + 04	8.197E − 01	9.794	324.61	16.61
5,000	255.17	5.408E + 04	7.368E − 01	9.791	320.56	16.28
6,000	249.22	4.725E + 04	6.605E − 01	9.788	316.47	15.95
7,000	242.73	4.113E + 04	5.903E − 01	9.785	312.32	15.61
8,000	236.24	3.567E + 04	5.260E − 01	9.782	308.12	15.27
9,000	229.75	3.081E + 04	4.672E − 01	9.779	303.86	14.93
10,000	223.26	2.651E + 04	4.136E − 01	9.776	299.54	14.58
12,000	216.65	1.940E + 04	3.119E − 01	9.770	295.07	14.22
14,000	216.65	1.417E + 04	2.279E − 01	9.764	295.07	14.22
16,000	216.65	1.035E + 04	1.665E − 01	9.757	295.07	14.22
20,000	216.65	5.529E + 03	8.891E − 02	9.745	295.07	14.22
22,000	218.57	4.049E + 03	6.454E − 02	9.739	296.38	14.32
24,000	220.55	2.974E + 03	4.697E − 02	9.733	297.72	14.43
26,000	222.54	2.190E + 03	3.429E − 02	9.727	299.05	14.54
28,000	224.52	1.617E + 03	2.510E − 02	9.721	300.38	14.65
30,000	226.51	1.198E + 03	1.842E − 02	9.715	301.71	14.75
32,000	228.49	8.891E + 02	1.356E − 02	9.709	303.02	14.86
35,000	236.50	5.750E + 02	8.471E − 03	9.700	308.29	15.29
40,000	250.32	2.875E + 02	4.001E − 03	9.684	317.17	16.01
45,000	264.15	1.492E + 02	1.968E − 03	9.669	325.82	16.71
50,000	270.65	7.978E + 01	1.027E − 03	9.654	329.80	17.04
55,000	260.80	4.258E + 01	5.687E − 04	9.639	323.74	16.54
60,000	247.06	2.201E + 01	3.103E − 04	9.624	315.10	15.84
80,000	198.61	1.054E + 00	1.848E − 05	9.564	282.52	13.21
86,000	186.87	3.732E − 01	6.957E − 06	9.547	274.04	12.53

Table 2.2. U.S. Standard Atmosphere- USCS Units

Altitude (ft)	Temperature (°R)	Pressure (psia)	Density (lb_m/ft^3)	Gravity (ft/s^2)	Sonic velocity (ft/s)	Dynamic viscosity $lb_f \cdot s/ft^2 \times 10^6$
0	518.67	1.470E+01	7.647E−02	32.17	1116.0	0.3737
1,000	515.11	1.417E+01	7.426E−02	32.17	1113.0	0.3717
2,000	511.55	1.367E+01	7.210E−02	32.17	1109.0	0.3697
3,000	507.99	1.317E+01	6.999E−02	32.16	1105.0	0.3677
4,000	504.43	1.270E+01	6.793E−02	32.16	1101.0	0.3657
5,000	500.87	1.223E+01	6.591E−02	32.16	1097.0	0.3637
6,000	497.31	1.178E+01	6.394E−02	32.16	1093.0	0.3616
8,000	490.19	1.092E+01	6.014E−02	32.15	1085.0	0.3576
10,000	483.07	1.011E+01	5.650E−02	32.14	1077.0	0.3534
20,000	447.47	6.763E+00	4.079E−02	32.11	1037.0	0.3325
30,000	411.87	4.375E+00	2.867E−02	32.08	994.9	0.3107
35,000	394.07	3.468E+00	2.375E−02	32.07	973.2	0.2995
40,000	389.97	2.730E+00	1.889E−02	32.05	968.1	0.2969
45,000	389.97	2.149E+00	1.487E−02	32.04	968.1	0.2969
50,000	389.97	1.692E+00	1.171E−02	32.02	968.1	0.2969
55,000	389.97	1.332E+00	9.219E−03	32.01	968.1	0.2969
60,000	389.97	1.049E+00	7.259E−03	31.99	968.1	0.2969
70,000	392.24	6.511E−01	4.480E−03	31.96	970.9	0.2984
80,000	397.68	4.067E−01	2.760E−03	31.93	977.6	0.3018
90,000	403.13	2.556E−01	1.711E−03	31.90	984.3	0.3052
100,000	408.57	1.617E−01	1.068E−03	31.87	990.9	0.3086
120,000	433.54	6.675E−02	4.155E−04	31.81	1021.0	0.3240
130,000	448.71	4.387E−02	2.639E−04	31.78	1038.0	0.3332
140,000	463.88	2.924E−02	1.701E−04	31.75	1056.0	0.3422
150,000	479.06	1.974E−02	1.112E−04	31.72	1073.0	0.3511
175,000	477.64	7.629E−03	4.311E−05	31.64	1071.0	0.3503
200,000	439.97	2.800E−03	1.718E−05	31.57	1028.0	0.3279
250,000	370.88	2.858E−04	2.080E−06	31.42	944.1	0.2846
282,152	336.37	5.413E−05	4.343E−07	31.32	899.1	0.2617

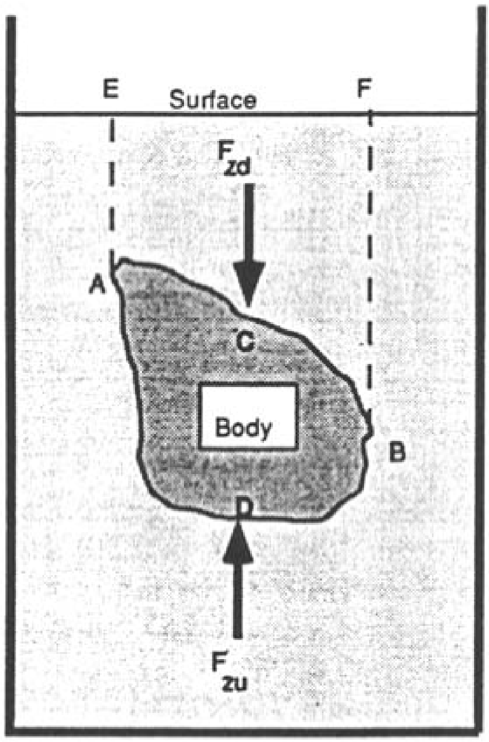


Figure 2.8 Notation for submerged bodies.

Thus, the buoyant force is the weight of the fluid displaced and always acts upward.

When an object floats as shown in Figure 2.9, the buoyant force F_B becomes the following:

$$F_B = \gamma V_{ABD} \tag{2.20}$$

The weight of the body F_g acts downward, so for vertical equilibrium the following can be written:

$$\sum F_z = 0 = F_g - F_B \tag{2.21}$$

or

$$F_B = F_g \tag{2.22}$$

Free-Body Analysis

The equation developed for flotation is a special case where the body is lighter than the fluid it can displace. A more general approach is that of the free-body diagram. If an object immersed in a liquid is heavier than the

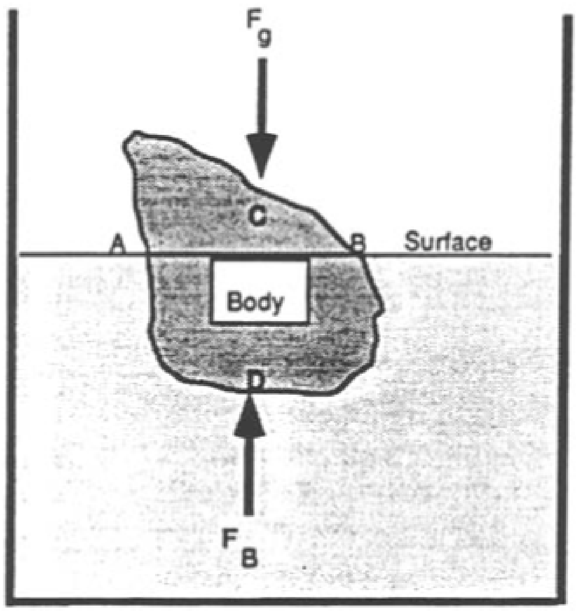


Figure 2.9 Notation for floating bodies.

fluid it can displace, it will sink to the bottom unless an upward force is applied to prevent it. A lighter-than-air ship or balloon will continue to rise unless a downward force is applied or unless it reaches an altitude where its density is the same as that of the atmosphere.

Figure 2.10 is a free-body diagram of an object immersed in a fluid. For vertical equilibrium, the following expression applies:

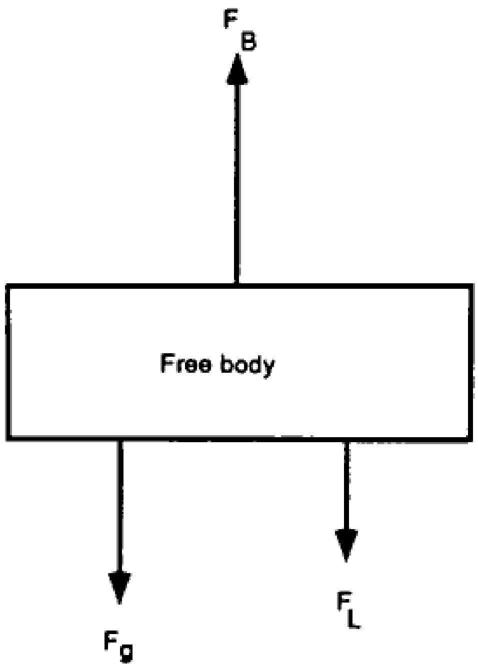


Figure 2.10 Free-body diagram.

$$\sum F_z = 0 = F_B - F_g - F_L \tag{2.23}$$

In equation (2.23), F_B is the buoyant force, F_g is the weight of the body, F_L is the force required to prevent the body from rising, and $(-F_L)$ is the force required to raise the object.

3

Equations of Fluid Motion

3.1 INTRODUCTION

This chapter concludes the introductory material of this book by examining the basic equations for velocity and energy in flow systems. Topics include fluid kinematics, fluid dynamics, and gas dynamics. This chapter may be skipped by those familiar with the material. The topics covered here provide a foundation for the derivation of equations in the remainder of the book.

3.2 FLUID KINEMATICS

Fluid kinematics is that branch of fluid mechanics that deals with the geometry of fluid motion without consideration of forces causing motion. It will be assumed that any fluid particle is very large in size with respect to a molecule and is, hence, continuous. A quantity such as velocity or fluid particle displacement must be measured relative to some convenient coordinate system. Two methods have been devised for representing fluid motion. One describes the behavior of a single fluid particle, the other is concerned with several fluid particles passing by certain points or sections of a fluid. The description of the behavior of individual fluid particles is called the *Lagrangian* method, after Joseph Louis Lagrange (1736–1813). This method of analysis involves establishing a coordinate system relative to a moving fluid particle as it moves through the continuum and measuring all quantities relative to the moving particle. The behavior of the individual fluid particle is of no practical importance in fluid mechanics, and this method is seldom used.

The establishment of a fixed coordinate system and the observation of the fluid passing through this system is called the *Eulerian* method, after Leonard Euler (1707–1783). The Eulerian method will be used for the most part throughout this book.

Velocity Profile

Volumetric Flow Rate

Figure 3.1 shows the steady flow of a fluid in a circular pipe. The velocity profile is obtained by plotting the velocity U of each streamline as it passes section A-A. The volume rate of flow Q past section A-A is given by the following:

$$Q = \int U \, dA \tag{3.1}$$

Average Velocity

In many engineering applications, the velocity profile is nearly a straight line or can be reduced to one, so the average velocity v may be used. The average velocity v is defined as follows:

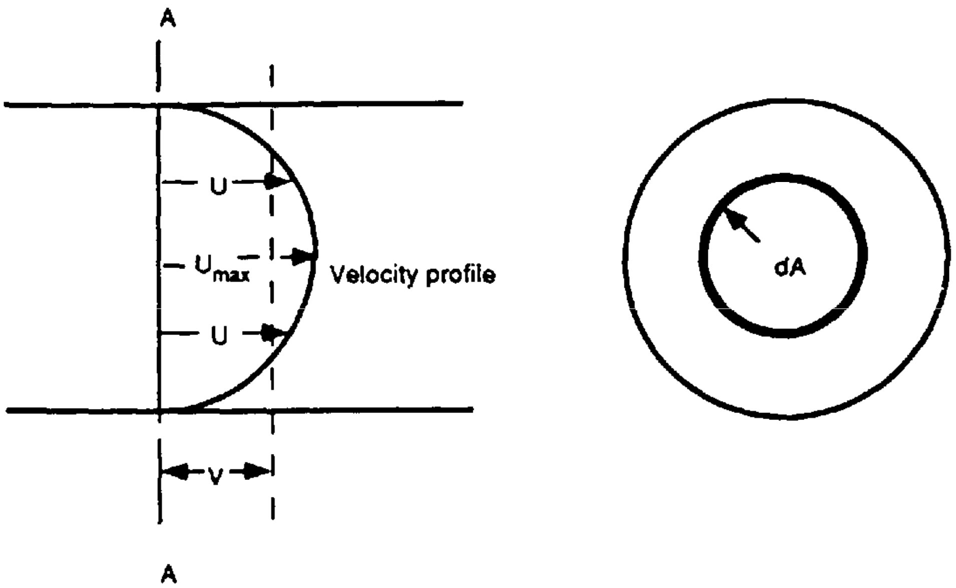


Figure 3.1 Velocity profile.

$$v = \frac{Q}{A} = \frac{1}{A}\int U\,dA \tag{3.2}$$

Continuity Equation

Mass Flow Rate

Consider the volume of fluid $ds\,dA$ moving in a streamtube with a velocity of U as shown in Figure 3.2. By definition, $m = \rho V$ or $dm = \rho\,ds\,dA$. Dividing by the time dt for this volume to move the distance ds and noting that, by definition, $U = ds/dt$ yields the following:

$$d\dot{m} = d\left(\frac{dm}{dt}\right) = \frac{\rho\,ds\,dA}{dt} = \rho U\,dA \tag{3.3}$$

Continuity Equation

The continuity equation is a special case of the general physical law of the conservation of mass. It may be stated simply that the mass flow rate entering a system is equal to the mass rate of storage in the system plus the mass flow rate leaving the system. Consider the flow system shown in Figure 3.3. Fluid is being supplied to the tank by means of the pipe at the rate $\dot{m}_1 = \rho_1 A_1 v_1$ and leaves the system at the rate of $\dot{m}_2 = \rho_2 A_2 v_2$. If the amount supplied is greater than that leaving, the tank level, z, will rise and fluid will

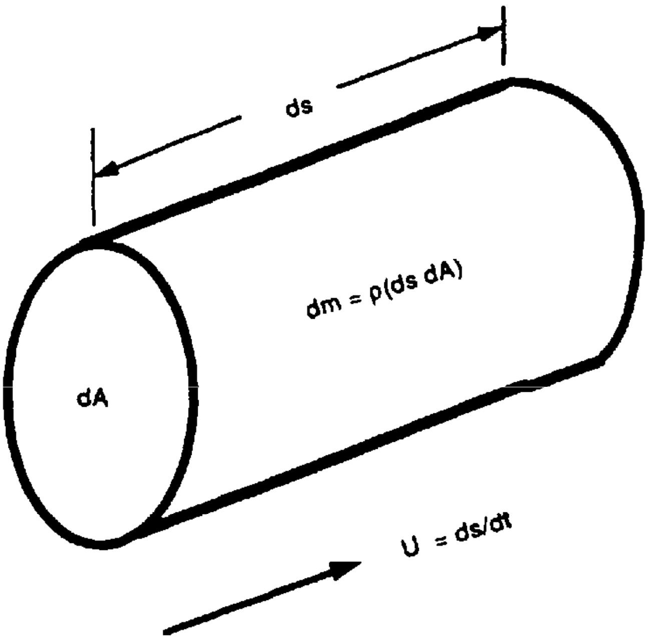


Figure 3.2 Mass flow rate.

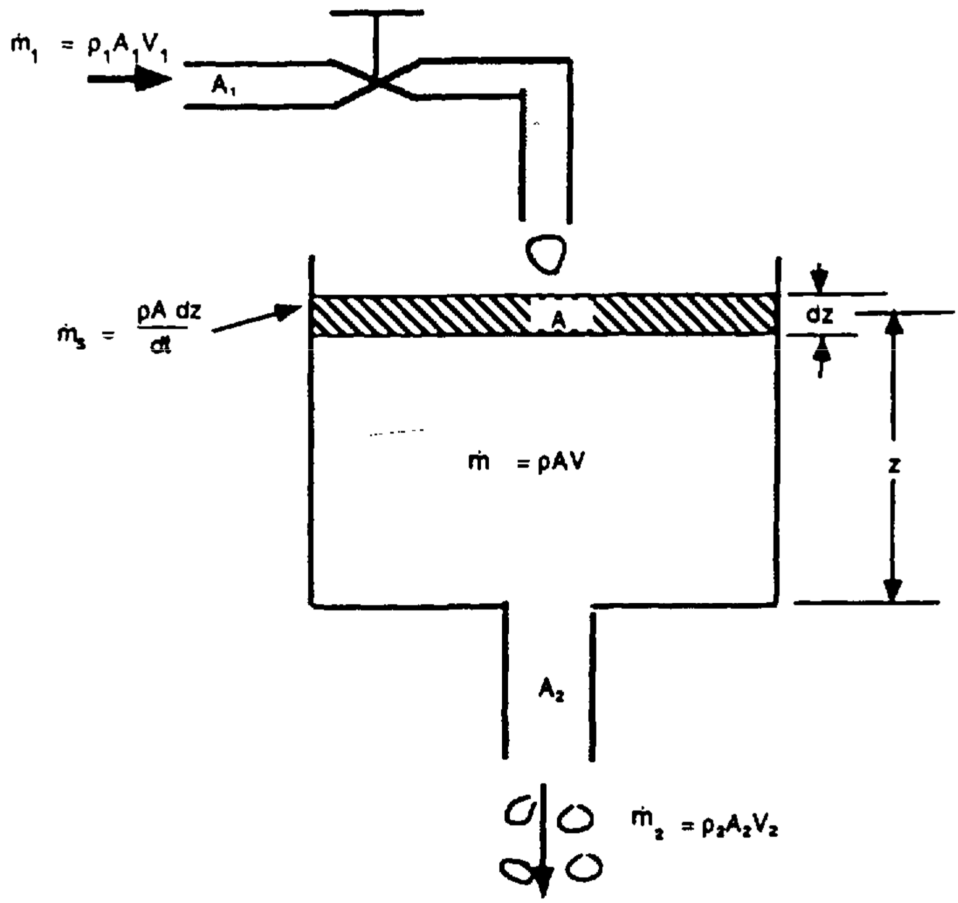


Figure 3.3 Continuity equation.

be stored in the tank at the rate of $\dot{m}_s = \rho A(dz/dt)$. Mathematically, this all can be stated as follows:

mass rate entering = mass rate of storage + mass rate leaving

$$\dot{m}_1 = \dot{m}_s + \dot{m}_2$$
$$\rho_1 A_1 v_1 = \rho A\left(\frac{dz}{dt}\right) + \rho_2 A_2 v_2 \tag{3.4}$$

Steady State

If the amount supplied is equal to the amount removed, then dz is zero or there is no storage. Equation (3.4) becomes the following:

$$\dot{m} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \ldots = \rho_n A_n v_n = \rho A v \tag{3.5}$$

The relationship of density to specific volume [$\rho = 1/\nu$ (Equation (1.15)] allows the equation of continuity to be written as the following:

$$\dot{m} = \rho A v = \frac{Av}{\nu} \tag{3.6}$$

The mass flow rate is constant any place in a steady-state system. For compressible fluids, it is sometimes convenient to use a differential form of equation (3.6), which may be obtained by writing it in logarithmic form and differentiating, noting that $\dot{m}$ is a constant.

$$\log_e v + \log_e A - \log_e \nu = \log_e \dot{m} \tag{3.7}$$

$$\frac{dv}{v} + \frac{dA}{A} - \frac{d\nu}{\nu} = 0 \tag{3.8}$$

Using the relationship $\nu = 1/\rho$ [Equation (1.15)] again, equation (3.8) may be written as follows:

$$\frac{dv}{v} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \tag{3.9}$$

3.3 FLUID DYNAMICS

Fluid dynamics is that branch of fluid mechanics that deals with energy and force. This topic includes the equation of motion, the energy equation, and the impulse-momentum equation. The continuity equation was developed in the previous section as a special case of the principle of the conservation of mass. The *equation of motion* is an application of Newton's second law to fluid flow in a streamtube. The *energy equation* is a special case of the principle of the conservation of energy. The *impulse-momentum equation* was developed in Chapter 1 as a special case of the equation of motion.

Equation of Motion

Derivation

Consider the fluid element flowing steadily in a streamtube shown in Figure 3.4. This element has a length of dL, an area normal to the motion of dA, and a perimeter of dP. The elemental mass is $\rho\, dA\, dL$. The increase in elevation of this mass is dz, and the motion of the element is upward.

Forces tending to change the velocity, U, of this fluid mass are:

Pressure forces on the ends of the element:

$$dF_p = p\, dA - (p + dp)dA = -dp\, dA \tag{3.10}$$

Gravity force due to the component of weight in the direction of motion:

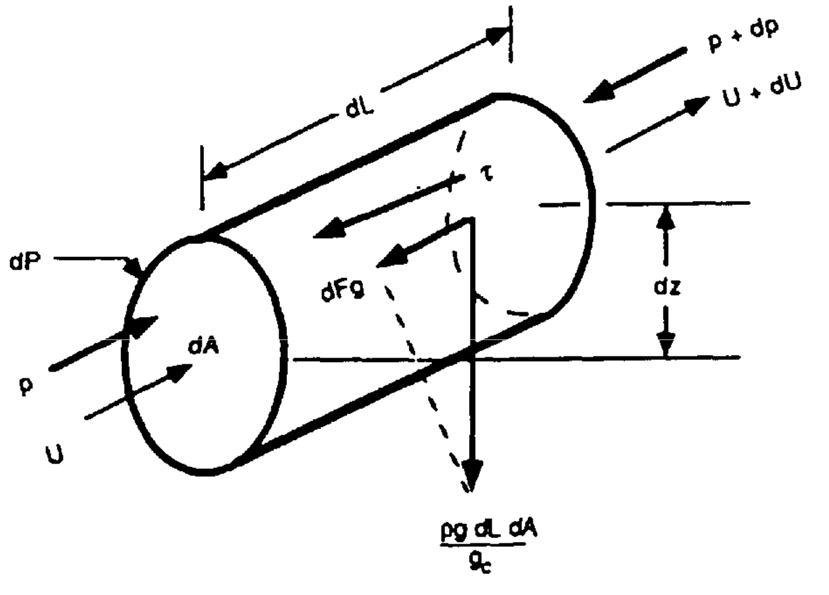


Figure 3.4 Elements of a streamtube.

$$dF_g = -\frac{\rho g\ dA\ dL}{g_c}\left(\frac{dz}{dL}\right) = -\frac{dA\ dz}{g_c} \tag{3.11}$$

Friction force on the outer surface of the element:

$$dF_f = -\tau\ dP\ dL \tag{3.12}$$

The *combined* force becomes the following:

$$\begin{aligned}\sum dF &= dF_p + dF_g + dF_f = -dP\ dA - \frac{\rho g\ dA\ dz}{g_c} - \tau\ dP\ dL\\ \sum dF &= -dA\left(dp + \frac{\rho g\ dz}{g_c} - \frac{\tau\ dP\ dL}{dA}\right)\end{aligned} \tag{3.13}$$

Application of Newton's second law [equation (1.11)] yields the following:

$$\sum dF = \frac{m\alpha}{g_c} = \frac{\rho\ dA\ dL}{g_c}\left(\frac{dU}{dt}\right) = \frac{\rho\ dA\ dU}{g_c}\left(\frac{dL}{dt}\right) = \frac{\rho\ dA\ U\ dU}{g_c} \tag{3.14}$$

Substituting from equation (3.13) for $\sum dF$, equation (3.14) may be written as follows:

$$\sum dF = -dA\left(dp + \frac{\rho g\ dz}{g_c} + \frac{\tau\ dP\ dL}{dA}\right) = \frac{\rho\ dA\ U\ dU}{g_c} \tag{3.15}$$

Simplifying equation (3.15) and setting it equal to zero results in the following:

$$\frac{g}{g_c}dz + \frac{U\,dU}{g_c} + \frac{dp}{\rho} + \frac{\tau\,dL}{\rho}\left(\frac{dP}{dA}\right) = 0 \tag{3.16}$$

Substituting $\nu = 1/\rho$ from equation (1.15) results in the following:

$$\frac{g}{g_c}dz + \frac{U\,dU}{g_c} + \nu\,dp + \nu\tau\,dL\left(\frac{dP}{dA}\right) = 0 \tag{3.17}$$

One-Dimensional Steady-Flow Equation of Motion

It will be shown later (Section 4.13) that dA/dP is equal to a quantity termed the *hydraulic radius* (R_h). For one-dimensional flow, $v = U$. Substituting this value of U and the definition of hydraulic radius into equation (3.17) yields the following:

$$\frac{g}{g_c}dz + \frac{v\,dv}{g_c} + \nu\,dp + \frac{\nu\tau}{R_h}dL = 0 \tag{3.18}$$

Integrating equation (3.18) between sections 1 and 2 leads to the following expression:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \int_1^2 \nu\,dp + \frac{1}{R_h}\int_1^2 \nu\tau\,dL = 0 \tag{3.19}$$

Let

$$H_f = \frac{1}{R_h}\int_1^2 \nu\tau\,dL \tag{3.20}$$

where H_f is the energy lost due to friction. Substituting this into equation (3.19) results in the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \int_1^2 \nu\,dp + H_f = 0 \tag{3.21}$$

For an *incompressible fluid* ($\nu_1 = \nu_2$), equation (3.21) becomes the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \nu(p_2 - p_1) + H_f = 0 \tag{3.22}$$

For frictionless flow of an incompressible fluid ($H_f = 0$), equation (3.22) reduces to the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \nu(p_2 - p_1) = 0 \tag{3.23}$$

Multiplying equation (3.23) by g_c/g and noting from equation (1.14) that $\nu = g/\gamma g_c$ results in the following:

$$(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g} + \frac{p_2 - p_1}{\gamma} = 0 \tag{3.24}$$

This is the equation proposed by Daniel Bernoulli in his *Hydrodynamica* published in 1738; thus, it is named the Bernoulli equation.

Specific Energy

Before developing the energy equation, a general discussion of energy is in order. Two types of energy will be considered. The first is the energy of the fluid at a section, and the second is the energy added to or taken from the fluid between sections. The total energy possessed by a fluid at a section is dependent upon the net energy added to or taken from the fluid between it and a prior section, but the individual energies are independent of their counterparts at the prior section. For this reason, fluid energies are called *point* functions. The energies added to or taken from the fluid between sections depend upon the manner or process, and these transitional energies are called *path* functions because of their dependence upon the process undergone. The total amount of energy in a system cannot be measured but must be referenced to some arbitrary datum. In fluid mechanics, we are interested in *energy* change, and any convenient datum may be used.

Specific Potential Energy

The potential energy of a fluid mass is the energy possessed by it due to its elevation relative to some arbitrary datum. It is equivalent to the work that would be required to lift it from the datum to its elevation in the absence of friction.

The change in specific potential energy (ΔPE) may be computed for a field of constant gravity as follows:

$$\Delta \mathrm{PE} = \frac{g}{g_c}\int_{z_1}^{z_2} dz = \frac{g}{g_c}(z_2 - z_1) \tag{3.25}$$

Note that equation (3.25) is the same as the first term of equation (3.19).

Specific Kinetic Energy

The kinetic energy of a fluid mass is the energy possessed by it due to its motion. It is equivalent to the work required to impart the motion from rest in the absence of friction. The change in specific kinetic energy (ΔKE) may be computed as follows:

$$\Delta KE = \frac{1}{g_c}\int_{v_1}^{v_2} v\, dv = \frac{v_2^2 - v_1^2}{2g_c} \tag{3.26}$$

Note that equation (3.26) is the same as the second term of equation (3.19).

Specific Internal Energy

The internal energy of a body is the sum total of the *kinetic* and *potential* energies of its molecules, apart from any kinetic or potential energy of the body as a whole. The total kinetic internal energy is due primarily to the translation, rotation, and vibration of its molecules. The potential internal energy is due to the bonding or attractive forces that hold the molecules in a phase.

The potential internal energy decreases as a substance changes from solid to liquid to gaseous phases as the bonding forces decrease. In the gas phase, the internal energy is mainly kinetic. As the ideal gas state is approached and molecular activity increases with temperature, the internal energy becomes all kinetic; thus, the internal energy of an ideal gas is a pure temperature function.

The symbol for specific internal energy is u, and the change in specific internal energy is given by the following:

$$\Delta u = \int_1^2 du = u_2 - u_1 \tag{3.27}$$

In SI, the units for internal energy are the joule per kilogram or newton-meter per kilogram. For USCS units, conventional practice is to use the British thermal unit per pound-mass (Btu/lbm). For fluid mechanics, it will be necessary to convert the Btu to ft·lbf (778.2 ft·lbf = 1 Btu).

Specific Flow Work

Flow work is the amount of mechanical energy required to force a flowing fluid across a section boundary. Consider the steady-flow system shown in Figure 3.5. Fluid enters the system at section 1, where the flow area is A_1 and the pressure is p_1, and leaves at section 2, where the flow area is A_2 and the pressure is p_2. The force acting to prevent the fluid from crossing a section boundary is

$$F = pA \tag{3.27}$$

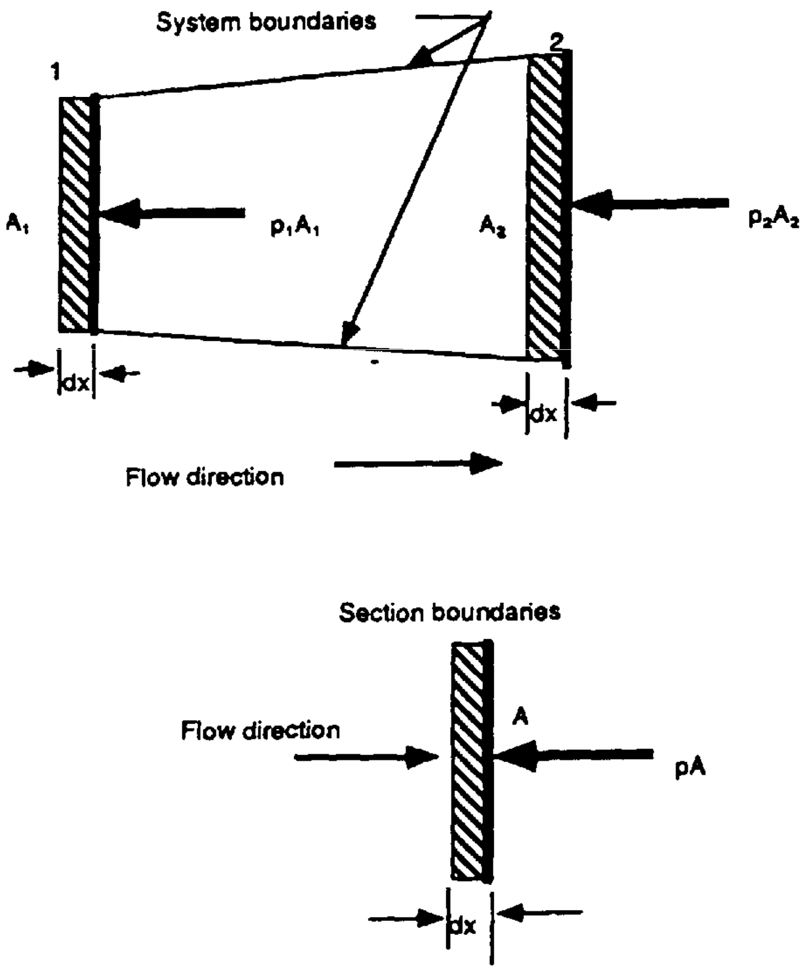


Figure 3.5 Flow work.

where p is the pressure at the section boundary and A is the flow area. The thermodynamic definition of specific work is given as equation (3.28):

$$W = \frac{1}{m}\int F\,dx \tag{3.28}$$

Substituting equation (3.27) in equation (3.28) yields the following:

$$\text{FW} = \frac{1}{m}\int F\,dx = \int \frac{pA}{m}\,dx \tag{3.29}$$

FW in equation (3.29) is the specific flow work. Noting that $A\,dx$ is the volume V of fluid being "pushed" across a section boundary and that by definition $V = mv$, equation (3.29) may be written as follows:

$$\text{FW} = \int_1^2 \frac{d(pV)}{m} = \int_1^2 \frac{d(pmv)}{m} = \int_1^2 d(pv) = p_2v_2 - p_1v_1 \tag{3.30}$$

ΔFW is the change in specific flow work.

Specific Enthalpy

It is sometimes desirable to combine certain fluid properties to obtain a new one. *Enthalpy* is a property combining internal energy, pressure, and specific volume.

The symbol for specific enthalpy is h; specific enthalpy is defined by the following equation:

$$h = u + pv \tag{3.31}$$

The change in specific enthalpy becomes

$$\Delta h = h_2 - h_1 = u_2 - u_1 + p_2 v_2 - p_1 v_1 \tag{3.32}$$

Specific enthalpy units in SI are the joule per kilogram or newton-meter per kilogram. For the USCS units, conventional practice is to use the British thermal unit per pound-mass (Btu/lbm). For fluid mechanics, it will be necessary to convert the Btu to ft·lbf (778.2 ft·lbf = 1 Btu).

Shaft Work

Definition

Shaft work is that form of mechanical energy that crosses the boundaries of a system, transmitted through the shaft of a machine. The result of this transmission is to increase or decrease the total amount of energy stored in a fluid.

Shaft work is mechanical energy in *transition* and cannot be stored as such in a fluid. For example, consider a pump that is pumping water from a lower level to a higher one. While the pump is in operation, shaft work is transmitted to the water; this increase in energy causes the water to rise to a higher elevation. After the pump has stopped, the amount of energy added to the fluid less losses is now stored in the water in the form of increased mechanical potential energy.

Because the first engines built by man were made to extract work from the fluid energy, conventional practice is to call shaft work done *by* a fluid *positive work*, and work done *on* a fluid *negative work*. Shaft work may also be classified as *steady flow* or *nonflow* according to the type of machine and process.

Nonflow Shaft Work

Consider the cylinder and piston arrangement shown in Figure 3.6. As the piston advances from the state point 1 to point 2, the fluid in the cylinder

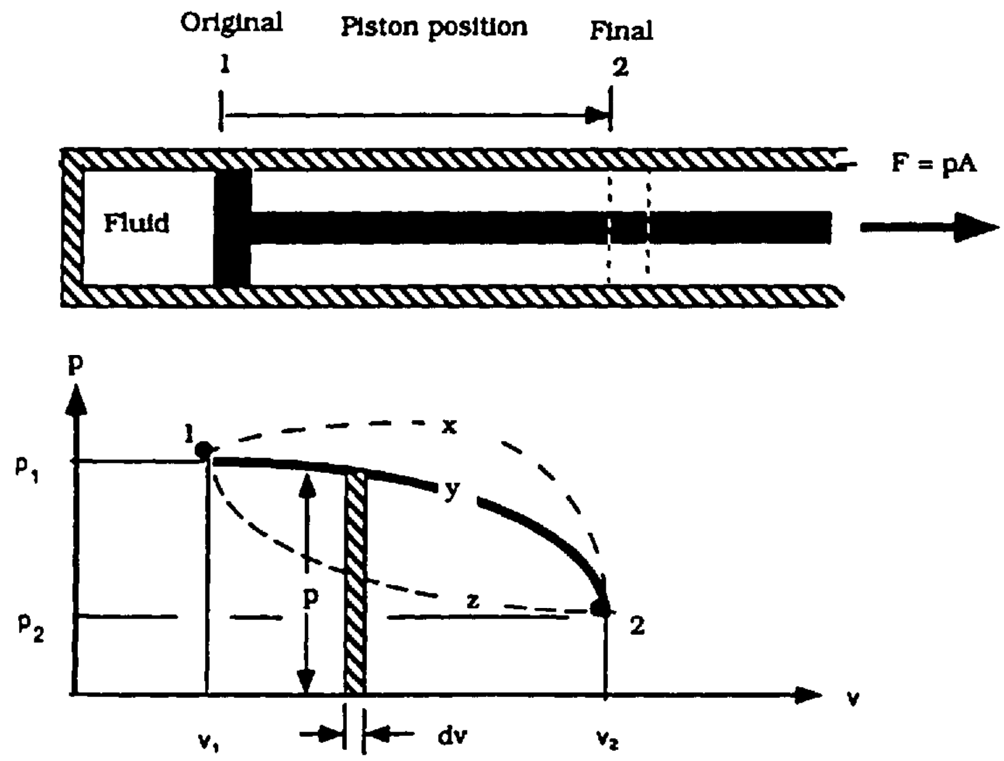


Figure 3.6 Nonflow shaft work.

expands and work is done *by the fluid*. If the piston were made to retract, then the fluid would be compressed and work would be done *on the fluid*.

The force exerted by the fluid on the piston of Figure 3.6 is given by the following:

$$F = pA \tag{3.33}$$

Substituting equation (3.33) in equation (3.28) and noting that the area of the piston, A, is a constant such that $A\ dx = dV$ and that by definition $V = mv$ results in the following:

$$W'_{nf} = \frac{1}{m}\int F\ dx = \int \frac{pA}{m}\ dx = \int_1^2 \frac{p\ dV}{m} = \int_1^2 p\ dv \tag{3.34}$$

W_{nf} in this equation is the specific shaft work.

Steady-Flow Shaft Work

The specific steady-flow shaft work may be expressed as follows:

$$W_{sf} = \int dW_{sf} \neq W_{sf2} - W_{sf1} \tag{3.35}$$

W_{sf} is the steady-flow shaft work per unit mass. Because the differential of shaft work is inexact, the Greek letter δ is used instead of d. Equation (3.35) may be correctly written as follows:

$$W_{sf} = \int \delta W_{sf} \tag{3.36}$$

Heat and Entropy

Heat is the form of thermal energy that crosses the boundaries of a system without the transfer of mass as a result of a difference in temperature between the system and its surroundings. The effect of this transfer is to increase or decrease the total amount of energy stored in a fluid. Heat is thermal energy in *transition*, and like shaft work it cannot be stored as such in a fluid. Because the first devices made by humans were to produce shaft work by adding heat, heat added to a substance is positive, and heat rejected is negative. *Entropy* is the fluid property required by the second law of thermodynamics to describe the path of a reversible process. Entropy is defined by the following equation:

$$q = \int T\, ds \tag{3.37}$$

In equation (3.37), q is the heat transferred per unit mass and s is the entropy per unit mass.

Heat may also be expressed as the following:

$$q = \int \delta\, dq \neq q_2 - q_1 \tag{3.38}$$

Note that the symbol δ is used in place of d, because the differential of heat transfer is inexact.

Equations (3.38) and (3.37) may be combined as follows:

$$q = \int \delta\, dq = \int T\, ds \tag{3.39}$$

Equation (3.39) is a mathematical statement; heat is the area under the temperature–entropy curve of Figure 3.7.

In SI, the joule per kilogram or newton-meter per kilogram is used for heat and the joule/kilogram Kelvin is used for entropy. In USCS units, the British thermal unit per pound mass is used for heat, and the British thermal unit/pound mass degree Rankine is used for entropy. For fluid mechanics, it will be necessary to convert the Btu to ft · lbf and the Btu/lbm·°R to ft · lbf/lbm·°F (778.2 ft · lbf = 1 Btu).

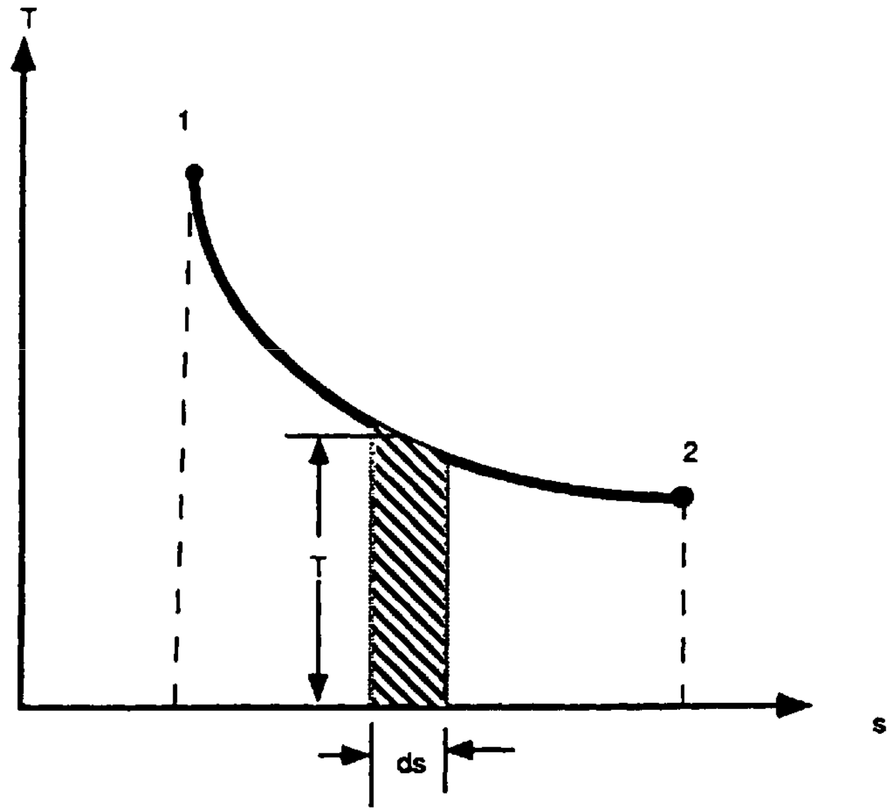


Figure 3.7 Temperature-entropy plane.

Steady-Flow Energy Equation

The steady-flow energy equation is readily derived by the application of the principles of conservation of energy to a thermodynamic system. The following forms of energy are considered.

Stored in Fluid:

Potential energy $$\Delta PE = \frac{g}{g_c}(z_2 - z_1) \tag{3.25}$$

Kinetic energy $$\Delta KE = \frac{v_2^2 - v_1^2}{2g_c} \tag{3.26}$$

Internal energy $$\Delta u = u_2 - u_1 \tag{3.27}$$

Flow work $$FW = p_2 v_2 - p_1 v_1 \tag{3.30}$$

In Transition:

Shaft work $$W_{sf} = \int \delta W_{sf} \tag{3.36}$$

Heat transfer $$q = \int \delta\, dq \tag{3.38}$$

The basic requirement for the satisfaction of the principle of conservation of energy may be stated as follows:

$$\begin{aligned} &\sum \text{energy entering system} - \sum \text{energy stored in system} \\ &\quad = \sum \text{energy leaving system} \end{aligned} \tag{3.40}$$

In a steady-flow system, the energy stored in the system does not change with time, so for any given period of time equation (3.40) reduces to equation (3.41):

$$\sum \text{energy in} = \sum \text{energy out} \tag{3.41}$$

Equation (3.41) may be modified to show the types of energy as follows:

energy stored in entering fluid + energy in transition added to system system = {energy in transition removed from system + energy stored in fluid leaving (3.42)

Consider the block diagram of Figure 3.8. The fluid enters the system through section 1, transporting with it its stored energy expressed as follows:

$$\frac{g}{g_c} z_1 \frac{v_1^2}{2g_c} + u_1 + p_1 v_1$$

The fluid leaves the system at section 2, removing its stored energy expressed as follows:

$$\frac{g}{g_c} z_2 \frac{v_2^2}{2g_c} + u_2 + p_2 v_2$$

Since heat (q) added to a system is considered positive, the arrow shows heat being added between sections 1 and 2. In a like manner, the steady-flow shaft work (W_{sf}) is shown to be leaving between sections 1 and 2, because work done by the fluid is considered positive.

Application of equation (3.42) to Figure 3.8 results in the following:

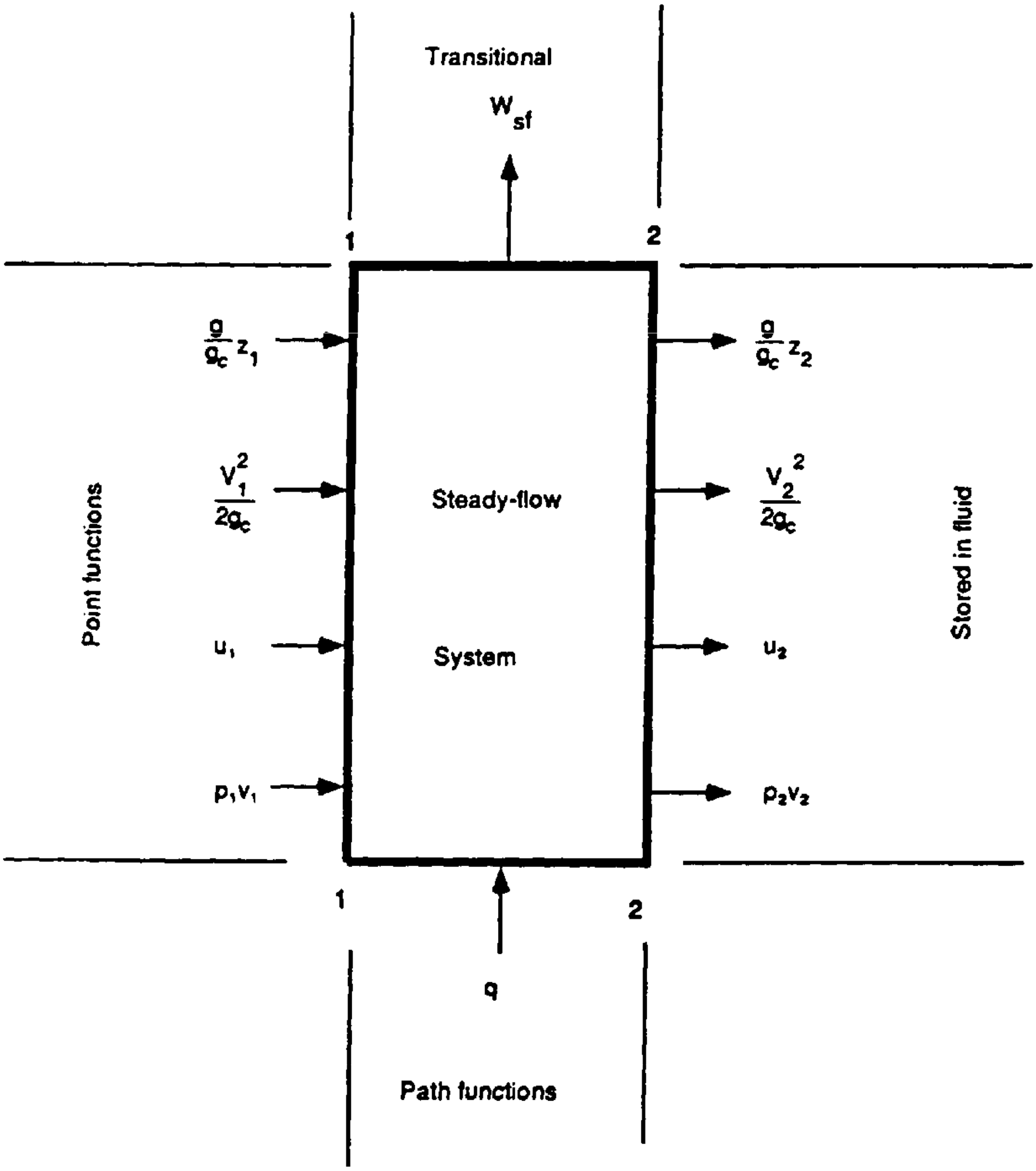


Figure 3.8 Steady-flow energy diagram.

$$\left(\frac{g}{g_c} z_1 + \frac{v_1^2}{2g_c} + u_1 + p_1 v_1\right) + q = W_{sf} + \left(\frac{g}{g_c} z_2 + \frac{v_2^2}{2g_c} + u_2 + p_2 v_2\right) \tag{3.43}$$

Equation (3.43) may be written as follows:

$$q = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + u_2 - u_1 + p_2 v_2 - p_1 v_1 \tag{3.44}$$

Equation (3.44) may be written as follows:

$$q = W_{sf} + \Delta\text{PE} + \Delta\text{KE} + \Delta u + \Delta\text{FW} \tag{3.45}$$

Relation of Motion Equations and Energy Equations

The equation of motion was derived earlier in Section 3.3 without consideration of steady-state shaft work. Had shaft work been considered, the resulting one-dimensional equation of motion (3.21) would have been the following:

$$W_{sf} = \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \int_1^2 v\, dp + H_f = 0 \tag{3.46}$$

Noting that

$$p_2 v_2 - p_1 v_1 = \int_1^2 d(pv) = \int_1^2 v\, dp + \int_1^2 p\, dv \tag{3.47}$$

the energy equation (3.44) may be written as follows:

$$q = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + u_2 - u_1 + \int_1^2 v\, dp + \int_1^2 p\, dv \tag{3.48}$$

As the equations are now written, the equation of motion (3.46) contains no thermal energy terms and the energy equation (3.48) contains no term for friction. If equation (3.46) is subtracted from equation (3.48) and solved for H_f, the following is obtained:

$$H_f = u_2 - u_1 + \int_1^2 p\, dv - q \tag{3.49}$$

For an incompressible fluid, $v_1 = v_2$, or $dv = 0$, so equation (3.49) reduces to the following:

$$H_f = u_2 - u_1 - q \tag{3.50}$$

Equation (3.49) indicates that energy is not "lost" due to friction but is simply converted into some other form that is either removed from the system as heat transfer or/and increases the internal energy of the fluid.

Nonflow vs. Steady-Flow Energy Equations

Consider the horizontal piston and cylinder arrangement shown in Figure 3.6. Fluid does not cross the system boundaries, so no flow work is performed; nor can there be any change in kinetic energy. Because the cylinder is horizontal, there is no change in potential energy. Of the six forms of energy considered earlier in Section 3.3 for the steady-flow equation, only three--internal energy, shaft work, and heat transfer - need to be considered for a nonflow system.

From Figure 3.9, application of the principle of conservation of energy leads to the following:

$$q = \Delta u + W_{nf} \tag{3.51}$$

Noting from equation (3.27) that

$$\Delta u = u_2 - u_1 \tag{3.27}$$

and from equation (3.34) that

$$W_{nf} = \int_1^2 p\, dv \tag{3.34}$$

the nonflow equation may be written as follows:

$$q = u_2 - u_1 + \int_1^2 p\, dv \tag{3.52}$$

If equation (3.52) is subtracted from equation (3.48), the following is obtained:

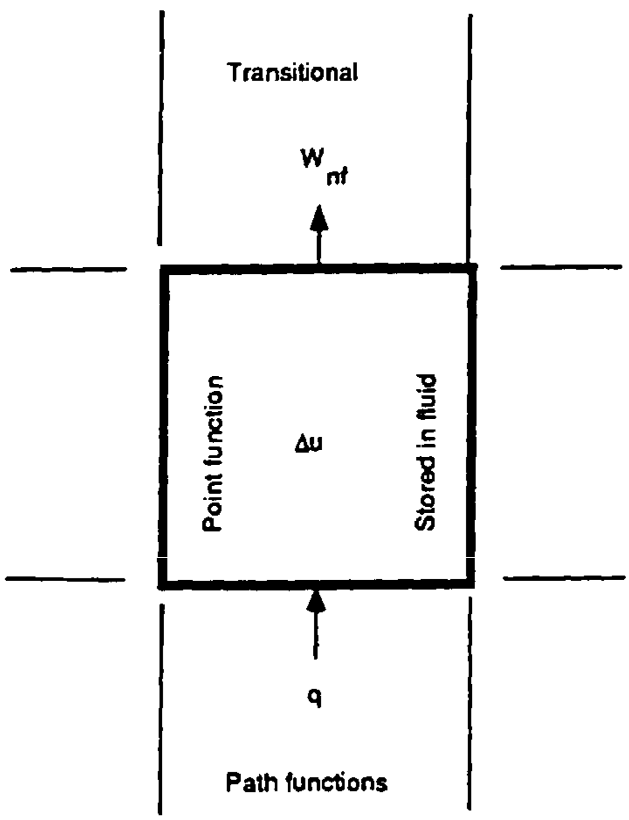


Figure 3.9 Nonflow energy diagram.

$$0 = W_{sf} + \frac{g}{g_c}(z_2 - z) + \frac{v^2 - v_1^2}{2g_c} + \int_1^2 v\, dp \tag{3.53}$$

or

$$-\int_1^2 v\, dp = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} \tag{3.54}$$

Equation (3.54) may also be written in the following form:

$$-\int_1^2 v\, dp = W_{sf} + \Delta\text{PE} + \Delta\text{KE} \tag{3.55}$$

From equation (1.32) $n = -(v\,dp/p\,dv)$, and from equation (3.34) $W_{nf} = \int p\,dv$. Using these relations and equation (3.55) results in the following:

$$n = \frac{-v\,dp}{p\,dv} = \frac{-\int_1^2 v\,dp}{\int_1^2 p\,dv} = \frac{W_{sf} + \Delta\text{PE} + \Delta\text{KE}}{W_{nf}} \tag{3.56}$$

Note that in the absence of potential and kinetic energy changes, the process path n is the ratio between the steady-flow work and the nonflow work for a reversible process. If $-\int v\,dp$ from equation (3.54), Δh from equation (3.32), and $\int T\,ds$ from equation (3.37) are substituted in equation (3.44), the following results:

$$\begin{aligned} q &= \left[W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c}\right] + (u_2 - u_1 + p_2v_2 - p_1v_1) \\ q &= -\int_1^2 v\,dp + \Delta h = \int_1^2 T\,ds \end{aligned} \tag{3.57}$$

Equation (3.57) may be written in differential form as follows:

$$\delta q = T\,ds = -v\,dp + dh \tag{3.58}$$

Equation (3.52) can be written in differential form as follows:

$$\delta q = T\,ds = p\,dv + du \tag{3.59}$$

Equation (3.58) was developed from the steady-flow energy equation and equation (3.59) from the nonflow equation. Noting that by definition $dh = du + d(pv) = u + p\,dv + v\,dp$ and substituting in equation (3.58) yields the following:

$$T\,ds = -v\,dp + \Delta h = -v\,dp + du + p\,dv + v\,dp = p\,dv + du \tag{3.60}$$

Ideal Gas Specific Heat and Energy Relations

Specific Heats

The specific heat of any substance is defined by the following equation:

$$c_x = \left(\frac{\partial q}{\partial T}\right)_x \tag{3.61}$$

where c_x is the specific heat for process x.

In SI, the joule per kilogram per Kelvin (J/(kg·K)) or the Newton-meter per kilogram per Kelvin is used for specific heat. For USCS units, conventional practice is to use the British thermal unit per pound-mass per degree Rankine (Btu/(lbm-°R). For fluid mechanics, it will be necessary to convert the Btu to ft·lbf (778.2 ft lbf = 1 Btu).

Constant-Volume Specific Heat

Note that if equation (3.59) is solved for a constant-volume process ($dv = 0$), the following results:

$$\delta q_v = p(0) + du = du \tag{3.62}$$

From equation (3.61), the following can be written:

$$c_x\left(\frac{\partial q}{\partial T}\right)_x = c_v = \left(\frac{\partial u}{\partial T}\right)_v \tag{3.63}$$

Since the internal energy of an ideal gas is a function of temperature only, the partial notation may be dropped; equation (3.63) may be then be written as follows:

$$du = c_v\,dT \tag{3.64}$$

Equation (3.64) may be used for any ideal gas process.

Constant-Pressure Specific Heat

Note that if equation (3.59) is solved for a constant-pressure process ($dp = 0$), the following results:

$$\delta q_p = -v(0) + dh = dh \tag{3.65}$$

From equation (3.61), the following can be written:

$$c_x = \left(\frac{\partial q}{\partial T}\right)_x = c_p = \left(\frac{\partial h}{\partial T}\right)_p \tag{3.66}$$

Since the enthalpy of an ideal gas is a function of temperature only, the partial notation may be dropped; equation (3.66) may be then be written as follows:

$$dh = c_p\, dT \tag{3.67}$$

Equation (3.67) may be used for any ideal gas process.

Ratio of Specific Heats

If the relation of equation (3.67) is substituted in equation (3.58), the following results:

$$T\, ds = -v dp + c_p\, dT \tag{3.68}$$

For an isentropic process ($ds = 0$), equation (3.68) reduces to the following:

$$v\, dp = c_p\, dT \qquad (s = c) \tag{3.69}$$

If the relation of equation (3.64) is substituted in equation (3.59), the following equation results:

$$T\, ds = p\, dv + c_v\, dT \tag{3.70}$$

For an isentropic process ($ds = 0$), equation (3.70) reduces to the following:

$$-p\, dv = c_v\, dT \qquad (s = c) \tag{3.71}$$

Substituting equations (3.69) and (3.71) in equation (1.32) yields the following:

$$n = \frac{-v\, dp}{p\, dv} = -\left(\frac{c_p\, dT}{c_v\, dT}\right) = \frac{cp}{c_v} = k \tag{3.72}$$

In equation (3.72), k, the ratio of specific heats, is the exponent for an isentropic process.

If the definition of enthalpy is written in differential form and from equation (1.37) $pv = RT$, equation (3.64) $du = c_v dT$, and equation (3.67) $dh = c_p\, dT$, the following can be written:

$$dh = du + d(pv) = c_p\, dT = c_v\, dT + R\, dT$$

or

$$c_p - c_v = R \tag{3.73}$$

Dividing equation (3.73) by c_v, yields the following:

$$\frac{c_p}{c_v} = \frac{c_v}{c_v} = \frac{R}{c_v} = k - 1 = \frac{R}{c_v}$$

or

$$c_v = \frac{R}{k-1} \tag{3.74}$$

In a like manner, the following can be written:

$$c_p = \frac{kR}{k-1} \tag{3.75}$$

Polytropic Specific Heat

Integrating equation (3.34) for a polytropic process using $pv^n = c$ [equation (1.31)] gives the following:

$$\int_1^2 p\,dv = c\int_1^2 \frac{dv}{v^n} = c\left(\frac{v^{1-n}}{a-n}\right)_1^2 = \frac{p_2v_2 - p_1v_1}{1-n} \qquad (n \neq 1) \tag{3.76}$$

If the ideal gas equation of state (1.37) $pv = RT$ is substituted in equation (3.76), the following results:

$$\int_1^2 p\,dv = \frac{p_2v_2 - p_1v_1}{1-n} = \frac{RT_2 - RT_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} \tag{3.77}$$

Writing equation (3.77) in differential form and substituting $c_p - c_v = R$ from equation (3.73) results in the following:

$$p\,dv = \frac{R\,dT}{1-n} = \frac{(c_p - c_v)dT}{1-n} \tag{3.78}$$

Substituting $c_n\,dT$ for $T\,ds$ and $p\,dv$ from equation (3.78) in equation (3.70), noting that $c_p/c_v = k$, and solving for c_n yields the following:

$$T\,ds = p\,dv + c_v\,dT = c_n\,dT = \frac{c_p - c_v}{1-n}dT + c_v\,dT$$

or

$$c_n = \frac{cp - nc_v}{1-n} + c_v\left(\frac{k-n}{1-n}\right) \tag{3.79}$$

Equation (3.79) may be used only for polytropic ideal gas processes where $n \neq 1$.

Isentropic Energy Relations

The path of frictionless adiabatic flow of an ideal gas, is from equation (1.33), pv^k = a constant. If the friction term ($v\tau\ DL/R_h$) of the equation of motion (3.18) is dropped, and the equation integrated between sections 1 and 2, the following equation can be written for frictionless flow:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \int_1^2 v\,dp = 0 \tag{3.80}$$

The third term of equation (3.80) may be integrated by noting from equation (1.42) that $v = v_1(p_1/p)^{1/k}$, so the following results:

$$\int_1^2 v\,dp = v_1 p_1^{1/k} \int_1^2 \frac{dp}{p^{1/k}} = v_1 p_1^{1/k}\left[\left(\frac{k}{k-1}\right)p^{(k-1)/k}\right]_1^2$$
$$\int_1^2 v\,dp = p_1 v_1\left(\frac{k}{k-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right] \tag{3.81}$$

Substituting equation (3.81) in equation (3.80) yields the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + p_1 v_1\left(\frac{k}{k-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right] = 0 \tag{3.82}$$

Substituting from the equation of state (1.37) $p_1 v_1 = RT_1$ and from equation (1.42) $T_2/T_1 = (p_2/p_1)^{(k-1)/k}$ in equation (3.82) results in the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + RT_1\left(\frac{k}{k-1}\right)\left(\frac{T_2}{T_1} - 1\right) =$$
$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \left(\frac{Rk}{k-1}\right)(T_2 - T_1) = 0 \tag{3.83}$$

Substituting from equation (3.75), $c_p = Rk/(k-1)$, and from equation (3.67), $dh = c_p dT$, in equation (3.83) results in the following:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + c_p[T_2 - T_1] = \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + h_2 - h_1 = 0 \tag{3.84}$$

The same result can be obtained from the energy equation. For an isentropic process, $q = 0$, for no shaft work, $W_{sf} = 0$, and by definition $u_2 - u_1 + p_2 v_2 = p_1 v_1 = h_2 - h_1$. Substituting these expressions in equation (3.44) yields the following:

$$q = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + u_2 - u_1 + p_2 v_2 - p_1 v_1 \tag{3.44}$$

$$0 = 0 + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + h_2 - h_1 \tag{3.85}$$

The general energy equation (3.44) may be written in the following form by substituting for $h_2 - h_1$ the value of $[kR/(k-1)](T_2 - T_1)$:

$$q = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + u_2 - u_1 + p_2 \nu_2 - p_1 \nu_1 \tag{3.44}$$

$$q = W_{sf} + \frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + \left(\frac{kR}{k-1}\right)(T_2 - T_1) \tag{3.86}$$

Impulse-Momentum Equation

The impulse-momentum equation is used to calculate the forces exerted on a solid boundary by a moving stream. It was derived in Section 1.6 as an application of Newton's second law. This resulted in the following equation:

$$F = \frac{ma}{g_c} = \frac{\dot{m}(v_2 - v_1)}{g_c} \tag{1.11}$$

Substituting ΣF (the summation of all forces acting) for F and from equation (3.6), $\dot{m} = \rho A v$, yields the following:

$$\sum F = \frac{\dot{m}(v_2 - v_1)}{g_c} = \frac{\rho A v}{g_c}(v_2 - v_1) \tag{3.87}$$

In the application of equation (3.87), it must be remembered that velocity is a vector and as such has both magnitude and direction. The impulse-momentum equation is often used in conjunction with the continuity and energy equations to solve engineering problems.

In general, the "free-body" method is used to compute the forces involved on the boundaries on a control volume. The symbol F is used for the force exerted by the boundaries on the fluid. There is an equal but opposite force exerted by the fluid on the boundaries.

3.4 GAS DYNAMICS

Gas dynamics is the branch of fluid dynamics concerned with the motion of gases and consequent effects. Gas dynamics combines the principles of fluid mechanics and thermodynamics. This subject is based on the application of the following four fundamentals:

Newton's second law of motion
The law of conservation of mass
The first law of thermodynamics
The second law of thermodynamics

Because the potential energy changes in ideal gas systems are usually small compared with other energy changes, all systems in this section are assumed to be *horizontal*, and thus $z_2 - z_1 = 0$. It is further assumed that the flow is *one-dimensional* and that all fluids are in the *ideal gas state*.

Area–Velocity Relations

In this section, the differences between incompressible and compressible flow area–velocity relations are developed.

Incompressible Fluids

Repeating the continuity equation in differential form from Section 3.2:

$$\frac{dv}{v} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \tag{3.9}$$

For an incompressible fluid, $d\rho = 0$, so equation (3.9) reduces to the following:

$$\frac{dv}{v} = -\frac{dA}{A} \tag{3.89}$$

Inspection of equation (3.89) indicates the following:

1. If area increases, velocity decreases.
2. If area decreases, velocity increases.
3. If area is constant, velocity is constant.
4. There are no critical values.

Compressible Fluids

The equation of motion (3.18) for a horizontal system ($dz = 0$) and for frictionless flow ($\tau = 0$) becomes the following:

$$\frac{v\,dv}{g_c} + \nu\,dp = 0 \tag{3.90}$$

Substituting equation (1.15), $\nu = 1/\rho$, in equation (3.90), yields the following:

$$\frac{v\,dv}{g_c} + \frac{dp}{\rho} = 0 \tag{3.91}$$

Substituting equation (1.57), $\rho = Eg_c/c^2$, and equation (1.45), $dp = -E\,d\nu/\nu$, in equation (3.91) and dividing by v^2 yields the following:

$$\frac{v\,dv}{v^2 g_c} + \frac{dp}{\rho v^2} = \frac{dv}{g_c v} + \frac{(-E\,d\nu/\nu)}{v^2(Eg_c/c^2)} = 0$$

or

$$\frac{d\nu}{\nu} = \left(\frac{v}{c}\right)^2 \frac{dv}{v} \tag{3.92}$$

Equation (3.8), the differential form of the continuity equation, can be written as follows:

$$\frac{dv}{v} + \frac{dA}{A} + \frac{d\nu}{\nu} = 0 \tag{3.8}$$

Substituting the relationship for $d\nu/\nu$ from equation (3.92) in equation (3.8) and solving for dA results in the following:

$$\frac{dA}{A} = \frac{d\nu}{\nu} - \frac{dv}{v} = \left(\frac{v}{c}\right)^2 \frac{dv}{v} - \frac{dv}{v} = \frac{dv}{v}\left[\left(\frac{v}{c}\right)^2 - 1\right] \tag{3.93}$$

The ratio of actual velocity, v, to the speed of sound c is known as the *Mach number*, M, named in honor of Ernst Mach, an Austrian physicist. For an ideal gas from equation (1.59), $c = (kg_cRT)^{1/2}$, so the following can be written:

$$M = \frac{v}{c} = \frac{v}{\sqrt{kg_cRT}} \tag{3.94}$$

Substituting equation (3.94) in equation (3.93) and rearranging terms yields the following:

$$\frac{dA}{A} = \frac{dv}{v}\left[\left(\frac{v}{c}\right)^2 - 1\right] = -\frac{dv}{v}(1 - M^2) \tag{3.95}$$

Analysis of equation (3.95) leads to the following conclusions:

1. $v < c$, $M < 1$	dA/A varies as $-dv/v$	*Velocity subsonic:* If area increases, velocity decreases. Same as for incompressible flow.
2. $v = c$, $M = 1$	dA/A equals zero	*Velocity sonic:* Sonic velocity can exist only where the change in area is zero, i.e., at the end of a convergent passage.

3. $v > c$, $M > 1$	dA/A varies as dv/v	*Velocity supersonic:* If area increases, velocity increases reverse of incompressible flow. Also, supersonic velocity can exist only in the expanding portion of a passage after a constriction where sonic (acoustic) velocity existed.

Frictionless Adiabatic (Isentropic) Flow of Ideal Gases in Horizontal Passages

General Considerations

Frictionless adiabatic (isentropic) compressible flow of an *ideal gas* must satisfy the following requirements:

1. *The ideal gas law.* The equation of state for an ideal gas is the following:

$$pv = RT \tag{1.37}$$

2. *The process relationship.* For an ideal gas undergoing an isentropic process:

$$pv^k = p_1 v_1^k = p_2 v_2^k \tag{1.33}$$

3. *Conservation of mass.* The continuity equation may be expressed as follows:

$$\dot{m} = \frac{Av}{v} = \frac{A_1 v_1}{v_1} = \frac{A_2 v_2}{v_2} \tag{3.3}$$

4. *Conservation of energy.* The sum of all the energy at a section is the same for all sections. Equation (3.84) for a horizontal passage may be written as follows:

$$h + \frac{v^2}{2g_c} = h_1 + \frac{v_1^2}{2g_c} = h_2 + \frac{v_2^2}{2g_c} \tag{3.96}$$

Derivation of equations

For an ideal gas, equation (3.83) may be written as:

$$\frac{RkT}{k-1} + \frac{v^2}{2g_c} = \frac{RkT_1}{k-1} + \frac{v_1^2}{2g_c} = \frac{RkT_2}{k-1} + \frac{v_2^2}{2g_c} \tag{3.97}$$

Substituting for Mach number from equation (3.94) $[M = v/(kg_c RT)^{1/2}]$ in equation (3.97) and simplifying results in the following:

$$\frac{RKT_1}{k-1} + \frac{M_1^2(kg_c RT_1)}{2g_c} = \frac{RkT_2}{k-1} + \frac{M_2^2(kg_c RT_2)}{2g_c}$$

which reduces to:

$$\frac{T_2}{T_1} = \frac{1 + \left(\frac{k-1}{2}\right) M_1^2}{1 + \left(\frac{k-1}{2}\right) M_2^2} \tag{3.98}$$

Stagnation Condition

The stagnation state exists when the velocity is zero and hence the Mach number is also zero (see Figure 3.10). Let T_0 represent the temperature when $M = 0$ ($v = 0$). T_0 is known as the *stagnation temperature*. Substituting in equation (3.98) T_0 for T_1, T for T_2, and M for M_2 results in the following:

$$\frac{T}{T_0} = \frac{1 + \left(\frac{k-1}{2}\right) 0_1^2}{1 + \left(\frac{k-1}{2}\right) M^2} = \left[1 + \left(\frac{k-1}{2}\right) M^2\right]^{-1} \tag{3.99}$$

Let p_0 represent the pressure when $M = 0$ ($v = 0$). p_0 is the *stagnation pressure*. Substituting the isentropic T–p relationship of equation (1.42) $(p/p_0 = (T/T_0)^{k/(k-1)})$ in equation (3.99) results in the following:

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{k/(k-1)} = \left\{\left[1 + \left(\frac{k-1}{2}\right) M^2\right]^{-1}\right\}^{k/(k-1)} = \left[1 + \left(\frac{k-1}{2}\right) M^2\right]^{k/(1-k)} \tag{3.100}$$

Let ρ_0 represent the density when $M = 0$ ($v = 0$). ρ_0 is the *stagnation density*. The ρ/ρ_0 relationship may be established by noting that the isentropic process of equation (1.33), $pv^k = c$, may be written as a density function, since, from equation (1.15) $\rho = 1/v$, or $\rho/\rho^k = c$. Substituting these values in equation (3.100) results in the following:

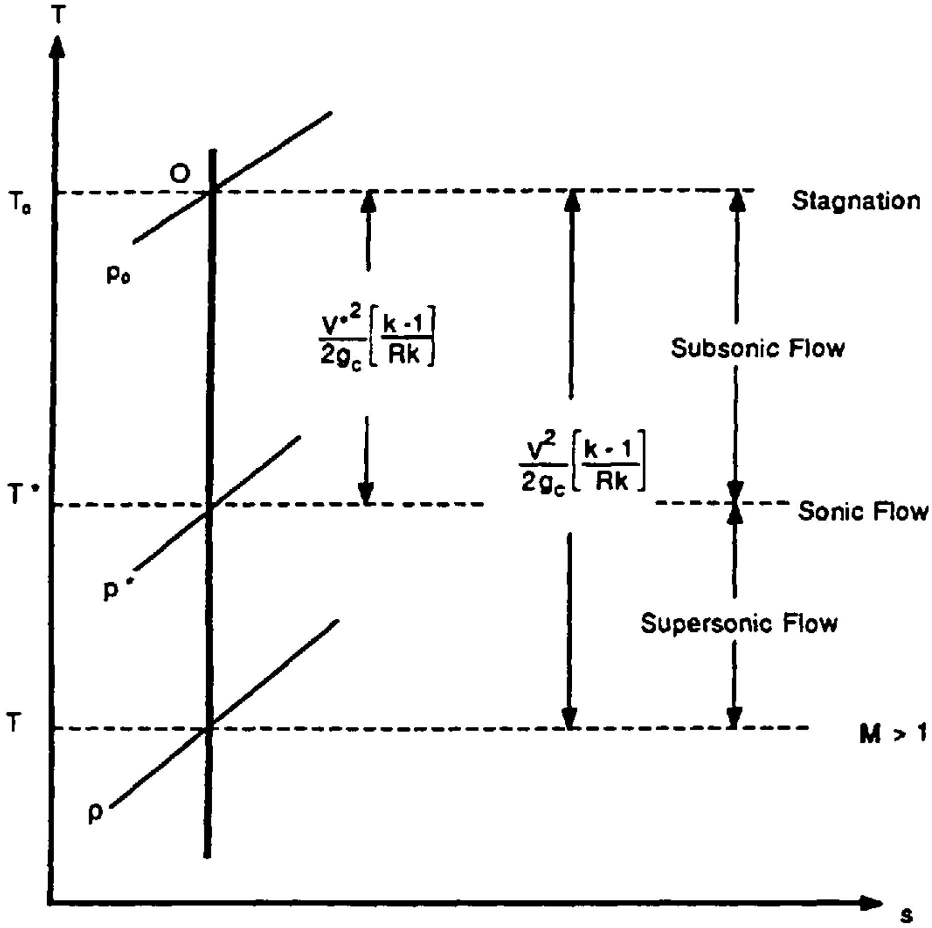


Figure 3.10 Notation for isentropic flow.

$$\frac{p}{p_0} = \left(\frac{p}{p_0}\right)^{1/k} = \left\{\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/(1-k)}\right\}^{1/k} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{1/(1-k)} \tag{3.101}$$

Critical Conditions

Critical conditions exist when the Mach number is unity. Let T^* represent the temperature when $M = 1$, where T^* is the *critical temperature*. Substituting in equation (3.99) T^* for T and M^* for M results in the following:

$$\frac{T^*}{T_0} = \left[1 + \left(\frac{k-1}{2}\right)M^{*2}\right]^{-1} = \left[1 + \left(\frac{k-1}{2}\right)1^2\right]^{-1} = \frac{2}{k+1} \tag{3.102}$$

Let p^* represent the pressure when $M = 1$, where p^* is the *critical pressure*. Substituting in equation (3.100) p^* for p and 1 for M results in the following:

$$\frac{p^*}{p_0} = \left[1 + \left(\frac{k-1}{2}\right)M^{*2}\right]^{k/(1-k)} = \left[1 + \left(\frac{k-1}{2}\right)1^2\right]^{k/(1-k)} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{3.103}$$

Let ρ^* represent the density when M = 1, where ρ^* is the *critical density*. Substituting in equation (3.101) ρ^* for ρ and 1 for M results in the following:

$$\frac{\rho^*}{\rho_0} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/(1-k)} = \left[1 + \left(\frac{k-1}{2}\right)1^2\right]^{k/(1-k)} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{3.104}$$

Let v^* represent the velocity when $M = 1$, where v^* is the *critical velocity*. From equation (3.94), $v = M(kg_cRT)^{1/2}$, so the following can be written:

$$M^* = \frac{v}{v^*} = M\sqrt{\frac{kg_cRT}{kg_cRT^*}} = M\sqrt{\frac{T}{T^*}}\sqrt{\frac{T_0}{T_0}} = M\sqrt{\frac{T}{T_0}}\sqrt{\frac{T_0}{T^*}} \tag{3.105}$$

Substituting from equation (3.99) for T/T_0 and from equation (3.102) for T_0/T^* into (3.105) yields the following:

$$M^* = \frac{v}{v*} = M\sqrt{\left[1 - \frac{(k-1)}{2}M^2\right]^{-1}}\sqrt{\frac{(k+1)}{2}} = M\sqrt{\frac{k+1}{2\left(1 + \frac{k-1}{2}M^2\right)}} \tag{3.106}$$

The *critical area* A^* is obtained by writing the continuity equation, $\rho A v = \rho^* A^* v^*$, as follows:

$$\frac{A}{A^*} = \left(\frac{\rho^*}{\rho}\right)\left(\frac{v^*}{v}\right) = \left(\frac{\rho^*}{\rho}\right)\left(\frac{\rho_0}{\rho_0}\right)\left(\frac{v^*}{v}\right) = \left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\left(\frac{v^*}{v}\right) \tag{3.107}$$

Substituting from equation (3.104) for ρ^*/ρ_0, equation (3.101) for ρ_0/ρ, and equation (3.106) for v^*/v into equation (3.107) results in the following:

$$\frac{A}{A^*} = \left(\frac{2}{k+1}\right)^{1/(k-1)}\left(1 + \frac{k-1}{2}M^2\right)^{1/(k-1)}\frac{1}{M}\sqrt{\frac{2\left(1 + \frac{k-1}{2}M^2\right)}{k+1}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/2(k-1)} \tag{3.108}$$

Note that A/A^* is always greater than 1 and that equation (3.108) has two solutions. For every area ratio except unity, there are two Mach numbers, one subsonic and one supersonic, that will satisfy equation (3.108).

Writing the continuity equation for an ideal gas [$\dot{m} = Avp/RT$] and substituting for T^* from equation (3.102), p^* from equation (3.103), and v^* from equation (3.106) results in the following:

$$\dot{m}^* = \frac{A^* v^* p^*}{RT^*} = \frac{A^* \sqrt{kg_c R\left(\frac{2T_0}{k+1}\right)} p_0 \left(\frac{2}{k+1}\right)^{k/(k-1)}}{R\left(\frac{2T_0}{k+1}\right)} \tag{3.109}$$

$$\dot{m}^* = \frac{A^* p_0}{\sqrt{T_0}} \sqrt{\frac{kg_c}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

In equation (3.109), m^* is the maximum mass flow rate.

Tabulated Values of Isentropic Flow Functions

It has been found useful to compute and tabulate certain standard isentropic functions. These functions are all dimensionless ratios and are functions of the Mach number. Table 3.1 contains the following ratios:

Function	*Equation*
$M^* = \frac{v}{v^*}$	3.106
$\frac{A}{A^*}$	3.108
$\frac{T}{T_0}$	3.99
$\frac{p}{p_0}$	3.100
$\frac{\rho}{\rho_0}$	3.101

In applying this table it should be noted that all data are based on the assumption that the gas is ideal and that the molecular weight, specific heats, and ratios of specific heats are constant. Table D.2 (see Appendix D) gives values of k for ideal gases as a function of temperature. When the temperature range is known before calculations, the average value of k

Table 3.1 Isentropic Flow Functions

		k = 1			
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.000E-02	6.066E+01	1.000E+00	1.000E+00	1.000E+00
0.02	2.000E-02	3.033E+01	1.000E+00	9.998E-01	9.998E-01
0.03	3.000E-02	2.023E+01	1.000E+00	9.996E-01	9.996E-01
0.04	4.000E-02	1.518E+01	1.000E+00	9.992E-01	9.992E-01
0.05	5.000E-02	1.215E+01	1.000E+00	9.988E-01	9.988E-01
0.06	6.000E-02	1.013E+01	1.000E+00	9.982E-01	9.982E-01
0.07	7.000E-02	8.686E+00	1.000E+00	9.976E-01	9.976E-01
0.08	8.000E-02	7.606E+00	1.000E+00	9.968E-01	9.968E-01
0.09	9.000E-02	6.767E+00	1.000E+00	9.960E-01	9.960E-01
0.10	1.000E-01	6.096E+00	1.000E+00	9.950E-01	9.950E-01
0.15	1.500E-01	4.089E+00	1.000E+00	9.888E-01	9.888E-01
0.20	2.000E-01	3.094E+00	1.000E+00	9.802E-01	9.802E-01
0.25	2.500E-01	2.503E+00	1.000E+00	9.692E-01	9.692E-01
0.30	3.000E-01	2.115E+00	1.000E+00	9.560E-01	9.560E-01
0.35	3.500E-01	1.842E+00	1.000E+00	9.406E-01	9.406E-01
0.40	4.000E-01	1.643E+00	1.000E+00	9.231E-01	9.231E-01
0.45	4.500E-01	1.491E+00	1.000E+00	9.037E-01	9.037E-01
0.50	5.000E-01	1.375E+00	1.000E+00	8.825E-01	8.825E-01
0.60	6.000E-01	1.210E+00	1.000E+00	8.353E-01	8.353E-01
0.70	7.000E-01	1.107E+00	1.000E+00	7.827E-01	7.827E-01
0.80	8.000E-01	1.044E+00	1.000E+00	7.261E-01	7.261E-01
0.90	9.000E-01	1.010E+00	1.000E+00	6.670E-01	6.670E-01
1.00	1.000E+00	1.000E+00	1.000E+00	6.065E-01	6.065E-01
1.10	1.100E+00	1.010E+00	1.000E+00	5.461E-01	5.461E-01
1.20	1.200E+00	1.038E+00	1.000E+00	4.868E-01	4.868E-01
1.30	1.300E+00	1.086E+00	1.000E+00	4.296E-01	4.296E-01
1.40	1.400E+00	1.154E+00	1.000E+00	3.753E-01	3.753E-01
1.50	1.500E+00	1.245E+00	1.000E+00	3.247E-01	3.247E-01
1.60	1.600E+00	1.363E+00	1.000E+00	2.780E-01	2.780E-01
1.70	1.700E+00	1.513E+00	1.000E+00	2.357E-01	2.357E-01
1.80	1.800E+00	1.703E+00	1.000E+00	1.979E-01	1.979E-01
1.90	1.900E+00	1.941E+00	1.000E+00	1.645E-01	1.645E-01

Table 3.1 (*continued*) Isentropic Flow Functions

			k = 1		
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
2.00	2.000E+00	2.241E+00	1.000E+00	1.353E-01	1.353E-01
2.10	2.100E+00	2.620E+00	1.000E+00	1.103E-01	1.103E-01
2.20	2.200E+00	3.100E+00	1.000E+00	8.892E-02	8.892E-02
2.30	2.300E+00	3.714E+00	1.000E+00	7.101E-02	7.101E-02
2.40	2.400E+00	4.502E+00	1.000E+00	5.613E-02	5.613E-02
2.50	2.500E+00	5.522E+00	1.000E+00	4.394E-02	4.394E-02
2.60	2.600E+00	6.852E+00	1.000E+00	3.405E-02	3.405E-02
2.70	2.700E+00	8.600E+00	1.000E+00	2.612E-02	2.612E-02
2.80	2.800E+00	1.092E+01	1.000E+00	1.984E-02	1.984E-02
2.90	2.900E+00	1.402E+01	1.000E+00	1.492E-02	1.492E-02
3.00	3.000E+00	1.820E+01	1.000E+00	1.111E-02	1.111E-02
3.10	3.100E+00	2.389E+01	1.000E+00	8.189E-03	8.189E-03
3.20	3.200E+00	3.172E+01	1.000E+00	5.976E-03	5.976E-03
3.30	3.300E+00	4.257E+01	1.000E+00	4.318E-03	4.318E-03
3.40	3.400E+00	5.776E+01	1.000E+00	3.089E-03	3.089E-03
3.50	3.500E+00	7.922E+01	1.000E+00	2.187E-03	2.187E-03
3.60	3.600E+00	1.098E+02	1.000E+00	1.534E-03	1.534E-03
3.70	3.700E+00	1.540E+02	1.000E+00	1.065E-03	1.065E-03
3.80	3.800E+00	2.181E+02	1.000E+00	7.318E-04	7.318E-04
3.90	3.900E+00	3.123E+02	1.000E+00	4.980E-04	4.980E-04
4.00	4.000E+00	4.520E+02	1.000E+00	3.355E-04	3.355E-04
4.50	4.500E+00	3.364E+03	1.000E+00	4.007E-05	4.007E-05
5.00	5.000E+00	3.255E+04	1.000E+00	3.727E-06	3.727E-06
5.50	5.500E+00	4.085E+05	1.000E+00	2.700E-07	2.700E-07
6.00	6.000E+00	6.637E+06	1.000E+00	1.523E-08	1.523E-08
6.50	6.500E+00	1.394E+08	1.000E+00	6.692E-10	6.692E-10
7.00	7.000E+00	3.784E+09	1.000E+00	2.290E-11	2.290E-11
7.50	7.500E+00	1.325E+11	1.000E+00	6.102E-13	6.102E-13
8.00	8.000E+00	5.987E+12	1.000E+00	1.266E-14	1.266E-14
9.00	9.000E+00	2.615E+16	1.000E+00	2.577E-18	2.577E-18
10	1.000E+01	3.145E+20	1.000E+00	1.929E-22	1.929E-22
20	2.000E+01	2.191E+85	1.000E+00	1.384E-87	1.384E-87
30	3.000E+01	5.473E+193	1.000E+00	3.694E-196	3.694E-196

		k = 1.1			
M	M* = v*/v	A/A*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.025E-02	5.991E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.049E-02	2.996E+01	1.000E+00	9.998E-01	9.998E-01
0.03	3.074E-02	1.998E+01	1.000E+00	9.995E-01	9.996E-01
0.04	4.099E-02	1.499E+01	9.999E-01	9.991E-01	9.992E-01
0.05	5.123E-02	1.200E+01	9.999E-01	9.986E-01	9.988E-01
0.06	6.148E-02	1.000E+01	9.998E-01	9.980E-01	9.982E-01
0.07	7.172E-02	8.581E+00	9.998E-01	9.973E-01	9.976E-01
0.08	8.196E-02	7.514E+00	9.997E-01	9.965E-01	9.968E-01
0.09	9.220E-02	6.685E+00	9.996E-01	9.956E-01	9.960E-01
0.10	1.024E-01	6.023E+00	9.995E-01	9.945E-01	9.950E-01
0.15	1.536E-01	4.042E+00	9.989E-01	9.877E-01	9.888E-01
0.20	2.047E-01	3.059E+00	9.980E-01	9.783E-01	9.802E-01
0.25	2.558E-01	2.476E+00	9.969E-01	9.663E-01	9.693E-01
0.30	3.067E-01	2.093E+00	9.955E-01	9.518E-01	9.561E-01
0.35	3.575E-01	1.825E+00	9.939E-01	9.350E-01	9.408E-01
0.40	4.082E-01	1.629E+00	9.921E-01	9.161E-01	9.234E-01
0.45	4.588E-01	1.480E+00	9.900E-01	8.951E-01	9.042E-01
0.50	5.092E-01	1.365E+00	9.877E-01	8.723E-01	8.832E-01
0.60	6.094E-01	1.204E+00	9.823E-01	8.218E-01	8.366E-01
0.70	7.087E-01	1.104E+00	9.761E-01	7.662E-01	7.850E-01
0.80	8.069E-01	1.042E+00	9.690E-01	7.072E-01	7.298E-01
0.90	9.041E-01	1.010E+00	9.611E-01	6.462E-01	6.723E-01
1.00	1.000E+00	1.000E+00	9.524E-01	5.847E-01	6.139E-01
1.10	1.095E+00	1.009E+00	9.430E-01	5.241E-01	5.558E-01
1.20	1.188E+00	1.036E+00	9.328E-01	4.654E-01	4.989E-01
1.30	1.279E+00	1.080E+00	9.221E-01	4.097E-01	4.443E-01
1.40	1.369E+00	1.142E+00	9.107E-01	3.576E-01	3.926E-01
1.50	1.457E+00	1.223E+00	8.989E-01	3.095E-01	3.443E-01
1.60	1.544E+00	1.326E+00	8.865E-01	2.658E-01	2.999E-01
1.70	1.628E+00	1.454E+00	8.737E-01	2.266E-01	2.593E-01
1.80	1.711E+00	1.610E+00	8.606E-01	1.917E-01	2.228E-01
1.90	1.792E+00	1.801E+00	8.471E-01	1.612E-01	1.903E-01

Table 3.1 (*continued*) Isentropic Flow Functions

M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
		k = 1.1			
2.00	1.871E+00	2.032E+00	8.333E-01	1.346E-01	1.615E-01
2.10	1.948E+00	2.312E+00	8.193E-01	1.117E-01	1.363E-01
2.20	2.023E+00	2.651E+00	8.052E-01	9.219E-02	1.145E-01
2.30	2.096E+00	3.061E+00	7.908E-01	7.566E-02	9.568E-02
2.40	2.167E+00	3.560E+00	7.764E-01	6.179E-02	7.959E-02
2.50	2.236E+00	4.165E+00	7.619E-01	5.022E-02	6.592E-02
2.60	2.303E+00	4.901E+00	7.474E-01	4.064E-02	5.438E-02
2.70	2.368E+00	5.799E+00	7.329E-01	3.276E-02	4.470E-02
2.80	2.432E+00	6.896E+00	7.184E-01	2.630E-02	3.661E-02
2.90	2.493E+00	8.237E+00	7.040E-01	2.104E-02	2.989E-02
3.00	2.553E+00	9.880E+00	6.897E-01	1.679E-02	2.434E-02
3.10	2.611E+00	1.190E+01	6.754E-01	1.335E-02	1.977E-02
3.20	2.667E+00	1.438E+01	6.614E-01	1.059E-02	1.601E-02
3.30	2.721E+00	1.743E+01	6.475E-01	8.382E-03	1.295E-02
3.40	2.773E+00	2.119E+01	6.337E-01	6.619E-03	1.045E-02
3.50	2.824E+00	2.583E+01	6.202E-01	5.218E-03	8.414E-03
3.60	2.874E+00	3.157E+01	6.068E-01	4.106E-03	6.767E-03
3.70	2.921E+00	3.866E+01	5.936E-01	3.227E-03	5.436E-03
3.80	2.967E+00	4.743E+01	5.807E-01	2.533E-03	4.362E-03
3.90	3.012E+00	5.829E+01	5.680E-01	1.986E-03	3.497E-03
4.00	3.055E+00	7.175E+01	5.556E-01	1.556E-03	2.801E-03
4.50	3.250E+00	2.058E+02	4.969E-01	4.559E-04	9.176E-04
5.00	3.416E+00	5.977E+02	4.444E-01	1.337E-04	3.007E-04
5.50	3.556E+00	1.731E+03	3.980E-01	3.970E-05	9.976E-05
6.00	3.674E+00	4.949E+03	3.571E-01	1.206E-05	3.376E-05
6.50	3.775E+00	1.388E+04	3.213E-01	3.765E-06	1.172E-05
7.00	3.862E+00	3.798E+04	2.899E-01	1.213E-06	4.186E-06
7.50	3.936E+00	1.012E+05	2.623E-01	4.043E-07	1.541E-06
8.00	4.000E+00	2.621E+05	2.381E-01	1.394E-07	5.855E-07
9.00	4.104E+00	1.614E+06	1.980E-01	1.836E-08	9.270E-08
10	4.183E+00	8.874E+06	1.667E-01	2.756E-09	1.654E-08
20	4.472E+00	2.290E+12	4.762E-02	2.855E-15	5.995E-14
30	4.532E+00	5.746E+15	2.174E-02	5.125E-19	2.357E-17

		k = 1.2			
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.049E-02	5.921E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.098E-02	2.961E+01	1.000E+00	9.998E-01	9.998E-01
0.03	3.146E-02	1.974E+01	9.999E-01	9.995E-01	9.996E-01
0.04	4.195E-02	1.481E+01	9.998E-01	9.990E-01	9.992E-01
0.05	5.243E-02	1.186E+01	9.998E-01	9.985E-01	9.988E-01
0.06	6.292E-02	9.887E+00	9.996E-01	9.978E-01	9.982E-01
0.07	7.340E-02	8.480E+00	9.995E-01	9.971E-01	9.976E-01
0.08	8.388E-02	7.426E+00	9.994E-01	9.962E-01	9.968E-01
0.09	9.435E-02	6.607E+00	9.992E-01	9.952E-01	9.960E-01
0.10	1.048E-01	5.953E+00	9.990E-01	9.940E-01	9.950E-01
0.15	1.571E-01	3.996E+00	9.978E-01	9.866E-01	9.888E-01
0.20	2.093E-01	3.026E+00	9.960E-01	9.763E-01	9.802E-01
0.25	2.614E-01	2.451E+00	9.938E-01	9.633E-01	9.693E-01
0.30	3.132E-01	2.073E+00	9.911E-01	9.477E-01	9.562E-01
0.35	3.649E-01	1.809E+00	9.879E-01	9.296E-01	9.409E-01
0.40	4.162E-01	1.615E+00	9.843E-01	9.092E-01	9.237E-01
0.45	4.673E-01	1.469E+00	9.802E-01	8.867E-01	9.046E-01
0.50	5.180E-01	1.356E+00	9.756E-01	8.623E-01	8.839E-01
0.60	6.183E-01	1.199E+00	9.653E-01	8.088E-01	8.379E-01
0.70	7.168E-01	1.100E+00	9.533E-01	7.505E-01	7.873E-01
0.80	8.134E-01	1.041E+00	9.398E-01	6.892E-01	7.333E-01
0.90	9.079E-01	1.010E+00	9.251E-01	6.267E-01	6.774E-01
1.00	1.000E+00	1.000E+00	9.091E-01	5.645E-01	6.209E-01
1.10	1.090E+00	1.009E+00	8.921E-01	5.039E-01	5.649E-01
1.20	1.177E+00	1.034E+00	8.741E-01	4.461E-01	5.104E-01
1.30	1.261E+00	1.075E+00	8.554E-01	3.918E-01	4.581E-01
1.40	1.343E+00	1.132E+00	8.361E-01	3.417E-01	4.086E-01
1.50	1.421E+00	1.205E+00	8.163E-01	2.959E-01	3.625E-01
1.60	1.497E+00	1.296E+00	7.962E-01	2.547E-01	3.199E-01
1.70	1.570E+00	1.407E+00	7.758E-01	2.180E-01	2.810E-01
1.80	1.641E+00	1.540E+00	7.553E-01	1.856E-01	2.458E-01
1.90	1.708E+00	1.697E+00	7.348E-01	1.573E-01	2.141E-01

Table 3.1 (*continued*) Isentropic Flow Functions

k = 1.2

M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	r/r_o
2.00	1.773E+00	1.884E+00	7.143E-01	1.328E-01	1.859E-01
2.10	1.835E+00	2.103E+00	6.940E-01	1.117E-01	1.609E-01
2.20	1.894E+00	2.359E+00	6.739E-01	9.363E-02	1.389E-01
2.30	1.951E+00	2.660E+00	6.540E-01	7.826E-02	1.197E-01
2.40	2.005E+00	3.011E+00	6.345E-01	6.526E-02	1.029E-01
2.50	2.057E+00	3.421E+00	6.154E-01	5.431E-02	8.825E-02
2.60	2.106E+00	3.898E+00	5.967E-01	4.512E-02	7.562E-02
2.70	2.154E+00	4.455E+00	5.784E-01	3.743E-02	6.472E-02
2.80	2.199E+00	5.103E+00	5.605E-01	3.102E-02	5.534E-02
2.90	2.242E+00	5.858E+00	5.432E-01	2.568E-02	4.729E-02
3.00	2.283E+00	6.735E+00	5.263E-01	2.126E-02	4.039E-02
3.10	2.322E+00	7.755E+00	5.099E-01	1.758E-02	3.448E-02
3.20	2.359E+00	8.940E+00	4.941E-01	1.455E-02	2.944E-02
3.30	2.395E+00	1.032E+01	4.787E-01	1.203E-02	2.514E-02
3.40	2.429E+00	1.191E+01	4.638E-01	9.957E-03	2.147E-02
3.50	2.461E+00	1.376E+01	4.494E-01	8.242E-03	1.834E-02
3.60	2.492E+00	1.590E+01	4.355E-01	6.826E-03	1.567E-02
3.70	2.521E+00	1.838E+01	4.221E-01	5.657E-03	1.340E-02
3.80	2.549E+00	2.124E+01	4.092E-01	4.692E-03	1.147E-02
3.90	2.576E+00	2.454E+01	3.967E-01	3.895E-03	9.821E-03
4.00	2.602E+00	2.836E+01	3.846E-01	3.237E-03	8.417E-03
4.50	2.714E+00	5.796E+01	3.306E-01	1.305E-03	3.948E-03
5.00	2.803E+00	1.163E+02	2.857E-01	5.440E-04	1.904E-03
5.50	2.875E+00	2.281E+02	2.484E-01	2.352E-04	9.466E-04
6.00	2.934E+00	4.359E+02	2.174E-01	1.055E-04	4.855E-04
6.50	2.982E+00	8.108E+02	1.914E-01	4.915E-05	2.568E-04
7.00	3.023E+00	1.469E+03	1.695E-01	2.371E-05	1.399E-04
7.50	3.056E+00	2.593E+03	1.509E-01	1.183E-05	7.836E-05
8.00	3.084E+00	4.467E+03	1.351E-01	6.090E-06	4.507E-05
9.00	3.129E+00	1.238E+04	1.099E-01	1.761E-06	1.602E-05
10	3.162E+00	3.162E+04	9.091E-02	5.645E-07	6.209E-06
20	3.276E+00	2.196E+07	2.439E-02	2.105E-10	8.631E-09
30	3.298E+00	1.175E+09	1.099E-02	1.761E-12	1.602E-10

		k = 1.3			
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.072E-02	5.853E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.145E-02	2.927E+01	9.999E-01	9.997E-01	9.998E-01
0.03	3.217E-02	1.952E+01	9.999E-01	9.994E-01	9.996E-01
0.04	4.289E-02	1.464E+01	9.998E-01	9.990E-01	9.992E-01
0.05	5.361E-02	1.172E+01	9.996E-01	9.984E-01	9.988E-01
0.06	6.433E-02	9.774E+00	9.995E-01	9.977E-01	9.982E-01
0.07	7.504E-02	8.384E+00	9.993E-01	9.968E-01	9.976E-01
0.08	8.575E-02	7.342E+00	9.990E-01	9.959E-01	9.968E-01
0.09	9.646E-02	6.533E+00	9.988E-01	9.948E-01	9.960E-01
0.10	1.072E-01	5.886E+00	9.985E-01	9.935E-01	9.950E-01
0.15	1.606E-01	3.952E+00	9.966E-01	9.855E-01	9.888E-01
0.20	2.138E-01	2.994E+00	9.940E-01	9.744E-01	9.803E-01
0.25	2.668E-01	2.426E+00	9.907E-01	9.604E-01	9.694E-01
0.30	3.196E-01	2.054E+00	9.867E-01	9.435E-01	9.563E-01
0.35	3.719E-01	1.793E+00	9.820E-01	9.241E-01	9.411E-01
0.40	4.239E-01	1.602E+00	9.766E-01	9.023E-01	9.240E-01
0.45	4.754E-01	1.459E+00	9.705E-01	8.784E-01	9.051E-01
0.50	5.264E-01	1.348E+00	9.639E-01	8.525E-01	8.845E-01
0.60	6.267E-01	1.193E+00	9.488E-01	7.962E-01	8.392E-01
0.70	7.245E-01	1.097E+00	9.315E-01	7.354E-01	7.895E-01
0.80	8.195E-01	1.040E+00	9.124E-01	6.722E-01	7.367E-01
0.90	9.114E-01	1.009E+00	8.917E-01	6.084E-01	6.823E-01
1.00	1.000E+00	1.000E+00	8.696E-01	5.457E-01	6.276E-01
1.10	1.085E+00	1.008E+00	8.464E-01	4.854E-01	5.735E-01
1.20	1.167E+00	1.032E+00	8.224E-01	4.285E-01	5.211E-01
1.30	1.245E+00	1.070E+00	7.978E-01	3.757E-01	4.709E-01
1.40	1.320E+00	1.123E+00	7.728E-01	3.273E-01	4.235E-01
1.50	1.391E+00	1.189E+00	7.477E-01	2.836E-01	3.793E-01
1.60	1.458E+00	1.271E+00	7.225E-01	2.446E-01	3.385E-01
1.70	1.523E+00	1.369E+00	6.976E-01	2.100E-01	3.011E-01
1.80	1.583E+00	1.484E+00	6.729E-01	1.797E-01	2.671E-01
1.90	1.641E+00	1.618E+00	6.487E-01	1.533E-01	2.363E-01

Table 3.1 (*continued*) Isentropic Flow Functions

k = 1.3

M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
2.00	1.696E+00	1.773E+00	6.250E-01	1.305E-01	2.087E-01
2.10	1.747E+00	1.951E+00	6.019E-01	1.108E-01	1.841E-01
2.20	1.796E+00	2.156E+00	5.794E-01	9.393E-02	1.621E-01
2.30	1.842E+00	2.388E+00	5.576E-01	7.955E-02	1.427E-01
2.40	1.885E+00	2.654E+00	5.365E-01	6.731E-02	1.255E-01
2.50	1.926E+00	2.954E+00	5.161E-01	5.692E-02	1.103E-01
2.60	1.965E+00	3.295E+00	4.965E-01	4.813E-02	9.693E-02
2.70	2.001E+00	3.681E+00	4.777E-01	4.070E-02	8.520E-02
2.80	2.036E+00	4.116E+00	4.596E-01	3.442E-02	7.490E-02
2.90	2.068E+00	4.607E+00	4.422E-01	2.913E-02	6.587E-02
3.00	2.099E+00	5.160E+00	4.255E-01	2.466E-02	5.796E-02
3.10	2.128E+00	5.781E+00	4.096E-01	2.090E-02	5.103E-02
3.20	2.155E+00	6.478E+00	3.943E-01	1.773E-02	4.496E-02
3.30	2.181E+00	7.259E+00	3.797E-01	1.506E-02	3.965E-02
3.40	2.205E+00	8.133E+00	3.658E-01	1.280E-02	3.499E-02
3.50	2.228E+00	9.110E+00	3.524E-01	1.090E-02	3.092E-02
3.60	2.250E+00	1.020E+01	3.397E-01	9.288E-03	2.734E-02
3.70	2.271E+00	1.142E+01	3.275E-01	7.929E-03	2.421E-02
3.80	2.290E+00	1.277E+01	3.159E-01	6.778E-03	2.146E-02
3.90	2.309E+00	1.427E+01	3.047E-01	5.803E-03	1.904E-02
4.00	2.326E+00	1.594E+01	2.941E-01	4.977E-03	1.692E-02
4.50	2.402E+00	2.739E+01	2.477E-01	2.363E-03	9.542E-03
5.00	2.460E+00	4.596E+01	2.105E-01	1.169E-03	5.551E-03
5.50	2.506E+00	7.522E+01	1.806E-01	6.011E-04	3.329E-03
6.00	2.543E+00	1.201E+02	1.563E-01	3.210E-04	2.055E-03
6.50	2.573E+00	1.872E+02	1.363E-01	1.775E-04	1.303E-03
7.00	2.598E+00	2.853E+02	1.198E-01	1.014E-04	8.467E-04
7.50	2.618E+00	4.258E+02	1.060E-01	5.965E-05	5.630E-04
8.00	2.635E+00	6.231E+02	9.434E-02	3.606E-05	3.822E-04
9.00	2.662E+00	1.266E+03	7.605E-02	1.417E-05	1.863E-04
10	2.681E+00	2.416E+03	6.250E-02	6.055E-06	9.689E-05
20	2.746E+00	2.042E+05	1.639E-02	1.835E-08	1.119E-06
30	2.759E+00	2.943E+06	7.353E-03	5.684E-10	7.730E-08

		k = 1.4			
M	M* = v*/v	A/A*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.095E-02	5.787E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.191E-02	2.894E+01	9.999E-01	9.997E-01	9.998E-01
0.03	3.286E-02	1.930E+01	9.998E-01	9.994E-01	9.996E-01
0.04	4.381E-02	1.448E+01	9.997E-01	9.989E-01	9.992E-01
0.05	5.476E-02	1.159E+01	9.995E-01	9.983E-01	9.988E-01
0.06	6.570E-02	9.666E+00	9.993E-01	9.975E-01	9.982E-01
0.07	7.664E-02	8.292E+00	9.990E-01	9.966E-01	9.976E-01
0.08	8.758E-02	7.262E+00	9.987E-01	9.955E-01	9.968E-01
0.09	9.851E-02	6.461E+00	9.984E-01	9.944E-01	9.960E-01
0.10	1.094E-01	5.822E+00	9.980E-01	9.930E-01	9.950E-01
0.15	1.639E-01	3.910E+00	9.955E-01	9.844E-01	9.888E-01
0.20	2.182E-01	2.964E+00	9.921E-01	9.725E-01	9.803E-01
0.25	2.722E-01	2.403E+00	9.877E-01	9.575E-01	9.694E-01
0.30	3.257E-01	2.035E+00	9.823E-01	9.395E-01	9.564E-01
0.35	3.788E-01	1.778E+00	9.761E-01	9.188E-01	9.413E-01
0.40	4.313E-01	1.590E+00	9.690E-01	8.956E-01	9.243E-01
0.45	4.833E-01	1.449E+00	9.611E-01	8.703E-01	9.055E-01
0.50	5.345E-01	1.340E+00	9.524E-01	8.430E-01	8.852E-01
0.60	6.348E-01	1.188E+00	9.328E-01	7.840E-01	8.405E-01
0.70	7.318E-01	1.094E+00	9.107E-01	7.209E-01	7.916E-01
0.80	8.251E-01	1.038E+00	8.865E-01	6.560E-01	7.400E-01
0.90	9.146E-01	1.009E+00	8.606E-01	5.913E-01	6.870E-01
1.00	1.000E+00	1.000E+00	8.333E-01	5.283E-01	6.339E-01
1.10	1.081E+00	1.008E+00	8.052E-01	4.684E-01	5.817E-01
1.20	1.158E+00	1.030E+00	7.764E-01	4.124E-01	5.311E-01
1.30	1.231E+00	1.066E+00	7.474E-01	3.609E-01	4.829E-01
1.40	1.300E+00	1.115E+00	7.184E-01	3.142E-01	4.374E-01
1.50	1.365E+00	1.176E+00	6.897E-01	2.724E-01	3.950E-01
1.60	1.425E+00	1.250E+00	6.614E-01	2.353E-01	3.557E-01
1.70	1.482E+00	1.338E+00	6.337E-01	2.026E-01	3.197E-01
1.80	1.536E+00	1.439E+00	6.068E-01	1.740E-01	2.868E-01
1.90	1.586E+00	1.555E+00	5.807E-01	1.492E-01	2.570E-01

Table 3.1 (*continued*) Isentropic Flow Functions

			k = 1.4		
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
2.00	1.633E+00	1.688E+00	5.556E-01	1.278E-01	2.300E-01
2.10	1.677E+00	1.837E+00	5.313E-01	1.094E-01	2.058E-01
2.20	1.718E+00	2.005E+00	5.081E-01	9.352E-02	1.841E-01
2.30	1.756E+00	2.193E+00	4.859E-01	7.997E-02	1.646E-01
2.40	1.792E+00	2.403E+00	4.647E-01	6.840E-02	1.472E-01
2.50	1.826E+00	2.637E+00	4.444E-01	5.853E-02	1.317E-01
2.60	1.857E+00	2.896E+00	4.252E-01	5.012E-02	1.179E-01
2.70	1.887E+00	3.183E+00	4.068E-01	4.295E-02	1.056E-01
2.80	1.914E+00	3.500E+00	3.894E-01	3.685E-02	9.463E-02
2.90	1.940E+00	3.850E+00	3.729E-01	3.165E-02	8.489E-02
3.00	1.964E+00	4.235E+00	3.571E-01	2.722E-02	7.623E-02
3.10	1.987E+00	4.657E+00	3.422E-01	2.345E-02	6.852E-02
3.20	2.008E+00	5.121E+00	3.281E-01	2.023E-02	6.165E-02
3.30	2.028E+00	5.629E+00	3.147E-01	1.748E-02	5.554E-02
3.40	2.047E+00	6.184E+00	3.019E-01	1.512E-02	5.009E-02
3.50	2.064E+00	6.790E+00	2.899E-01	1.311E-02	4.523E-02
3.60	2.081E+00	7.450E+00	2.784E-01	1.138E-02	4.089E-02
3.70	2.096E+00	8.169E+00	2.675E-01	9.903E-03	3.702E-02
3.80	2.111E+00	8.951E+00	2.572E-01	8.629E-03	3.355E-02
3.90	2.125E+00	9.799E+00	2.474E-01	7.532E-03	3.044E-02
4.00	2.138E+00	1.072E+01	2.381E-01	6.586E-03	2.766E-02
4.50	2.194E+00	1.656E+01	1.980E-01	3.455E-03	1.745E-02
5.00	2.236E+00	2.500E+01	1.667E-01	1.890E-03	1.134E-02
5.50	2.269E+00	3.687E+01	1.418E-01	1.075E-03	7.578E-03
6.00	2.295E+00	5.318E+01	1.220E-01	6.334E-04	5.194E-03
6.50	2.316E+00	7.513E+01	1.058E-01	3.855E-04	3.643E-03
7.00	2.333E+00	1.041E+02	9.259E-02	2.416E-04	2.609E-03
7.50	2.347E+00	1.418E+02	8.163E-02	1.554E-04	1.904E-03
8.00	2.359E+00	1.901E+02	7.246E-02	1.024E-04	1.414E-03
9.00	2.377E+00	3.272E+02	5.814E-02	4.739E-05	8.150E-04
10	2.390E+00	5.359E+02	4.762E-02	2.356E-05	4.948E-04
20	2.434E+00	1.538E+04	1.235E-02	2.091E-07	1.694E-05
30	2.443E+00	1.144E+05	5.525E-03	1.254E-08	2.269E-06

			k = 1.5		
M	M* = v*/v	A/A*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.118E-02	5.725E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.236E-02	2.863E+01	9.999E-01	9.997E-01	9.998E-01
0.03	3.354E-02	1.909E+01	9.998E-01	9.993E-01	9.996E-01
0.04	4.471E-02	1.433E+01	9.996E-01	9.988E-01	9.992E-01
0.05	5.588E-02	1.147E+01	9.994E-01	9.981E-01	9.988E-01
0.06	6.705E-02	9.562E+00	9.991E-01	9.973E-01	9.982E-01
0.07	7.821E-02	8.203E+00	9.988E-01	9.963E-01	9.976E-01
0.08	8.937E-02	7.184E+00	9.984E-01	9.952E-01	9.968E-01
0.09	1.005E-01	6.393E+00	9.980E-01	9.939E-01	9.960E-01
0.10	1.117E-01	5.760E+00	9.975E-01	9.925E-01	9.950E-01
0.15	1.672E-01	3.870E+00	9.944E-01	9.833E-01	9.888E-01
0.20	2.225E-01	2.934E+00	9.901E-01	9.706E-01	9.803E-01
0.25	2.774E-01	2.380E+00	9.846E-01	9.546E-01	9.695E-01
0.30	3.317E-01	2.017E+00	9.780E-01	9.354E-01	9.565E-01
0.35	3.855E-01	1.764E+00	9.703E-01	9.135E-01	9.415E-01
0.40	4.385E-01	1.579E+00	9.615E-01	8.890E-01	9.246E-01
0.45	4.908E-01	1.439E+00	9.518E-01	8.623E-01	9.060E-01
0.50	5.423E-01	1.332E+00	9.412E-01	8.337E-01	8.858E-01
0.60	6.425E-01	1.183E+00	9.174E-01	7.722E-01	8.417E-01
0.70	7.387E-01	1.092E+00	8.909E-01	7.070E-01	7.936E-01
0.80	8.305E-01	1.037E+00	8.621E-01	6.407E-01	7.432E-01
0.90	9.176E-01	1.009E+00	8.316E-01	5.751E-01	6.916E-01
1.00	1.000E+00	1.000E+00	8.000E-01	5.120E-01	6.400E-01
1.10	1.078E+00	1.008E+00	7.678E-01	4.526E-01	5.894E-01
1.20	1.150E+00	1.029E+00	7.353E-01	3.975E-01	5.407E-01
1.30	1.219E+00	1.063E+00	7.030E-01	3.474E-01	4.942E-01
1.40	1.282E+00	1.108E+00	6.711E-01	3.023E-01	4.504E-01
1.50	1.342E+00	1.165E+00	6.400E-01	2.621E-01	4.096E-01
1.60	1.397E+00	1.232E+00	6.098E-01	2.267E-01	3.718E-01
1.70	1.448E+00	1.311E+00	5.806E-01	1.957E-01	3.370E-01
1.80	1.496E+00	1.402E+00	5.525E-01	1.686E-01	3.052E-01
1.90	1.540E+00	1.504E+00	5.256E-01	1.452E-01	2.763E-01

Table 3.1 (*continued*) Isentropic Flow Functions

k = 1.5

M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
2.00	1.581E+00	1.619E+00	5.000E-01	1.250E-01	2.500E-01
2.10	1.619E+00	1.747E+00	4.756E-01	1.076E-01	2.262E-01
2.20	1.655E+00	1.889E+00	4.525E-01	9.265E-02	2.047E-01
2.30	1.687E+00	2.046E+00	4.306E-01	7.982E-02	1.854E-01
2.40	1.718E+00	2.218E+00	4.098E-01	6.884E-02	1.680E-01
2.50	1.746E+00	2.407E+00	3.902E-01	5.943E-02	1.523E-01
2.60	1.772E+00	2.613E+00	3.717E-01	5.137E-02	1.382E-01
2.70	1.797E+00	2.838E+00	3.543E-01	4.447E-02	1.255E-01
2.80	1.820E+00	3.082E+00	3.378E-01	3.856E-02	1.141E-01
2.90	1.841E+00	3.347E+00	3.223E-01	3.349E-02	1.039E-01
3.00	1.861E+00	3.633E+00	3.077E-01	2.913E-02	9.467E-02
3.10	1.879E+00	3.943E+00	2.939E-01	2.539E-02	8.638E-02
3.20	1.896E+00	4.278E+00	2.809E-01	2.216E-02	7.890E-02
3.30	1.912E+00	4.638E+00	2.686E-01	1.939E-02	7.217E-02
3.40	1.927E+00	5.025E+00	2.571E-01	1.699E-02	6.608E-02
3.50	1.941E+00	5.441E+00	2.462E-01	1.491E-02	6.059E-02
3.60	1.955E+00	5.886E+00	2.358E-01	1.312E-02	5.562E-02
3.70	1.967E+00	6.363E+00	2.261E-01	1.156E-02	5.113E-02
3.80	1.979E+00	6.874E+00	2.169E-01	1.021E-02	4.705E-02
3.90	1.990E+00	7.419E+00	2.082E-01	9.028E-03	4.336E-02
4.00	2.000E+00	8.000E+00	2.000E-01	8.000E-03	4.000E-02
4.50	2.043E+00	1.151E+01	1.649E-01	4.488E-03	2.721E-02
5.00	2.076E+00	1.620E+01	1.379E-01	2.624E-03	1.902E-02
5.50	2.101E+00	2.233E+01	1.168E-01	1.593E-03	1.364E-02
6.00	2.121E+00	3.017E+01	1.000E-01	1.000E-03	1.000E-02
6.50	2.137E+00	4.004E+01	8.649E-02	6.469E-04	7.480E-03
7.00	2.150E+00	5.226E+01	7.547E-02	4.299E-04	5.696E-03
7.50	2.161E+00	6.721E+01	6.639E-02	2.926E-04	4.408E-03
8.00	2.169E+00	8.526E+01	5.882E-02	2.035E-04	3.460E-03
9.00	2.183E+00	1.324E+02	4.706E-02	1.042E-04	2.215E-03
10	2.193E+00	1.973E+02	3.846E-02	5.690E-05	1.479E-03
20	2.225E+00	2.934E+03	9.901E-03	9.706E-07	9.803E-05
30	2.231E+00	1.465E+04	4.425E-03	8.663E-08	1.958E-05

		k = 5/3			
M	M* = v*/v	A/A*	T/T_o	p/p_o	ρ/ρ_o
0.00	0.000E+00	∞	1.000E+00	1.000E+00	1.000E+00
0.01	1.155E-02	5.625E+01	1.000E+00	9.999E-01	1.000E+00
0.02	2.309E-02	2.813E+01	9.999E-01	9.997E-01	9.998E-01
0.03	3.464E-02	1.876E+01	9.997E-01	9.993E-01	9.996E-01
0.04	4.618E-02	1.408E+01	9.995E-01	9.987E-01	9.992E-01
0.05	5.771E-02	1.127E+01	9.992E-01	9.979E-01	9.988E-01
0.06	6.924E-02	9.398E+00	9.988E-01	9.970E-01	9.982E-01
0.07	8.076E-02	8.062E+00	9.984E-01	9.959E-01	9.976E-01
0.08	9.228E-02	7.061E+00	9.979E-01	9.947E-01	9.968E-01
0.09	1.038E-01	6.284E+00	9.973E-01	9.933E-01	9.960E-01
0.10	1.153E-01	5.663E+00	9.967E-01	9.917E-01	9.950E-01
0.15	1.726E-01	3.806E+00	9.926E-01	9.815E-01	9.889E-01
0.20	2.294E-01	2.888E+00	9.868E-01	9.674E-01	9.803E-01
0.25	2.857E-01	2.345E+00	9.796E-01	9.498E-01	9.695E-01
0.30	3.413E-01	1.989E+00	9.709E-01	9.288E-01	9.566E-01
0.35	3.961E-01	1.741E+00	9.608E-01	9.048E-01	9.417E-01
0.40	4.500E-01	1.560E+00	9.494E-01	8.782E-01	9.250E-01
0.45	5.029E-01	1.424E+00	9.368E-01	8.493E-01	9.067E-01
0.50	5.547E-01	1.320E+00	9.231E-01	8.186E-01	8.869E-01
0.60	6.547E-01	1.176E+00	8.929E-01	7.533E-01	8.437E-01
0.70	7.494E-01	1.088E+00	8.596E-01	6.851E-01	7.970E-01
0.80	8.386E-01	1.035E+00	8.242E-01	6.167E-01	7.482E-01
0.90	9.222E-01	1.008E+00	7.874E-01	5.502E-01	6.987E-01
1.00	1.000E+00	1.000E+00	7.500E-01	4.871E-01	6.495E-01
1.10	1.072E+00	1.007E+00	7.126E-01	4.286E-01	6.015E-01
1.20	1.139E+00	1.027E+00	6.757E-01	3.753E-01	5.554E-01
1.30	1.201E+00	1.058E+00	6.397E-01	3.272E-01	5.116E-01
1.40	1.257E+00	1.098E+00	6.048E-01	2.845E-01	4.704E-01
1.50	1.309E+00	1.148E+00	5.714E-01	2.468E-01	4.320E-01
1.60	1.357E+00	1.208E+00	5.396E-01	2.139E-01	3.963E-01
1.70	1.401E+00	1.275E+00	5.093E-01	1.851E-01	3.635E-01
1.80	1.441E+00	1.352E+00	4.808E-01	1.603E-01	3.334E-01
1.90	1.478E+00	1.437E+00	4.539E-01	1.388E-01	3.058E-01

Table 3.1 (*continued*) Isentropic Flow Functions

		k = 5/3			
M	$M^* = v^*/v$	A/A^*	T/T_o	p/p_o	ρ/ρ_o
2.00	1.512E+00	1.531E+00	4.286E-01	1.202E-01	2.806E-01
2.10	1.543E+00	1.634E+00	4.049E-01	1.043E-01	2.576E-01
2.20	1.571E+00	1.746E+00	3.827E-01	9.058E-02	2.367E-01
2.30	1.598E+00	1.868E+00	3.619E-01	7.878E-02	2.177E-01
2.40	1.622E+00	1.998E+00	3.425E-01	6.863E-02	2.004E-01
2.50	1.644E+00	2.139E+00	3.243E-01	5.990E-02	1.847E-01
2.60	1.664E+00	2.290E+00	3.074E-01	5.238E-02	1.704E-01
2.70	1.683E+00	2.451E+00	2.915E-01	4.589E-02	1.574E-01
2.80	1.701E+00	2.623E+00	2.768E-01	4.029E-02	1.456E-01
2.90	1.717E+00	2.806E+00	2.629E-01	3.545E-02	1.348E-01
3.00	1.732E+00	3.000E+00	2.500E-01	3.125E-02	1.250E-01
3.10	1.746E+00	3.206E+00	2.379E-01	2.761E-02	1.160E-01
3.20	1.759E+00	3.424E+00	2.266E-01	2.444E-02	1.079E-01
3.30	1.771E+00	3.654E+00	2.160E-01	2.168E-02	1.004E-01
3.40	1.782E+00	3.897E+00	2.060E-01	1.927E-02	9.353E-02
3.50	1.793E+00	4.153E+00	1.967E-01	1.716E-02	8.725E-02
3.60	1.802E+00	4.422E+00	1.880E-01	1.532E-02	8.150E-02
3.70	1.811E+00	4.705E+00	1.797E-01	1.370E-02	7.621E-02
3.80	1.820E+00	5.003E+00	1.720E-01	1.227E-02	7.134E-02
3.90	1.828E+00	5.314E+00	1.647E-01	1.102E-02	6.687E-02
4.00	1.835E+00	5.641E+00	1.579E-01	9.906E-03	6.274E-02
4.50	1.867E+00	7.508E+00	1.290E-01	5.981E-03	4.635E-02
5.00	1.890E+00	9.800E+00	1.071E-01	3.758E-03	3.507E-02
5.50	1.908E+00	1.256E+01	9.023E-02	2.445E-03	2.710E-02
6.00	1.922E+00	1.584E+01	7.692E-02	1.641E-03	2.133E-02
6.50	1.933E+00	1.969E+01	6.630E-02	1.132E-03	1.707E-02
7.00	1.941E+00	2.414E+01	5.769E-02	7.995E-04	1.386E-02
7.50	1.949E+00	2.925E+01	5.063E-02	5.769E-04	1.139E-02
8.00	1.955E+00	3.507E+01	4.478E-02	4.242E-04	9.475E-03
9.00	1.964E+00	4.900E+01	3.571E-02	2.410E-04	6.749E-03
10	1.971E+00	6.631E+01	2.913E-02	1.448E-04	4.971E-03
20	1.993E+00	5.075E+02	7.444E-03	4.781E-06	6.423E-04
30	1.997E+00	1.699E+03	3.322E-03	6.362E-07	1.915E-04

should be used. If one of the temperatures is not known, use the k value for the known temperature, and check for variation after the other is computed.

Adiabatic Expansion Factor Y

The adiabatic expansion factor, Y, is the ratio of the mass flow rate of a compressible fluid to that of an incompressible fluid under the same conditions. This factor is important in the flow of compressible fluids in some metering devices, such as the flow nozzle and the Venturi meter. For applications of this factor, refer to Chapter 6.

Consider conditions at the nozzle inlet section 1 of Figure 3.11. At this section both the area A_1 and the velocity v_1 are finite. Equation (3.97) can be written in terms of the kinetic energy change between sections 1 and 2 as follows:

$$\frac{v_2^2}{2g_c} - \frac{v_1^2}{2g_c} = \frac{Rk(T_1 - T_2)}{k-1} \tag{3.110}$$

Combining the continuity equation with the isentropic pressure relations derived in Chapter 1 results in the following:

$$v_1 = v_2\left(\frac{A_2}{A_1}\right)\left(\frac{p_2}{p_1}\right)^{1/k} \tag{3.111}$$

Substituting for v_1 from equation (3.111) in equation (3.110) and solving for v_2 results in the following:

$$\begin{aligned} \frac{v_2^2}{2g_c} - \frac{v_2^2}{2g_c}\left(\frac{A_2}{A_1}\right)^2\left(\frac{p_2}{p_1}\right)^{2/k} &= \frac{Rk(T_1 - T_2)}{k-1} \\ v_2 &= \sqrt{\frac{2g_c kRT_1(1 - T_2/T_1)}{(k-1)[1 - (A_2/A_1)^2(p_2/p_1)^{2/k}]}} \end{aligned} \tag{3.112}$$

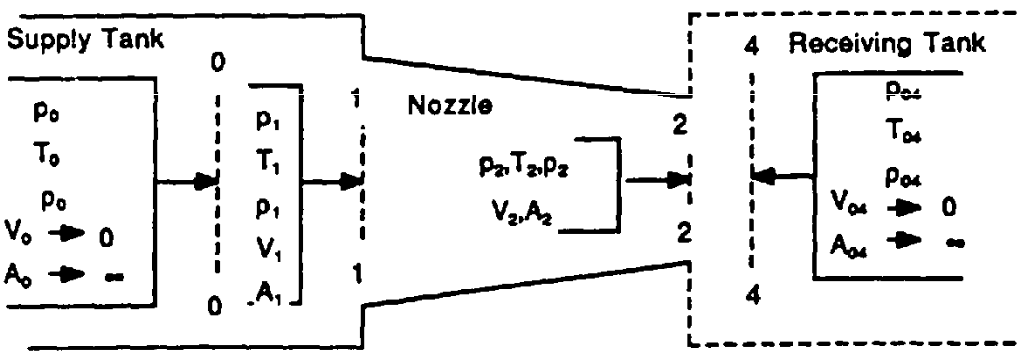


Figure 3.11 Notation for convergent nozzle study.

Substituting the value of v_2 from equation (3.112) in the equation of continuity and noting from the equation of state (1.37) that $RT_1 = p_1 \nu_1$ results in the following:

$$\dot{m} = \frac{A_2 v_2}{\nu_2} = A_2 \sqrt{\frac{2g_c k p_1 \nu_1 [1 - (T_2/T_1)]}{(k-1)[1 - (A_2/A_1)^2 (p_2/p_1)^{2/k}]\nu_2^2}} \tag{3.113}$$

From equation (1.41) $\nu_2/\nu_1 = (p_1/p_2)^{1/k}$ and from equation (1.40) $T_2/T_1 = (p_2/p_1)^{(k-1)/k}$. Substituting these relations in equation (3.113) yields the following:

$$\dot{m} = A_2 \sqrt{\left(\frac{2g_c k}{(k-1)}\right)\left(\frac{p_1}{\nu_1}\right)\frac{(p_2/p_1)^{2/k}[1 - (p_2/p_1)^{k-1/k}]}{[1 - (A_2/A_1)^2 (p_2/p_1)^{2/k}]}} \tag{3.114}$$

Figure 3.12 shows a plot of mass flow rate vs. pressure ratio for a convergent nozzle. As the pressure ratio p_2/p_1 is decreased, the mass flow rate from equation (3.114) increases until the pressure ratio p_2^*/p_1 is attained. The other mathematical solution of equation (3.114) is shown as a dotted line. The maximum flow rate is given by equation (3.113) and is known as *choked* flow.

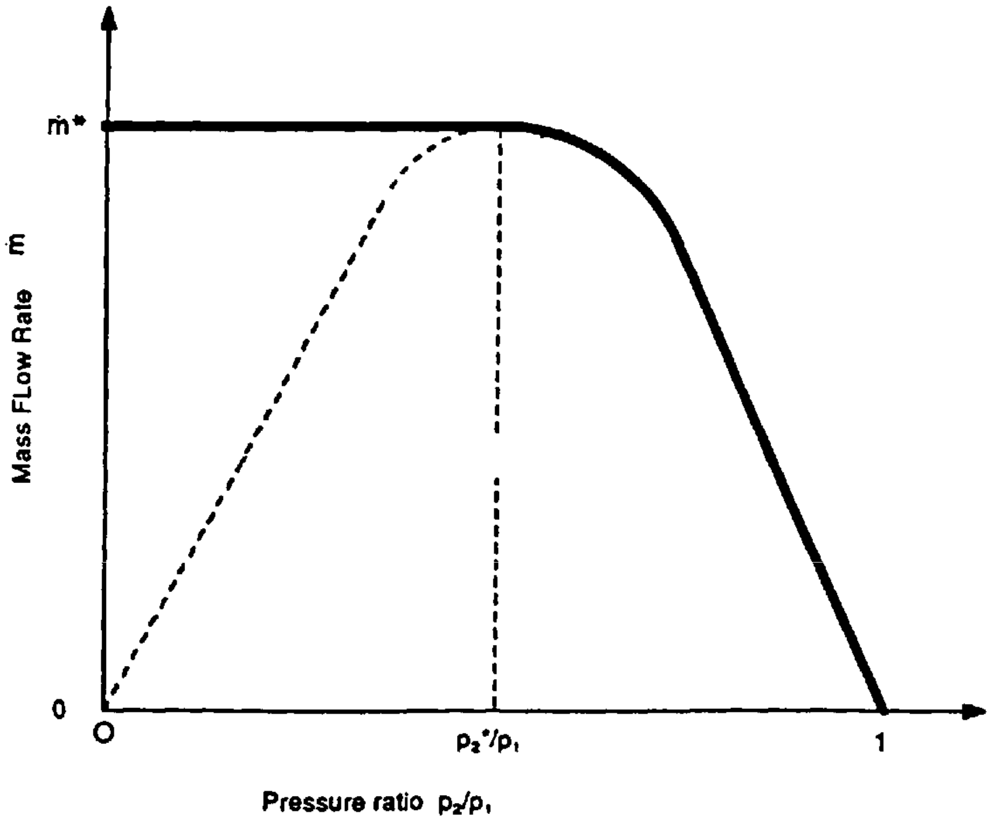


Figure 3.12 Mass flow rate vs. pressure ratio for a convergent nozzle.

Differentiating equation (3.114) with respect to p_2/p_1 and setting $d\dot{m}/d(p_2/p_1) = 0$ yields the following:

$$\left(\frac{p^*}{p_1}\right)^{(1-k)/k} + \left(\frac{2}{k-1}\right)\left(\frac{A_2}{A_1}\right)^2\left(\frac{p^*}{p_1}\right)^{2/k} = \frac{k+1}{2} \tag{3.115}$$

For the special case of $A_2/A_1 = 0$, equation (3.115) reduces to the following:

$$\frac{p^*}{p_0} = \frac{p^*}{p_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad \text{for } \frac{A_2}{A_1} = 0 \tag{3.116}$$

When an incompressible fluid flows without friction through a horizontal nozzle, the mass flow rate, $\dot{m}$, may be obtained by writing the Bernoulli equation (3.23) for a horizontal passage as follows:

$$\frac{v_2^2}{2g_c} - \frac{v_1^2}{2g_c} = \nu_1(p_1 - P_2) \tag{3.117}$$

From the continuity equation (3.6) $\dot{m} = v_1 A_1/\nu_1 = v_2 A_2 \nu_2$. Since for incompressible flow $\nu_2 = \nu_1$, the incompressible mass flow is $\dot{m}_i = v_1 A_1/\text{v}_1 = v_2 A_2 \nu_1$. Substituting these expressions in equation (3.117) results in the following:

$$\frac{(\dot{m}_i \nu_1/A_2)^2 - (\dot{m}_i \nu_1/A_1)^2}{2g_c} = \nu_1(p_1 - p_2)$$

which reduces to:

$$\dot{m}_i = A_2\sqrt{\frac{2g_c(p_1 - p_2)}{\nu_1[1 - (A_2/A_1)]^2}} \tag{3.118}$$

The *adiabatic expansion factor*, Y, is defined as follows:

$$Y = \frac{\dot{m}}{\dot{m}_i} = \frac{\text{mass flow rate of a compressible fluid}}{\text{mass flow rate of an incompressible fluid}} \tag{3.119}$$

Substituting equation (3.114) for $\dot{m}$ and equation (3.118) for $\dot{m}_i$ in equation (3.119) and simplifying results in the following:

$$Y = \frac{\dot{m}}{\dot{m}_i} = \sqrt{\frac{k(p_2/p_1)^{2/k}[1 - (p_2/p_1)^{(k-1)/k}][1 - (A_2/A)^2]}{(k-1)(1 - p_2/p_1)[1 - (A_2/A_1)^2(p_2/p_1)^{2/k}]}} \tag{3.120}$$

Values of the adiabatic expansion factor Y are given in Table 3.2. In this table, the diameter ratio (β) is used, which is defined as follows:

$$\beta = \frac{D_2}{D_1}; \qquad \beta^4 = \left(\frac{D_2}{D_1}\right)^4 = \left(\frac{A_2}{A_1}\right)^2 \tag{3.121}$$

Table 3.2 Adiabatic Expansion Factor *Y*

Beta Ratio β	Specific Heat Ratio k	Critical Values p^*/p_1	Critical Values Y^*	Adiabatic Expansion Factor Y p_2/p_1 0.60	p_2/p_1 0.70	p_2/p_1 0.80	p_2/p_1 0.90	p_2/p_1 1.00
0.00	1.00	0.6065	0.6837	-	0.7633	0.8450	0.9238	
	1.10	0.5847	0.6895	0.7021	0.7821	0.8580	0.9305	
	1.20	0.5645	0.6949	0.7229	0.7981	0.8689	0.9361	
	1.30	0.5457	0.7000	0.7409	0.8120	0.8783	0.9408	
	1.40	0.5283	0.7050	0.7568	0.8241	0.8865	0.9449	
	1.50	0.5120	0.7097	0.7709	0.8347	0.8936	0.9485	
	1.67	0.4871	0.7170	0.7910	0.8498	0.9037	0.9535	
0.10	1.00	0.6065	0.6837	-	0.7632	0.8450	0.9238	
	1.10	0.5847	0.6894	0.7021	0.7820	0.8580	0.9305	
	1.20	0.5645	0.6949	0.7228	0.7981	0.8689	0.9361	
	1.30	0.5457	0.7000	0.7409	0.8120	0.8783	0.9408	
	1.40	0.5283	0.7050	0.7568	0.8240	0.8865	0.9449	
	1.50	0.5120	0.7096	0.7709	0.8347	0.8936	0.9485	
	1.67	0.4872	0.7170	0.7910	0.8498	0.9037	0.9535	
0.20	1.00	0.6067	0.6835	-	0.7630	0.8448	0.9237	
	1.10	0.5849	0.6893	0.7018	0.7818	0.8578	0.9303	
	1.20	0.5647	0.6947	0.7225	0.7978	0.8687	0.9359	
	1.30	0.5459	0.6998	0.7406	0.8117	0.8781	0.9407	1.0000
	1.40	0.5285	0.7048	0.7565	0.8238	0.8863	0.9448	
	1.50	0.5122	0.7095	0.7706	0.8344	0.8934	0.9484	
	1.67	0.4873	0.7168	0.7907	0.8496	0.9035	0.9534	
0.25	1.00	0.6070	0.6833	-	0.7625	0.8444	0.9235	
	1.10	0.5851	0.6890	0.7013	0.7813	0.8574	0.9302	
	1.20	0.5649	0.6944	0.7220	0.7974	0.8684	0.9358	
	1.30	0.5462	0.6996	0.7401	0.8113	0.8778	0.9405	
	1.40	0.5288	0.7045	0.7560	0.8234	0.8860	0.9447	
	1.50	0.5125	0.7092	0.7701	0.8341	0.8932	0.9482	
	1.67	0.4876	0.7165	0.7903	0.8492	0.9033	0.9533	
0.30	1.00	0.6074	0.6827	-	0.7617	0.8438	0.9231	
	1.10	0.5856	0.6885	0.7004	0.7805	0.8568	0.9298	
	1.20	0.5654	0.6939	0.7212	0.7967	0.8678	0.9354	
	1.30	0.5467	0.6991	0.7393	0.8106	0.8773	0.9402	
	1.40	0.5293	0.7040	0.7552	0.8227	0.8855	0.9444	
	1.50	0.5130	0.7087	0.7693	0.8334	0.8927	0.9480	
	1.67	0.4882	0.7161	0.7895	0.8486	0.9028	0.9530	

Beta Ratio β	Specific Heat Ratio k	Critical Values		Adiabatic Expansion Factor Y				
		p^*/p_1	Y^*	p_2/p_1 0.60	p_2/p_1 0.70	p_2/p_1 0.80	p_2/p_1 0.90	p_2/p_1 1.00
	1.00	0.6094	0.6806	-	0.7582	0.8411	0.9215	
	1.10	0.5877	0.6864	0.6966	0.7772	0.8542	0.9283	
	1.20	0.5676	0.6918	0.7175	0.7935	0.8654	0.9341	
0.40	1.30	0.5489	0.6970	0.7357	0.8075	0.8750	0.9390	
	1.40	0.5315	0.7019	0.7517	0.8198	0.8833	0.9432	
	1.50	0.5153	0.7066	0.7659	0.8306	0.8906	0.9469	
	1.67	0.4905	0.7140	0.7863	0.8460	0.9009	0.9520	
	1.00	0.6137	0.6760	-	0.7506	0.8351	0.9180	
	1.10	0.5921	0.6818	0.6884	0.7699	0.8486	0.9251	
	1.20	0.5721	0.6872	0.7094	0.7865	0.8601	0.9311	
0.50	1.30	0.5536	0.6924	0.7278	0.8008	0.8700	0.9362	
	1.40	0.5363	0.6974	0.7441	0.8133	0.8785	0.9405	
	1.50	0.5201	0.7020	0.7585	0.8244	0.8860	0.9444	
	1.67	0.4954	0.7094	0.7792	0.8401	0.8967	0.9498	
	1.00	0.6219	0.6672	-	0.7358	0.8232	0.9110	
	1.10	0.6006	0.6730	-	0.7557	0.8374	0.9186	
	1.20	0.5809	0.6785	0.6939	0.7728	0.8495	0.9250	
0.60	1.30	0.5625	0.6836	0.7126	0.7876	0.8599	0.9305	1.0000
	1.40	0.5454	0.6886	0.7292	0.8006	0.8690	0.9352	
	1.50	0.5294	0.6933	0.7440	0.8121	0.8770	0.9394	
	1.67	0.5050	0.7008	0.7653	0.8286	0.8883	0.9452	
	1.00	0.6249	0.6641	-	0.7305	0.8189	0.9084	
	1.10	0.6037	0.6699	-	0.7505	0.8333	0.9162	
	1.20	0.5840	0.6753	0.6882	0.7677	0.8456	0.9228	
0.625	1.30	0.5658	0.6805	0.7071	0.7828	0.8562	0.9284	
	1.40	0.5487	0.6854	0.7238	0.7960	0.8655	0.9333	
	1.50	0.5328	0.6902	0.7387	0.8076	0.8736	0.9375	
	1.67	0.5084	0.6976	0.7603	0.8244	0.8851	0.9435	
	1.00	0.6368	0.6513	-	0.7083	0.8007	0.8973	
	1.10	0.6160	0.6570	-	0.7290	0.8161	0.9059	
	1.20	0.5968	0.6625	0.6651	0.7470	0.8292	0.9131	
0.70	1.30	0.5789	0.6677	0.6844	0.7627	0.8406	0.9193	
	1.40	0.5621	0.6727	0.7016	0.7765	0.8506	0.9247	
	1.50	0.5464	0.6774	0.7170	0.7889	0.8593	0.9294	
	1.67	0.5224	0.6849	0.7393	0.8066	0.8719	0.9361	

Table 3.2 (*continued*) Adiabatic Expansion Factor *Y*

Beta Ratio β	Specific Heat Ratio k	Critical Values p^*/p_1	Critical Values Y^*	Adiabatic Expansion Factor Y p_2/p_1 0.60	p_2/p_1 0.70	p_2/p_1 0.80	p_2/p_1 0.90	p_2/p_1 1.00
0.75	1.00	0.6482	0.6389	-	0.6865	0.7824	0.8857	
	1.10	0.6279	0.6447	-	0.7078	0.7986	0.8951	
	1.20	0.6091	0.6502	-	0.7263	0.8125	0.9030	
	1.30	0.5915	0.6554	0.6622	0.7426	0.8246	0.9098	
	1.40	0.5750	0.6603	0.6797	0.7571	0.8353	0.9158	
	1.50	0.5596	0.6651	0.6955	0.7700	0.8447	0.9210	
	1.67	0.5359	0.6726	0.7185	0.7886	0.8582	0.9283	
0.80	1.00	0.6638	0.6220	-	0.6560	0.7559	0.8683	
	1.10	0.6441	0.6277	-	0.6779	0.7732	0.8788	
	1.20	0.6258	0.6332	-	0.6971	0.7882	0.8878	
	1.30	0.6087	0.6384	-	0.7141	0.8013	0.8955	
	1.40	0.5927	0.6433	0.6491	0.7292	0.8129	0.9022	
	1.50	0.5776	0.6481	0.6653	0.7428	0.8231	0.9081	
	1.67	0.5544	0.6556	0.6890	0.7627	0.8380	0.9165	
0.85	1.00	0.6857	0.5980	-	0.6117	0.7159	0.8407	
	1.10	0.6670	0.6037	-	0.6341	0.7346	0.8528	
	1.20	0.6495	0.6090	-	0.6540	0.7509	0.8632	
	1.30	0.6331	0.6142	-	0.6717	0.7653	0.8722	1.0000
	1.40	0.6177	0.6191	-	0.6877	0.7781	0.8801	
	1.50	0.6033	0.6239	-	0.7021	0.7895	0.8871	
	1.67	0.5809	0.6313	0.6458	0.7234	0.8062	0.8971	
0.90	1.00	0.7184	0.5614	-	-	0.6506	0.7914	
	1.10	0.7012	0.5670	-	-	0.6707	0.8060	
	1.20	0.6850	0.5722	-	0.5860	0.6886	0.8187	
	1.30	0.6699	0.5773	-	0.6043	0.7045	0.8298	
	1.40	0.6556	0.5822	-	0.6209	0.7189	0.8396	
	1.50	0.6421	0.5868	-	0.6361	0.7318	0.8483	
	1.67	0.6211	0.5942	-	0.6587	0.7509	0.8609	
0.95	1.00	0.7743	0.4963	-	-	0.5260	0.6821	
	1.10	0.7599	0.5015	-	-	0.5465	0.7003	
	1.20	0.7463	0.5065	-	-	0.5652	0.7164	
	1.30	0.7335	0.5113	-	-	0.5822	0.7309	
	1.40	0.7213	0.5158	-	-	0.5979	0.7439	
	1.50	0.7097	0.5202	-	-	0.6123	0.7557	
	1.67	0.6917	0.5272	-	0.5344	0.6340	0.7730	

The use of the expansion factor from Table 3.2 facilitates computation. An expression for compressible flow may be obtained by substituting equation (3.118) for $\dot{m}_i$ in equation (3.119) and solving for $\dot{m}$, resulting in the following:

$$\dot{m} = YA_2\sqrt{\frac{2g_c(p_1 - p_2)}{v_1[1 - (A_2/A_1)^2]}} = YA_2\sqrt{\frac{2g_c(p_1 - p_2)}{v_1(1 - \beta^4)}} \tag{3.122}$$

Convergent-Divergent Nozzles

Area Pressure Relations

The mass flow rate through any section of the convergent-divergent nozzle shown in Figure 3.13 may be determined by modifying equation (3.114) for stagnation conditions ($A_2 = A_x$, $A_1 = A_0$, $A_x/A_0 \approx 0$, and $p_1 = p_0$, $v_1 = v_0$, $v_2 = v_x$) :

$$\dot{m} = A_x\sqrt{\frac{2g_c k}{(k-1)}\left(\frac{p_0}{v_0}\right)(p_x/p_0)^{2/k}[1 - (p_x/p_0)^{(k-1)/k}]} \tag{3.123}$$

The area pressure relations may be established by squaring equation (3.123) and equating for sections 2 and 3 as follows:

$$\frac{\dot{m}^2}{\dot{m}^2} = \frac{A_2^2[2g_c(k/k-1)](p_0/v_0)(p_2/p_0)^{2/k}[1 - (p_2/p_0)]^{(k-1)/k}}{A_3^2[2g_c(k/k-1)](p_0/v_0)(p_3/p_0)^{2/k}[1 - (p_3/p_0)]^{k-1)/k}}$$

which reduces to

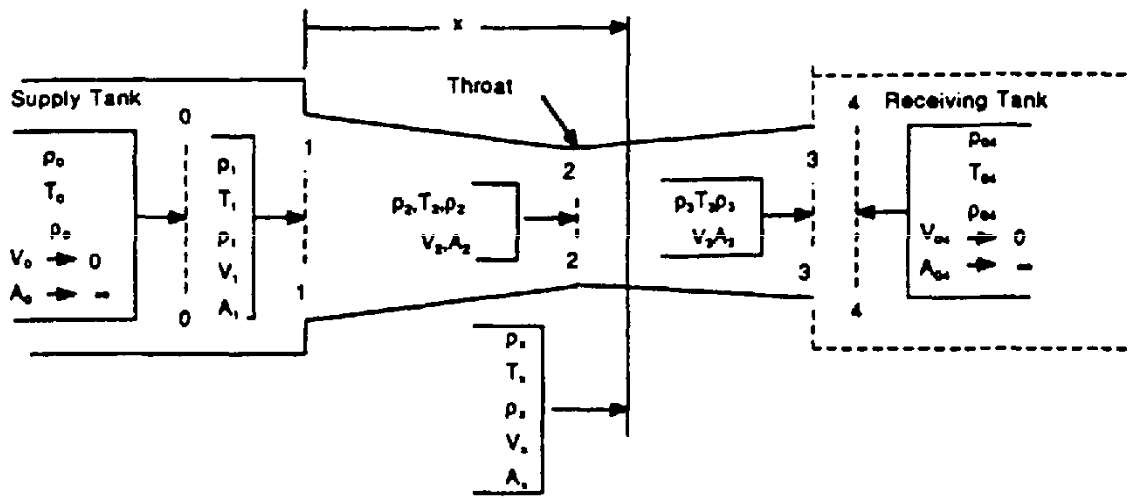


Figure 3.13 Notation for convergent-divergent nozzle study.

$$\frac{A_3}{A_2} = \frac{(p_2/p_0)^{2/k} - (p_2/p_0)^{(k+1)/k}}{(p_3/p_0)^{2/k} - (p_3/p_0)^{(k+1)/k}} \tag{3.124}$$

If the velocity in the throat is sonic, then from equation (3.103) the following can be written:

$$\frac{p_2}{p_0} = \frac{p^*}{p_0}\left(\frac{2}{k+1}\right)^{k/(k-1)} \quad \text{and} \quad A_2 - A^* \tag{3.125}$$

Substituting equation (3.103) in equation (3.124) yields the following:

$$\frac{A_3}{A^*} = \sqrt{\frac{\left(\frac{2}{k+1}\right)^{2/(k-1)} - \left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}{(p_3/p_0)^{2/k} - (p_3/p_0)^{(k+1)/k}}} \tag{3.126}$$

Note that equation (3.126) has two solutions, one for isentropic compression (subsonic flow) and the other for isentropic expansion (supersonic flow).

Flow Through a Convergent-Divergent Nozzle

Consider the arrangement shown in Figure 3.14. The supply tank pressure p_0 is maintained constant and the receiving tank pressure p_{04} may be lowered from p_0 to zero. As soon as p_{04} is below p_0, flow begins.

Path A represents the flow for any p_3 higher than p_{3B}. Since the flow in the throat for path A is subsonic (p_2A > p^*), the flow throughout the nozzle must be subsonic. In the convergent section the process is an isentropic expansion; in the divergent section the process is an isentropic compression. Path A, for example, represents compressible flow through an ideal Venturi meter (Chapter 6).

Path B represents an isentropic expansion in the convergent portion and an isentropic compression in the divergent section after sonic flow in the throat. Except for the throat, the flow in both portions of the nozzle is subsonic. Pressure p_{3B} is the pressure calculated from the subsonic solution of equation (3.126).

Path C represents an isentropic expansion in the both the convergent and divergent sections of the nozzle. The flow in the convergent section is subsonic; in the divergent section it is supersonic. Pressure p_{3C} is the pressure calculated from the supersonic solution of equation (3.126). Note that any receiving tank pressure (p_{04}) lower than p_{3C} will have no effect on this process.

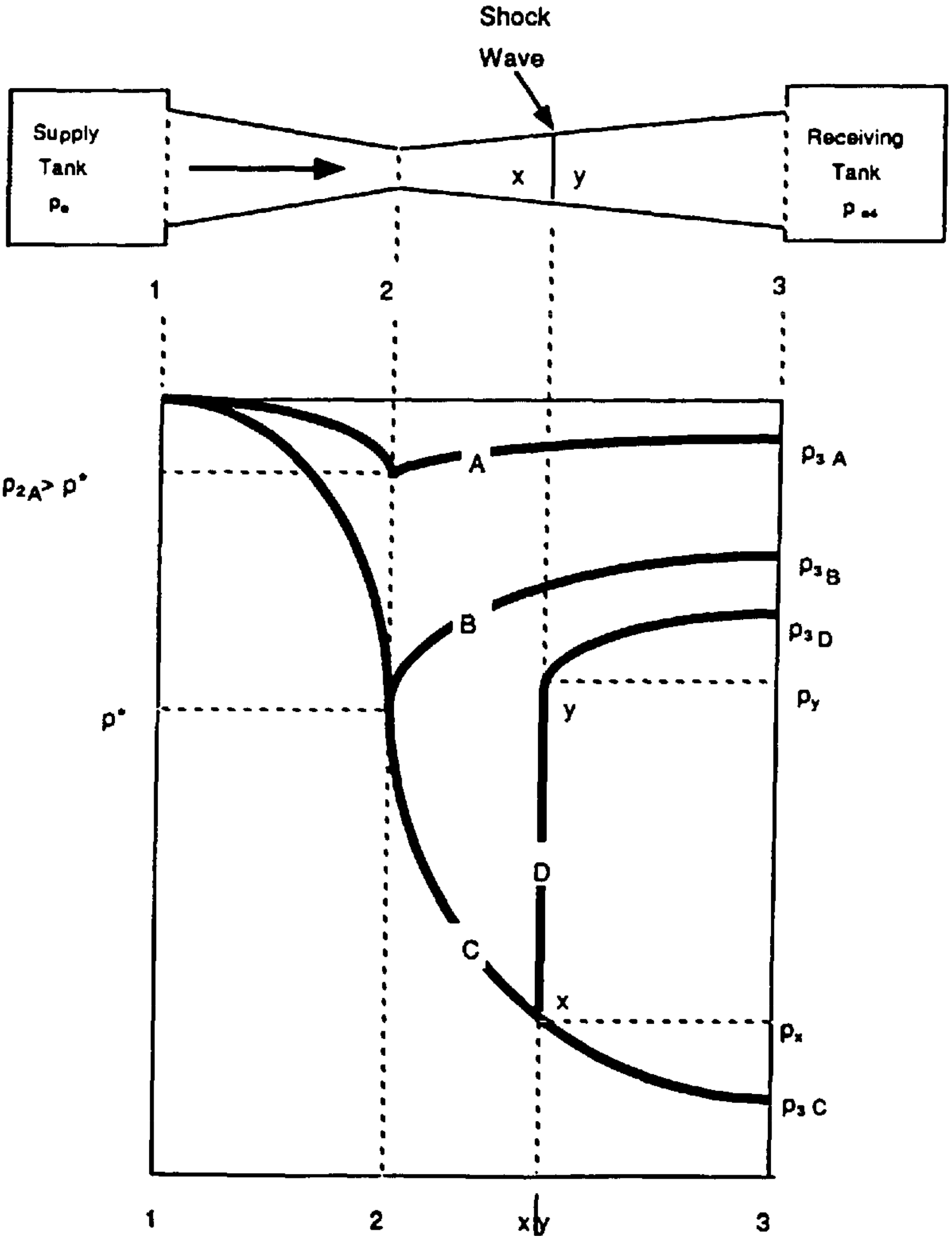


Figure 3.14 Pressures in a convergent-divergent nozzle.

Path D represents any pressure between p_{3B} and p_{3C}. The gas expands along an isentropic path to the throat and continues along path C until the distance x in the divergent portion of the nozzle is reached. At this point a shock wave is formed and the pressure (and other properties) essentially jump to point y. From point y to the exit path D is one of isentropic compression. The flow in the divergent portion is supersonic to point x and subsonic from point y.

Equation (3.114) or (3.122) may be used to calculate the mass rate of flow through the nozzle for path A. For all other paths, equation (3.109) should be used.

Normal Shock Functions

Compression Shock Wave

The discussion in this section continues with path D (Figure 3.14). When sonic flow exists in the throat and supersonic flow begins in the diverging section of a convergent-divergent nozzle, and the exit pressure p_3 is between that required for isentropic compression (Path B) p_{3B} and that for isentropic expansion (Path B) p_{3C}, a compression shock wave will be formed. This wave satisfies the requirements for the conservation of mass and energy. This type of wave is associated with large and sudden rises in pressure, density, temperature, and entropy. Figure 3.15 shows this phenomenon on the T–s plot. The shock wave is so thin that for computation purposes it may be considered as a single line, as shown in Figure 3.14.

Temperature Mach number velocity relations for a normal shock are also shown in Figure 3.15.

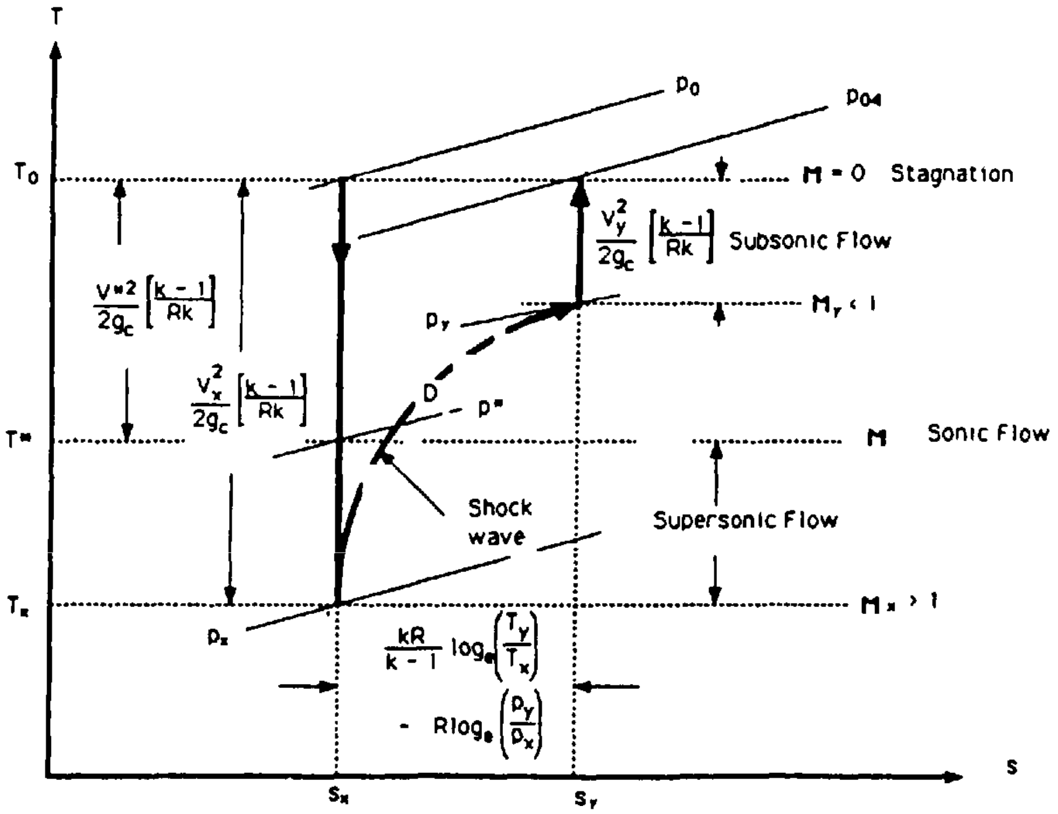


Figure 3.15 Notation for shock wave study.

Conservation of Energy

The formation of a shock wave does not change the total energy of the system, so energy relations may be established by writing equation (3.97) in terms of temperatures before the shock wave (T_x) and after the shock wave (T_y):

$$T_{0x} = \frac{RT_x}{k-1} + \frac{v_x^2}{2g_c} = \frac{RT_y}{k-1} + \frac{v_y^2}{2g_c} = T_{0y} \tag{3.127}$$

Substituting the value of acoustic velocity from equation (1.59) and that for Mach number from equation (3.94), as was done in the development of equation (3.98), results in the following:

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{2}{k-1}\right)M_x^2}{1 + \left(\frac{2}{k-1}\right)M_y^2} \tag{3.128}$$

Conservation of Mass

The continuity equation for an ideal gas $\dot{m} = Avp/RT$ may be written in terms of Mach number by noting that the definition of Mach number from equation (3.94) is $M = v/(kg_c RT)^{1/2}$. Substituting these values into the ideal gas continuity equation for before and after the shock wave yields the following:

$$\frac{\dot{m}}{A} = \frac{vp}{RT} = \frac{(M\sqrt{kg_c RT})p}{RT} = Mp\sqrt{\frac{kg_c}{RT}} = M_x p_x\sqrt{\frac{kg_c}{RT_x}} = M_y p_y\sqrt{\frac{kg_c}{RT_y}} \tag{3.129}$$

Solving for M_y leads to the following:

$$M_y = M_x \frac{p_x}{p_y}\sqrt{\frac{T_y}{T_x}} \tag{3.130}$$

Impulse-Momentum Concept

The impulse-momentum equation (3.87), when applied to the shock wave of Figure 3.14, yields the following:

$$(p_x - p_y)A = \frac{\rho_x A v_x}{g_c}(v_y - v_x) = \frac{\rho_y A v_y}{g_c}(v_y - v_x) \tag{3.87}$$

which reduces to:

$$p_y + \frac{\rho_y v_y^2}{g_c} = p_x + \frac{\rho_x v_x^2}{g_c} \tag{3.131}$$

Substituting in this equation the definition of Mach number [equation (3.94)] $M = v/(kg_c RT)^{1/2}$ and from the equation of state (1.37) $\rho = p/RT$ results in the following:

$$p_y + \frac{\left(\dfrac{p_y}{RT_y}\right)[M_y^2(kg_c RT_y)]}{g_c} = p_x + \frac{\left(\dfrac{p_x}{RT_x}\right)[M_x^2(kg_c RT_x)]}{g_c}$$

which reduces to:

$$\frac{p_y}{p_x} = \frac{1 + kM_x^2}{1 + kM_y^2} \tag{3.132}$$

Equations (3.128), (3.130), and (3.132) involve three unknowns, T_y, p_y, and M_y, and may be combined to yield a relationship between M_y and M_x as follows:

If equation (3.130) is solved for T_y/T_x, the following results:

$$\frac{T_y}{T_x} = \left(\frac{p_y}{p_x}\right)^2 \left(\frac{M_y}{M_x}\right)^2 \tag{3.133}$$

Equating equation (3.128) and equation (1.33) results in the following:

$$\frac{T_y}{T_x} = \left(\frac{p_y}{p_x}\right)^2 \left(\frac{M_y}{M_x}\right)^2 = \frac{1 + \left(\dfrac{2}{k-1}\right)M_x^2}{1 + \left(\dfrac{2}{k-1}\right)M_y^2}$$

which reduces to:

$$\frac{p_y}{p_x} = \frac{M_x}{M_y}\sqrt{\frac{1 + \left(\dfrac{2}{k-1}\right)M_x^2}{1 + \left(\dfrac{2}{k-1}\right)M_y^2}} \tag{3.134}$$

Equating equation (3.132) and equation (3.134) yields the following:

$$\frac{p_y}{p_x} = \frac{M_x}{M_y}\sqrt{\frac{1+\left(\dfrac{2}{k-1}\right)M_x^2}{1+\left(\dfrac{2}{k-1}\right)M_y^2}} = \frac{1+kM_x^2}{1+kM_y^2}$$

which reduces to:

$$\frac{M_x\sqrt{1+\left(\dfrac{2}{k-1}\right)M_x^2}}{1+kM_x^2} = \frac{M_y\sqrt{1+\left(\dfrac{2}{k-1}\right)M_y^2}}{1+kM_y^2} \tag{3.135}$$

Equation (3.135) may be arranged in quadratic form and solved directly for M_y^2. When this is done, the two solutions are as follows:

$$M_y = M_x$$

and

$$M_y^2 = \frac{M_x^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_x^2 - 1} \tag{3.136}$$

The first solution is trivial because the Mach number must decrease after a shock wave. Therefore, equation (3.136) represents the physical solution. Solving equation (3.136) for M_y results in the following:

$$M_y = \frac{(k-1)M_x^2 + 2}{2kM_x^2 - k + 1} \tag{3.137}$$

Temperature ratios are obtained by substituting M_y^2 from equation (3.136) in equation (3.128):

$$\frac{T_y}{T_x} = \frac{1+\left(\dfrac{2}{k-1}\right)M_x^2}{1+\left(\dfrac{2}{k-1}\right)M_y^2} = \frac{1+\left(\dfrac{2}{k-1}\right)M_x^2}{1+\dfrac{2}{k-1}\left[\dfrac{M_x^2 + 2/(k-1)}{\dfrac{2k}{k-1}M_x^2 - 1}\right]}$$

$$\frac{T_y}{T_x} = \frac{\left(1+\dfrac{k-1}{2}M_x^2\right)\left(\dfrac{2k}{k-1}M_x^2 - 1\right)}{\dfrac{(k+1)^2}{2(k-1)}M_x^2} \tag{3.138}$$

Pressure ratios are obtained by substituting M_y^2 from equation (3.136) in equation (3.132):

$$\frac{p_y}{p_x} = \frac{1 + kM_x^2}{1 + kM_y^2} = \frac{1 + kM_x^2}{1 + k\left(\dfrac{M_x^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_x^2 - 1}\right)} = \frac{2k}{k+1}M_x^2 - \frac{k+1}{k-1} \tag{3.139}$$

Density ratios may be obtained using the equation of state (1.37):

$$\frac{\rho_y}{\rho_x} = \frac{\dfrac{p_y}{RT_y}}{\dfrac{p_x}{RT_x}} = \left(\frac{p_y}{p_x}\right)\left(\frac{T_x}{T_y}\right) \tag{3.140}$$

Stagnation pressure ratios may be obtained by first expressing p_{0y}/p_{0x} in terms of equations (3.100) and (3.139) and substituting for M_y from equation (3.137):

$$\frac{p_{0y}}{p_{0x}} = \left(\frac{p_{0y}}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p_{0x}}\right) = \left[1 + \frac{k-1}{2}\left(\frac{M_x^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_x^2 - 1}\right)\right]^{k/(k-1)}$$

$$\times\left(\frac{2k}{k+1}M_x^2 - \frac{k-1}{k+1}\right)\left(1 + \frac{k-1}{2}M_x^2\right)^{k/(1-k)}$$

$$\frac{p_{0y}}{p_{0x}} = \left(\frac{[(k+1)/2]M_x^2}{1 + [(k-1)/2]M_x^2}\right)^{k/(k-1)}\left(\frac{2k}{k+1}M_x^2 - \frac{k-1}{k+1}\right)^{1/(1-k)} \tag{3.141}$$

The ratio of the stagnation pressure after the shock wave to the pressure just before p_0y/p_0x may be obtained following the method just used to obtain equation (3.141):

$$\frac{p_{0y}}{p_{0x}} = \left(\frac{p_{0y}}{p_y}\right)\left(\frac{p_y}{p_x}\right) = \left[1 + \frac{k-1}{2}\left(\frac{M_x^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_x^2 - 1}\right)\right]^{k/(k-1)}\left(\frac{2k}{k+1}M_x^2 - \frac{k-1}{k+1}\right)$$

$$\frac{p_{0y}}{p_x} = \left(\frac{k+1}{2}M_x^2\right)^{k/(k-1)}\left(\frac{2}{k+1}M_x^2 - \frac{k-1}{k+1}\right)^{1/(1-k)} \tag{3.142}$$

The velocity ratio across a shock wave may be determined from the continuity equation (3.6) as follows:

$$\dot{m} = \rho_x A v_x = \rho_y A v_y$$
$$\frac{v_x}{v_y} = \frac{\rho_y}{\rho_x} \tag{3.143}$$

Entropy Increase Across a Normal Shock Wave

The entropy change of an ideal gas was derived in Section 3.3 by equation (3.68). We can write this equation in differential form:

$$ds = \frac{c_p\,dT}{T} - \frac{v\,dp}{T} \tag{3.68}$$

Substituting from equation (3.75) $c_p = kR/(k-1)$ and from the equation of state of an ideal gas (1.37) $v/T = R/p$ in equation (3.68) results in the following:

$$ds = \frac{c_p\,dT}{T} - \frac{v\,dp}{T} = \left(\frac{kR}{k-1}\right)\frac{dT}{T} - \left(\frac{R}{p}\right)dp = \left(\frac{kR}{k-1}\right)\frac{dT}{T} - R\frac{dp}{p} \tag{3.144}$$

Integrating equation (3.144) for constant specific heat ratios between the limits of x and y results in the following:

$$\int_{s_x}^{s_y} ds = \frac{kR}{k-1}\int_{T_x}^{T_y}\frac{dT}{T} - R\int_{p_x}^{p_y}\frac{dp}{p}$$
$$s_y - s_x = \frac{kR}{k-1}\log_e\left(\frac{T_y}{T_x}\right) - R\log_e\left(\frac{p_y}{p_x}\right) \tag{3.145}$$

Equation (3.145) may be expressed in dimensionless form by dividing both sides by R and substituting for T_y/T_x from equation (3.138) and for p_y/p_x from equation (3.139), with the following result:

$$\frac{s_y - s_x}{R} = \frac{k}{k-1}\log_e\left(\frac{\left(1 + \frac{k-1}{2}M_x^2\right)\left(\frac{2k}{k-1}M_x^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)}M_x^2}\right) - \log_e\left(\frac{2k}{k-1}M_x^2 - \frac{k+1}{k-1}\right) \tag{3.146}$$

Tabulated Values of Normal Shock Functions

As in the case of isentropic flow functions, it has been found useful to compute and tabulate certain standard normal shock functions. These functions are all dimensionless ratios and are functions of the Mach number, M_x, just upstream of the shock wave. Table 3.3 contains the following ratios:

Function	*Equation(s)*
M_y	3.137
$\frac{p_y}{p_x}$	3.139
$\frac{T_y}{T_x}$	3.138
$\frac{\rho_y}{\rho_x} = \frac{v_x}{v_y}$	3.140, 3.143
$\frac{p_{0y}}{p_{0x}}$	3.141
$\frac{p_{0y}}{p_x}$	3.142

In using Table 3.3 it should again be noted, as in Table 3.1, that all data are based on the assumption that the gas is ideal and that the molecular weight, specific heats, and ratios of specific heats are constant. Table D.2 gives values of k for ideal gases as a function of temperature. When the temperature range is known before calculations, the average value of k should be used. If one of the temperatures is not known, use the k value for the known temperature, and check for variation after the other is computed.

Adiabatic Flow in Constant-Area Ducts with Friction—Fanno Line

The flow of fluids in most industrial and power piping applications may be assumed to be adiabatic. The primary reasons for this assumption are:

1. The piping lengths are relatively short (and hence heat transfer areas small) with respect to large mass flow rates, so the heat transfer is negligible.
2. The pipes are insulated.

In adiabatic flow with friction, the gas may enter the pipe either with subsonic or supersonic velocity, as shown in Figure 3.16. In case (a) the gas enters the pipe with a subsonic velocity. The second law of thermodynamics

Table 3.3 Normal Shock Functions

k = 1

M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.649E+00
1.05	9.524E-01	1.103E+00	1.000E+00	1.103E+00	9.998E-01	1.735E+00
1.10	9.091E-01	1.210E+00	1.000E+00	1.210E+00	9.988E-01	1.829E+00
1.15	8.696E-01	1.323E+00	1.000E+00	1.323E+00	9.964E-01	1.930E+00
1.20	8.333E-01	1.440E+00	1.000E+00	1.440E+00	9.919E-01	2.038E+00
1.25	8.000E-01	1.563E+00	1.000E+00	1.563E+00	9.851E-01	2.152E+00
1.30	7.692E-01	1.690E+00	1.000E+00	1.690E+00	9.759E-01	2.272E+00
1.35	7.407E-01	1.823E+00	1.000E+00	1.823E+00	9.640E-01	2.398E+00
1.40	7.143E-01	1.960E+00	1.000E+00	1.960E+00	9.494E-01	2.530E+00
1.45	6.897E-01	2.103E+00	1.000E+00	2.103E+00	9.321E-01	2.667E+00
1.50	6.667E-01	2.250E+00	1.000E+00	2.250E+00	9.122E-01	2.810E+00
1.60	6.250E-01	2.560E+00	1.000E+00	2.560E+00	8.653E-01	3.112E+00
1.70	5.882E-01	2.890E+00	1.000E+00	2.890E+00	8.100E-01	3.436E+00
1.80	5.556E-01	3.240E+00	1.000E+00	3.240E+00	7.482E-01	3.781E+00
1.90	5.263E-01	3.610E+00	1.000E+00	3.610E+00	6.820E-01	4.146E+00
2.00	5.000E-01	4.000E+00	1.000E+00	4.000E+00	6.134E-01	4.533E+00
2.10	4.762E-01	4.410E+00	1.000E+00	4.410E+00	5.446E-01	4.939E+00
2.20	4.545E-01	4.840E+00	1.000E+00	4.840E+00	4.772E-01	5.367E+00
2.30	4.348E-01	5.290E+00	1.000E+00	5.290E+00	4.129E-01	5.814E+00
2.40	4.167E-01	5.760E+00	1.000E+00	5.760E+00	3.527E-01	6.282E+00
2.50	4.000E-01	6.250E+00	1.000E+00	6.250E+00	2.975E-01	6.771E+00
3.00	3.333E-01	9.000E+00	1.000E+00	9.000E+00	1.057E-01	9.514E+00
3.50	2.857E-01	1.225E+01	1.000E+00	1.225E+01	2.791E-02	1.276E+01
4.00	2.500E-01	1.600E+01	1.000E+00	1.600E+01	5.538E-03	1.651E+01
4.50	2.222E-01	2.025E+01	1.000E+00	2.025E+01	8.316E-04	2.076E+01
5.00	2.000E-01	2.500E+01	1.000E+00	2.500E+01	9.505E-05	2.551E+01
6.00	1.667E-01	3.600E+01	1.000E+00	3.600E+01	5.559E-07	3.650E+01
7.00	1.429E-01	4.900E+01	1.000E+00	4.900E+01	1.133E-09	4.950E+01
8.00	1.250E-01	6.400E+01	1.000E+00	6.400E+01	8.169E-13	6.450E+01
9.00	1.111E-01	8.100E+01	1.000E+00	8.100E+01	2.100E-16	8.150E+01
10.00	1.000E-01	1.000E+02	1.000E+00	1.000E+02	1.938E-20	1.005E+02
20.00	5.000E-02	4.000E+02	1.000E+00	4.000E+02	5.543E-85	4.005E+02
30.00	3.333E-02	9.000E+02	1.000E+00	9.000E+02	3.326E-193	9.005E+02

Table 3.3 (*continued*) Normal Shock Functions

k = 1.1

M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.710E+00
1.05	9.526E-01	1.107E+00	1.009E+00	1.097E+00	1.003E+00	1.810E+00
1.10	9.099E-01	1.220E+00	1.018E+00	1.198E+00	9.989E-01	1.906E+00
1.15	8.712E-01	1.338E+00	1.027E+00	1.303E+00	9.954E-01	2.013E+00
1.20	8.360E-01	1.461E+00	1.036E+00	1.410E+00	9.918E-01	2.131E+00
1.25	8.038E-01	1.589E+00	1.044E+00	1.522E+00	9.874E-01	2.259E+00
1.30	7.743E-01	1.723E+00	1.053E+00	1.636E+00	9.760E-01	2.382E+00
1.35	7.471E-01	1.862E+00	1.061E+00	1.755E+00	9.638E-01	2.516E+00
1.40	7.221E-01	2.006E+00	1.070E+00	1.875E+00	9.505E-01	2.658E+00
1.45	6.989E-01	2.155E+00	1.079E+00	1.997E+00	9.358E-01	2.810E+00
1.50	6.773E-01	2.310E+00	1.088E+00	2.123E+00	9.155E-01	2.958E+00
1.60	6.386E-01	2.634E+00	1.105E+00	2.384E+00	8.753E-01	3.293E+00
1.70	6.048E-01	2.980E+00	1.124E+00	2.651E+00	8.242E-01	3.637E+00
1.80	5.750E-01	3.347E+00	1.143E+00	2.928E+00	7.678E-01	4.004E+00
1.90	5.487E-01	3.734E+00	1.163E+00	3.211E+00	7.099E-01	4.405E+00
2.00	5.252E-01	4.143E+00	1.184E+00	3.499E+00	6.480E-01	4.815E+00
2.10	5.042E-01	4.572E+00	1.205E+00	3.794E+00	5.874E-01	5.258E+00
2.20	4.853E-01	5.023E+00	1.228E+00	4.090E+00	5.265E-01	5.711E+00
2.30	4.682E-01	5.494E+00	1.251E+00	4.392E+00	4.689E-01	6.197E+00
2.40	4.527E-01	5.987E+00	1.275E+00	4.696E+00	4.136E-01	6.693E+00
2.50	4.385E-01	6.500E+00	1.300E+00	5.000E+00	3.627E-01	7.222E+00
3.00	3.837E-01	9.381E+00	1.439E+00	6.519E+00	1.707E-01	1.017E+01
3.50	3.466E-01	1.279E+01	1.603E+00	7.979E+00	7.102E-02	1.361E+01
4.00	3.203E-01	1.671E+01	1.791E+00	9.330E+00	2.758E-02	1.773E+01
4.50	3.009E-01	2.117E+01	2.003E+00	1.057E+01	1.013E-02	2.221E+01
5.00	2.863E-01	2.614E+01	2.241E+00	1.166E+01	3.659E-03	2.738E+01
6.00	2.661E-01	3.767E+01	2.790E+00	1.350E+01	4.718E-04	3.912E+01
7.00	2.531E-01	5.129E+01	3.439E+00	1.491E+01	6.440E-05	5.308E+01
8.00	2.443E-01	6.700E+01	4.188E+00	1.600E+01	9.651E-06	6.923E+01
9.00	2.381E-01	8.481E+01	5.036E+00	1.684E+01	1.606E-06	8.749E+01
10.00	2.336E-01	1.047E+02	5.984E+00	1.750E+01	2.978E-07	1.080E+02
20.00	2.185E-01	4.190E+02	2.095E+01	2.000E+01	1.228E-12	4.301E+02
30.00	2.156E-01	9.428E+02	4.589E+01	2.054E+01	4.957E-16	9.673E+02

k = 1.2						
M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.772E+00
1.05	9.528E-01	1.112E+00	1.018E+00	1.092E+00	9.990E-01	1.871E+00
1.10	9.106E-01	1.229E+00	1.035E+00	1.187E+00	9.993E-01	1.983E+00
1.15	8.726E-01	1.352E+00	1.052E+00	1.285E+00	9.958E-01	2.098E+00
1.20	8.383E-01	1.480E+00	1.069E+00	1.384E+00	9.924E-01	2.224E+00
1.25	8.071E-01	1.614E+00	1.086E+00	1.486E+00	9.850E-01	2.354E+00
1.30	7.787E-01	1.753E+00	1.102E+00	1.591E+00	9.770E-01	2.493E+00
1.35	7.527E-01	1.897E+00	1.119E+00	1.695E+00	9.678E-01	2.643E+00
1.40	7.288E-01	2.047E+00	1.136E+00	1.802E+00	9.548E-01	2.794E+00
1.45	7.067E-01	2.203E+00	1.153E+00	1.911E+00	9.385E-01	2.949E+00
1.50	6.864E-01	2.364E+00	1.170E+00	2.021E+00	9.213E-01	3.113E+00
1.60	6.501E-01	2.702E+00	1.205E+00	2.242E+00	8.819E-01	3.462E+00
1.70	6.186E-01	3.062E+00	1.241E+00	2.467E+00	8.360E-01	3.835E+00
1.80	5.912E-01	3.444E+00	1.279E+00	2.693E+00	7.852E-01	4.230E+00
1.90	5.671E-01	3.847E+00	1.319E+00	2.917E+00	7.322E-01	4.654E+00
2.00	5.458E-01	4.273E+00	1.360E+00	3.142E+00	6.765E-01	5.094E+00
2.10	5.268E-01	4.720E+00	1.402E+00	3.367E+00	6.213E-01	5.563E+00
2.20	5.099E-01	5.189E+00	1.446E+00	3.589E+00	5.668E-01	6.054E+00
2.30	4.947E-01	5.680E+00	1.492E+00	3.807E+00	5.139E-01	6.567E+00
2.40	4.810E-01	6.193E+00	1.540E+00	4.021E+00	4.635E-01	7.102E+00
2.50	4.686E-01	6.727E+00	1.590E+00	4.231E+00	4.163E-01	7.665E+00
3.00	4.214E-01	9.727E+00	1.867E+00	5.210E+00	2.298E-01	1.081E+01
3.50	3.904E-01	1.327E+01	2.192E+00	6.054E+00	1.199E-01	1.455E+01
4.00	3.690E-01	1.736E+01	2.565E+00	6.768E+00	6.102E-02	1.885E+01
4.50	3.536E-01	2.200E+01	2.988E+00	7.363E+00	3.093E-02	2.370E+01
5.00	3.421E-01	2.718E+01	3.460E+00	7.855E+00	1.586E-02	2.916E+01
6.00	3.267E-01	3.918E+01	4.551E+00	8.609E+00	4.409E-03	4.177E+01
7.00	3.170E-01	5.336E+01	5.841E+00	9.135E+00	1.344E-03	5.668E+01
8.00	3.106E-01	6.973E+01	7.329E+00	9.514E+00	4.497E-04	7.385E+01
9.00	3.061E-01	8.827E+01	9.016E+00	9.790E+00	1.644E-04	9.337E+01
10.00	3.029E-01	1.090E+02	1.090E+01	1.000E+01	6.499E-05	1.151E+02
20.00	2.923E-01	4.363E+02	4.065E+01	1.073E+01	9.662E-08	4.590E+02
30.00	2.903E-01	9.817E+02	9.024E+01	1.088E+01	1.818E-09	1.033E+03

Table 3.3 (*continued*) Normal Shock Functions

k = 1.3

M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.832E+00
1.05	9.530E-01	1.116E+00	1.026E+00	1.088E+00	9.995E-01	1.940E+00
1.10	9.112E-01	1.237E+00	1.051E+00	1.177E+00	1.000E+00	2.060E+00
1.15	8.739E-01	1.365E+00	1.075E+00	1.270E+00	9.955E-01	2.181E+00
1.20	8.403E-01	1.497E+00	1.100E+00	1.361E+00	9.934E-01	2.318E+00
1.25	8.100E-01	1.636E+00	1.124E+00	1.456E+00	9.863E-01	2.456E+00
1.30	7.825E-01	1.780E+00	1.148E+00	1.551E+00	9.786E-01	2.605E+00
1.35	7.575E-01	1.930E+00	1.172E+00	1.647E+00	9.681E-01	2.759E+00
1.40	7.346E-01	2.085E+00	1.197E+00	1.742E+00	9.566E-01	2.923E+00
1.45	7.136E-01	2.246E+00	1.222E+00	1.838E+00	9.425E-01	3.092E+00
1.50	6.942E-01	2.413E+00	1.247E+00	1.935E+00	9.262E-01	3.266E+00
1.60	6.599E-01	2.763E+00	1.299E+00	2.127E+00	8.896E-01	3.637E+00
1.70	6.304E-01	3.137E+00	1.353E+00	2.319E+00	8.462E-01	4.029E+00
1.80	6.048E-01	3.532E+00	1.409E+00	2.507E+00	8.003E-01	4.453E+00
1.90	5.825E-01	3.950E+00	1.467E+00	2.693E+00	7.513E-01	4.900E+00
2.00	5.629E-01	4.391E+00	1.527E+00	2.876E+00	7.007E-01	5.371E+00
2.10	5.455E-01	4.855E+00	1.591E+00	3.052E+00	6.498E-01	5.865E+00
2.20	5.301E-01	5.341E+00	1.656E+00	3.225E+00	5.999E-01	6.387E+00
2.30	5.163E-01	5.850E+00	1.725E+00	3.391E+00	5.514E-01	6.931E+00
2.40	5.040E-01	6.381E+00	1.796E+00	3.553E+00	5.050E-01	7.503E+00
2.50	4.929E-01	6.935E+00	1.869E+00	3.711E+00	4.609E-01	8.097E+00
3.00	4.511E-01	1.004E+01	2.280E+00	4.404E+00	2.825E-01	1.145E+01
3.50	4.241E-01	1.372E+01	2.763E+00	4.966E+00	1.676E-01	1.539E+01
4.00	4.058E-01	1.796E+01	3.318E+00	5.413E+00	9.926E-02	1.995E+01
4.50	3.927E-01	2.276E+01	3.946E+00	5.768E+00	5.940E-02	2.514E+01
5.00	3.832E-01	2.813E+01	4.648E+00	6.052E+00	3.613E-02	3.092E+01
6.00	3.704E-01	4.057E+01	6.271E+00	6.469E+00	1.422E-02	4.429E+01
7.00	3.625E-01	5.526E+01	8.189E+00	6.748E+00	6.098E-03	6.014E+01
8.00	3.573E-01	7.222E+01	1.040E+01	6.944E+00	2.827E-03	7.839E+01
9.00	3.536E-01	9.143E+01	1.291E+01	7.082E+00	1.404E-03	9.912E+01
10.00	3.510E-01	1.129E+02	1.571E+01	7.187E+00	7.405E-04	1.223E+02
20.00	3.426E-01	4.520E+02	5.994E+01	7.541E+00	8.948E-06	4.877E+02
30.00	3.410E-01	1.017E+03	1.337E+02	7.607E+00	6.237E-07	1.097E+03

k = 1.4						
M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.893E+00
1.05	9.531E-01	1.120E+00	1.033E+00	1.084E+00	9.989E-01	2.006E+00
1.10	9.118E-01	1.245E+00	1.065E+00	1.169E+00	9.989E-01	2.133E+00
1.15	8.750E-01	1.376E+00	1.097E+00	1.254E+00	9.971E-01	2.267E+00
1.20	8.422E-01	1.513E+00	1.128E+00	1.341E+00	9.933E-01	2.409E+00
1.25	8.126E-01	1.656E+00	1.159E+00	1.429E+00	9.874E-01	2.558E+00
1.30	7.860E-01	1.805E+00	1.191E+00	1.516E+00	9.794E-01	2.714E+00
1.35	7.618E-01	1.960E+00	1.223E+00	1.603E+00	9.692E-01	2.876E+00
1.40	7.397E-01	2.120E+00	1.255E+00	1.689E+00	9.582E-01	3.049E+00
1.45	7.196E-01	2.286E+00	1.287E+00	1.776E+00	9.451E-01	3.229E+00
1.50	7.011E-01	2.458E+00	1.320E+00	1.862E+00	9.301E-01	3.414E+00
1.60	6.684E-01	2.820E+00	1.388E+00	2.032E+00	8.952E-01	3.805E+00
1.70	6.405E-01	3.205E+00	1.458E+00	2.198E+00	8.557E-01	4.224E+00
1.80	6.165E-01	3.613E+00	1.532E+00	2.358E+00	8.129E-01	4.671E+00
1.90	5.956E-01	4.045E+00	1.608E+00	2.516E+00	7.674E-01	5.142E+00
2.00	5.774E-01	4.500E+00	1.688E+00	2.666E+00	7.209E-01	5.640E+00
2.10	5.613E-01	4.978E+00	1.770E+00	2.812E+00	6.743E-01	6.166E+00
2.20	5.471E-01	5.480E+00	1.857E+00	2.951E+00	6.281E-01	6.716E+00
2.30	5.344E-01	6.005E+00	1.947E+00	3.084E+00	5.833E-01	7.294E+00
2.40	5.231E-01	6.553E+00	2.040E+00	3.212E+00	5.402E-01	7.898E+00
2.50	5.130E-01	7.125E+00	2.138E+00	3.333E+00	4.990E-01	8.526E+00
3.00	4.752E-01	1.033E+01	2.679E+00	3.856E+00	3.286E-01	1.207E+01
3.50	4.512E-01	1.413E+01	3.315E+00	4.262E+00	2.128E-01	1.623E+01
4.00	4.350E-01	1.850E+01	4.047E+00	4.571E+00	1.388E-01	2.107E+01
4.50	4.236E-01	2.346E+01	4.875E+00	4.812E+00	9.168E-02	2.653E+01
5.00	4.152E-01	2.900E+01	5.800E+00	5.000E+00	6.172E-02	3.265E+01
6.00	4.042E-01	4.183E+01	7.941E+00	5.268E+00	2.966E-02	4.682E+01
7.00	3.974E-01	5.700E+01	1.047E+01	5.444E+00	1.535E-02	6.355E+01
8.00	3.929E-01	7.450E+01	1.339E+01	5.564E+00	8.488E-03	8.287E+01
9.00	3.898E-01	9.433E+01	1.669E+01	5.652E+00	4.964E-03	1.048E+02
10.00	3.876E-01	1.165E+02	2.039E+01	5.714E+00	3.045E-03	1.292E+02
20.00	3.804E-01	4.665E+02	7.872E+01	5.926E+00	1.078E-04	5.155E+02
30.00	3.790E-01	1.050E+03	1.759E+02	5.969E+00	1.453E-05	1.159E+03

Table 3.3 (*continued*) Normal Shock Functions

k = 1.5

M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.953E+00
1.05	9.533E-01	1.123E+00	1.039E+00	1.081E+00	9.999E-01	2.075E+00
1.10	9.123E-01	1.252E+00	1.078E+00	1.161E+00	9.990E-01	2.207E+00
1.15	8.761E-01	1.387E+00	1.116E+00	1.243E+00	9.968E-01	2.348E+00
1.20	8.438E-01	1.528E+00	1.154E+00	1.324E+00	9.930E-01	2.498E+00
1.25	8.150E-01	1.675E+00	1.193E+00	1.404E+00	9.875E-01	2.656E+00
1.30	7.890E-01	1.828E+00	1.231E+00	1.485E+00	9.801E-01	2.821E+00
1.35	7.655E-01	1.987E+00	1.270E+00	1.565E+00	9.709E-01	2.995E+00
1.40	7.442E-01	2.152E+00	1.309E+00	1.644E+00	9.600E-01	3.176E+00
1.45	7.248E-01	2.323E+00	1.349E+00	1.722E+00	9.473E-01	3.364E+00
1.50	7.071E-01	2.500E+00	1.389E+00	1.800E+00	9.331E-01	3.560E+00
1.60	6.759E-01	2.872E+00	1.472E+00	1.951E+00	9.006E-01	3.973E+00
1.70	6.494E-01	3.268E+00	1.558E+00	2.098E+00	8.637E-01	4.414E+00
1.80	6.266E-01	3.688E+00	1.648E+00	2.238E+00	8.237E-01	4.884E+00
1.90	6.069E-01	4.132E+00	1.742E+00	2.372E+00	7.816E-01	5.382E+00
2.00	5.898E-01	4.600E+00	1.840E+00	2.500E+00	7.384E-01	5.907E+00
2.10	5.747E-01	5.092E+00	1.942E+00	2.622E+00	6.951E-01	6.461E+00
2.20	5.615E-01	5.608E+00	2.049E+00	2.737E+00	6.523E-01	7.041E+00
2.30	5.497E-01	6.148E+00	2.159E+00	2.848E+00	6.106E-01	7.649E+00
2.40	5.393E-01	6.712E+00	2.275E+00	2.950E+00	5.703E-01	8.285E+00
2.50	5.299E-01	7.300E+00	2.394E+00	3.049E+00	5.318E-01	8.948E+00
3.00	4.953E-01	1.060E+01	3.062E+00	3.462E+00	3.691E-01	1.267E+01
3.50	4.734E-01	1.450E+01	3.847E+00	3.769E+00	2.547E-01	1.708E+01
4.00	4.588E-01	1.900E+01	4.750E+00	4.000E+00	1.773E-01	2.216E+01
4.50	4.486E-01	2.410E+01	5.772E+00	4.175E+00	1.253E-01	2.792E+01
5.00	4.412E-01	2.980E+01	6.914E+00	4.310E+00	9.018E-02	3.437E+01
6.00	4.313E-01	4.300E+01	9.556E+00	4.500E+00	4.928E-02	4.928E+01
7.00	4.253E-01	5.860E+01	1.268E+01	4.621E+00	2.877E-02	6.691E+01
8.00	4.214E-01	7.660E+01	1.628E+01	4.705E+00	1.776E-02	8.726E+01
9.00	4.186E-01	9.700E+01	2.036E+01	4.764E+00	1.150E-02	1.103E+02
10.00	4.167E-01	1.198E+02	2.492E+01	4.807E+00	7.743E-03	1.361E+02
20.00	4.104E-01	4.798E+02	9.692E+01	4.950E+00	5.270E-04	5.430E+02
30.00	4.092E-01	1.080E+03	2.169E+02	4.979E+00	1.058E-04	1.221E+03

k = 5/3

M_x	M_y	p_y/p_x	T_y/T_x	$\rho_y/\rho_x = v_x/v_y$	p_{oy}/p_{ox}	p_{oy}/p_x
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	2.055E+00
1.05	9.535E-01	1.128E+00	1.050E+00	1.074E+00	1.000E+00	2.189E+00
1.10	9.131E-01	1.263E+00	1.099E+00	1.149E+00	9.986E-01	2.332E+00
1.15	8.776E-01	1.403E+00	1.147E+00	1.223E+00	9.974E-01	2.488E+00
1.20	8.463E-01	1.550E+00	1.196E+00	1.296E+00	9.937E-01	2.651E+00
1.25	8.184E-01	1.704E+00	1.244E+00	1.370E+00	9.879E-01	2.821E+00
1.30	7.935E-01	1.863E+00	1.293E+00	1.441E+00	9.814E-01	3.002E+00
1.35	7.711E-01	2.029E+00	1.343E+00	1.511E+00	9.727E-01	3.191E+00
1.40	7.509E-01	2.201E+00	1.393E+00	1.580E+00	9.626E-01	3.387E+00
1.45	7.325E-01	2.379E+00	1.445E+00	1.646E+00	9.512E-01	3.593E+00
1.50	7.158E-01	2.564E+00	1.497E+00	1.713E+00	9.379E-01	3.804E+00
1.60	6.866E-01	2.951E+00	1.604E+00	1.840E+00	9.089E-01	4.255E+00
1.70	6.620E-01	3.364E+00	1.716E+00	1.960E+00	8.755E-01	4.734E+00
1.80	6.410E-01	3.802E+00	1.833E+00	2.074E+00	8.395E-01	5.244E+00
1.90	6.229E-01	4.265E+00	1.955E+00	2.182E+00	8.019E-01	5.784E+00
2.00	6.073E-01	4.753E+00	2.083E+00	2.282E+00	7.634E-01	6.354E+00
2.10	5.936E-01	5.266E+00	2.216E+00	2.376E+00	7.248E-01	6.954E+00
2.20	5.817E-01	5.804E+00	2.355E+00	2.465E+00	6.865E-01	7.584E+00
2.30	5.711E-01	6.367E+00	2.499E+00	2.548E+00	6.492E-01	8.243E+00
2.40	5.617E-01	6.954E+00	2.650E+00	2.624E+00	6.131E-01	8.935E+00
2.50	5.534E-01	7.567E+00	2.806E+00	2.697E+00	5.783E-01	9.654E+00
3.00	5.227E-01	1.101E+01	3.678E+00	2.993E+00	4.282E-01	1.369E+01
3.50	5.036E-01	1.507E+01	4.704E+00	3.204E+00	3.178E-01	1.848E+01
4.00	4.910E-01	1.976E+01	5.885E+00	3.358E+00	2.385E-01	2.399E+01
4.50	4.822E-01	2.508E+01	7.221E+00	3.473E+00	1.816E-01	3.024E+01
5.00	4.758E-01	3.102E+01	8.714E+00	3.560E+00	1.407E-01	3.723E+01
6.00	4.674E-01	4.478E+01	1.217E+01	3.680E+00	8.832E-02	5.341E+01
7.00	4.623E-01	6.104E+01	1.625E+01	3.756E+00	5.855E-02	7.253E+01
8.00	4.589E-01	7.981E+01	2.096E+01	3.808E+00	4.059E-02	9.459E+01
9.00	4.566E-01	1.011E+02	2.630E+01	3.844E+00	2.919E-02	1.196E+02
10.00	4.550E-01	1.248E+02	3.226E+01	3.869E+00	2.168E-02	1.476E+02
20.00	4.497E-01	5.001E+02	1.264E+02	3.956E+00	2.885E-03	5.889E+02
30.00	4.487E-01	1.126E+03	2.834E+02	3.973E+00	8.679E-04	1.324E+03

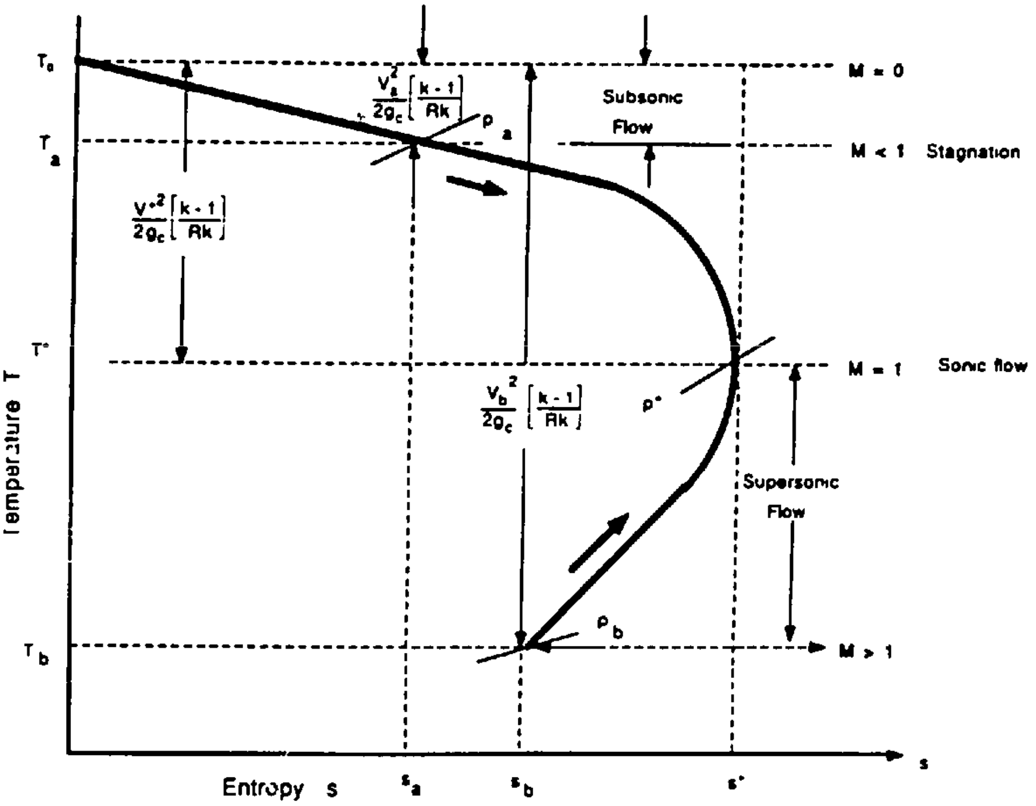


Figure 3.16 Notation for Fanno flow study.

requires that, for an adiabatic process, the entropy may not decrease. The effect of friction is to limit the expansion of the gas from p_a to p^* and sonic velocity. For this reason supersonic flow cannot exist in a pipe if the initial flow is subsonic. In case (b) the gas enters the pipe with a supersonic velocity. Again, the second law of thermodynamics requires that, for an adiabatic process, the entropy may not decrease. The effect of friction is to limit the compression of the gas from p_b to p^* and sonic velocity. For this reason subsonic flow cannot exist in a pipe if the initial flow is supersonic. The limiting velocity in either case is sonic.

General Considerations

Adiabatic compressible flow of an ideal gas with friction in a constant-area duct must satisfy the following requirements:

1. *The ideal gas law*: The equation of state for an ideal gas is equation (1.37):

$$pv = RT \tag{1.37}$$

2. *Constant-area duct*: The flow area must be the same at all sections:

$$A = A_1 = A_2 = \cdots = A_n$$

3. *Conservation of mass*: The continuity equation may be expressed as follows:

$$\dot{m} = \frac{Av}{\nu} = \frac{Av_1}{\nu_1} = \frac{Av_2}{\nu_2} \tag{3.6}$$

4 *Conservation of energy*: The sum of all the energy at a section is the same for all sections:

$$\frac{RkT}{k-1} + \frac{v^2}{2g_c} = \frac{RkT_1}{k-1} + \frac{v_1^2}{2g_c} = \frac{RkT_2}{k-1} + \frac{v_2^2}{2g_c} \tag{3.97}$$

5. *Equation of motion*: Writing equation (3.18) for a horizontal pipe results in the following:

$$\frac{v\,dv}{g_c} + \nu\,dp + \frac{\nu\tau}{R_h}\,dL = 0 \tag{3.147}$$

6. *Constant-friction factor*: In Chapter 4 the variation of friction factor with various parameters is presented. Conventional engineering practice is to use a friction factor f to calculate friction losses in pipes. Methods for the computation of numerical values of friction factor are given in Chapter 4. The friction factor f is defined as follows:

$$H_f = \frac{f}{D}\frac{v^2}{2g_c}\int_1^2 dL \tag{3.148}$$

The factor defined by equation (3.148) is known as the D'Arcy-Weisbach friction factor. There is another friction factor used in some texts, known as the Fanning friction factor. The numerical value of the Fanning friction factor is 1/4 that of the D'Arcy–Weisbach, so care must be used when selecting a friction factor from another source.

In Section 3.3, the energy lost due to friction, H_f, was defined by equation (3.20) as follows:

$$H_f = \frac{1}{R_h}\int_1^2 \nu\tau\,dL \tag{3.20}$$

Setting equation (3.148) equal to equation (3.20) and solving for τ results in the following:

$$\tau = \frac{R_h f v^2}{2gcD\nu} \tag{3.149}$$

Substituting the value of τ from equation (3.149) in equation (3.147) results in the following:

$$\frac{v\,dv}{g_c} + \nu\,dp + \frac{\nu}{R_h}\left(\frac{R_h f v^2}{2g_c D\nu}\right) dL = 0$$

which reduces to:

$$\frac{v\,dv}{g_c} + \nu\,dp + \frac{f}{D}\frac{v^2}{2g_c}\,dL = 0 \tag{3.150}$$

Dividing equation (3.150) by $p\nu$ results in the following:

$$\frac{v\,dv}{(p\nu)g_c} + \frac{\nu\,dp}{p\nu} + \frac{fv^2}{2g_c D(p\nu)}\,dL = 0 \tag{3.151}$$

For an ideal gas from the equation of state (1.37), $p\nu = RT$. Substituting for $p\nu$ in equation (3.151) yields the following:

$$\frac{v\,dv}{RTg_c} + \frac{dp}{p} + \frac{f}{2D}\frac{v^2}{RTg_c}\,dL = 0 \tag{3.152}$$

Derivation of equations

All of the terms of equation (3.152) can be expressed as functions of Mach number. If the first term of equation (3.152) is multiplied by v/v and the relation $v^2 = kg_cRTM^2$ from equation (3.94) is substituted, the following expression results:

$$\frac{v}{RTg_c}\,dv = \left(\frac{v}{v}\right)\frac{v}{RTg_c}\,dv = \frac{v^2}{RTg_c}\frac{dv}{v} = \frac{kg_cRTM^2}{RTg_c}\frac{dv}{v} = kM^2\,\frac{dv}{v} \tag{3.153}$$

The energy equation for an ideal gas (3.97) may be written as follows:

$$\frac{RkT_0}{k-1} = \frac{RkT}{k-1} + \frac{v^2}{2g_c} \tag{3.154}$$

Differentiating equation (3.154) yields the following:

$$\frac{Rk}{k-1}\,dT + \frac{v\,dv}{2g_c} = 0 = \frac{kg_cR}{k-1}\,dT + v\,dv = 0 \tag{3.155}$$

Dividing equation (3.155) by $v^2 = kg_cRTM^2$ yields the following:

$$\frac{[kg_cR/(k-1)dT}{(kg_cRTM^2)} = -\frac{v\,dv}{v^2} = \frac{1}{(k-1)M^2}\frac{dT}{T} = -\frac{dv}{v}$$

or

$$\frac{dT}{T} = -(k-1)M^2\,\frac{dv}{v} \tag{3.156}$$

Writing $v^2 = kg_c RTM^2$ in logarithmic form, differentiating, and solving for dT/T, results in the following:

$$2 \log_e v = \log_e(kg_c R) + \log_e T + 2 \log_3 M$$

$$2 \frac{dv}{v} = \frac{dT}{T} + 2 \frac{dM}{M} \qquad \text{or} \qquad \frac{dT}{T} = 2 \frac{dv}{v} - 2 \frac{dM}{M} \tag{3.157}$$

Substituting for dT/T from equation (3.157) in equation (3.156) and simplifying results in the following:

$$\frac{dT}{T} = -(k-1)M^2 \frac{dv}{v} - 2 \frac{dM}{M}$$

which reduces to:

$$\frac{dv}{v} = \frac{1}{1 + [(k-1)/2]M^2} \frac{dM}{M} \tag{3.158}$$

Eliminating dv/v from equation (3.153) by substitution of equation (3.158) and simplifying results in the expression of the first term of equation (3.152) as a function of Mach number:

$$\frac{v}{RTg_c} dv = kM^2 \frac{dv}{v} = kM^2 \left[\left(\frac{1}{1 + \frac{k-1}{2} M^2} \right) \frac{dM}{M} = \frac{kM}{1 + [(k-1)/2]M^2} dM \right. \tag{3.159}$$

Evaluation of the middle term of equation (3.152) starts by expressing the continuity equation for an ideal gas ($\dot{m} = Avp/RT$) in logarithmic form and differentiating, noting that, for a constant-area duct, A is a constant:

$$\log_e \dot{m} = \log_e A + \log_e v + \log_e p - \log_e R - \log_e T$$

$$0 = 0 + \frac{dv}{v} + \frac{dp}{p} - 0 - \frac{dT}{T}$$

or

$$\frac{dp}{p} = \frac{dT}{T} - \frac{dv}{v} \tag{3.160}$$

Substituting dT/T from equation (3.156) and dv/v from equation (3.158) in equation (3.160) and simplifying results in the following:

$$\frac{dp}{p} = -[(k-1)M^2]\left[\frac{1}{1 + [(k-1)/2]M^2}\right] \frac{dM}{M} - \left[\frac{1}{1 + [(k-1)/2]M^2}\right] \frac{dM}{M}$$

which reduces to:

$$\frac{dp}{p} = -\left[\frac{k+1)M^2+1}{1+[(k-1)/2]M^2}\right]\frac{dM}{M} \tag{3.161}$$

The last term of equation (3.152) may be converted to a Mach number relation by substitution of $v^2 = kg_c RTM^2$.

$$\frac{f}{2D}\frac{v^2}{RTg_c}\,dL = \frac{f}{2D}\frac{kg_c RTM^2}{RTg_c}\,dL = \frac{f}{2D}kM^2\,dL \tag{3.162}$$

Substituting equations (3.159), (3.160), and (3.161) for the first, second, and third terms of equation (3.152), respectively, results in the following:

$$\frac{v}{RTg_c}\,dv + \frac{dp}{p} + \frac{f}{2D}\frac{v^2}{RTg_c}\,dL = 0$$

$$\frac{kM}{1+\frac{(k-1)}{2}M^2}\,dM - \frac{(k-1)M^2+1}{1+\frac{(k-1)}{2}M^2}\frac{dM}{M} + \frac{f}{2D}kM^2\,dL = 0 \tag{3.163}$$

Solving equation (3.163) for fdL/D, results in the following:

$$\frac{f}{D}\,dL = \frac{2(1-M^2)}{kM^3\left(1+\frac{(k-1)}{2}M^2\right)}\,dM$$

$$\frac{f}{D}\,dL = \frac{2}{k}\frac{dM}{M^3} - \frac{k+1}{k}\left[\frac{1}{\left(1+\frac{(k-1)}{2}M^2\right)}\right]\frac{dM}{M} \tag{3.164}$$

Integrating equation (3.164) between the limits of L_1 and L_2 and M_1 and M_2 yields the following:

$$\frac{f}{D}\int_1^2 dL = \frac{2}{k}\int_1^2 \frac{dM}{M^3} - \frac{k+1}{k}\int_1^2\left[\frac{1}{\left(1+\frac{k-1}{2}M^2\right)}\right]\frac{dM}{M}$$

$$\frac{f}{D}(L_2-L_1) = \frac{1}{k}\left(\frac{1}{M_1^2}-\frac{1}{M_2^2}\right) + \frac{k+1}{2k}\log_e\left[\frac{M_1^2\,(k-1)M_2^2+2}{M_2^2\,(k-1)M_1^2+2}\right] \tag{3.165}$$

The maximum length L^* is obtained at the point in the pipe where the velocity is sonic. Substituting in equation (3.165) L^* for $L_2 - L_1$, M for M_1, and 1 for M_2 results in the following:

$$\frac{fL^*}{D}=\frac{1}{k}\left(\frac{1}{M^2}-\frac{1}{1}\right)+\frac{k+1}{2k}\log_e\left[\frac{M^2}{1}\frac{(k-1)\times 1^2+2}{(k-1)M^2+2}\right]$$

$$\frac{fL^*}{D}=\frac{1-M^2}{kM^2}+\frac{k+1}{2k}\log_e\left[\frac{(k+1)M^2}{2\left(1+\frac{k-1}{2}M^2\right)}\right] \tag{3.166}$$

Texts and reference sources that use the Fanning friction factor express the first term of equation (3.166) as $4fL^*/D$.

For adiabatic flow, the total energy at each section is a constant whether or not friction is involved, so equation (3.98) may be applied. Expressing this equation for $T_1 = T$, $T_2 = T^*$, $M_1 = M$, and $M_2 = 1$ results in the following:

$$\frac{T_2}{T_1}=\frac{1+\left(\frac{k-1}{2}\right)M_1^2}{1+\left(\frac{k-1}{2}\right)M_2^2}=\frac{T^*}{T}=\frac{1+\left(\frac{k-1}{2}\right)M^2}{1+\left(\frac{k-1}{2}\right)\times 1^2}$$

or

$$\frac{T}{T^*}=\frac{k+1}{2\left[1+\left(\frac{k-1}{2}\right)M^2\right]} \tag{3.167}$$

Again writing the continuity equation for an ideal gas ($\dot{m} = Avp/RT$) and noting from equation (3.94) that $v = M(kg_cRT)^{1/2}$ leads to the following:

$$\frac{Av_1p_1}{RT_1}=\frac{Av_2p_2}{RT_2}$$

Solving for $p_2/p_1 = v_1T_2/v_2T_1$ results in the following:

$$\frac{p_2}{p_1}=\frac{(M_1\sqrt{kg_cRT_1})T_2}{(M2\sqrt{kg_cRT)_2})T_1}=\frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{1/2} \tag{3.168}$$

Substituting T_2/T_1 from equation (3.98) in equation (3.168) results in the following:

$$\frac{p_2}{p_1}=\frac{M_1}{M_2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_1^2}{1+\left(\frac{k-1}{2}\right)M_2^2}\right]^{1/2} \tag{3.169}$$

Substituting in equation (3.169) for $p_1 = p, p_2 = p^*$, $M_1 = M$, and $M_2 = 1$ results in the following:

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_1^2}{1+\left(\frac{k-1}{2}\right)M_2^2}\right]^{1/2} = \frac{p^*}{p} = \frac{M}{1}\left[\frac{1+\left(\frac{k-1}{2}\right)M_1^2}{1+\left(\frac{k-1}{2}\right)\times 1^2}\right]^{1/2}$$

or

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{k+1}{2\left[1+\left(\frac{k-1}{2}\right)M^2\right]}} \qquad (3.170)$$

If equation (3.144) is written in dimensionless form and integrated between the limits of 1 and 2, the following results:

$$\frac{1}{R}\int_1^2 ds = \frac{k}{k-1}\int_1^2 \frac{dT}{T} - \int_1^2 \frac{dp}{p} = \frac{s_2 - s_1}{R} = \frac{k}{k-1}\log_e\left(\frac{T_2}{T_1}\right) - \log_e\left(\frac{p_2}{p_1}\right)$$

Substituting for T_2/T_1 from equation (3.98) and for p_2/p_1 from equation (3.169) yields the following:

$$\frac{s_2 - s_1}{R} = \frac{k}{k-1}\log_e\left[\frac{1+\frac{(k-1)}{2}M_1^2}{1+\frac{(k-1)}{2}M_2^2}\right] - \log_e\left[\frac{M_1}{M_2}\left(\frac{1+\frac{(k-1)}{2}M_1^2}{1+\frac{(k-1)}{2}M_2^2}\right)^{1/2}\right]$$

$$\frac{s_2 - s_1}{R} = \log_e\left[\frac{M_2}{M_1}\left(\frac{1+\frac{(k-1)}{2}M_1^2}{1+\frac{(k-1)}{2}M_2^2}\right)^{\frac{k+1}{2(k-1)}}\right] \qquad (3.171)$$

Substituting in equation (3.171) for $s_2 - s_1 = s^*$, $M_1 = M$, and $M_2 = 1$ results in the following:

$$\frac{s^*}{R} = \log_e\left[\frac{1}{M}\left(\frac{1+\frac{(k-1)}{2}M^2}{1+\frac{(k-1)}{2}\times 1^2}\right)^{\frac{k+1}{2(k-1)}}\right] = \log_e\left[\frac{1}{M}\left(\frac{1+\frac{(k-1)}{2}M^2}{\frac{(k+1)}{2}}\right)^{\frac{k+1}{2(k-1)}}\right] \qquad (3.172)$$

The stagnation pressure ratio is obtained by multiplying the stagnation pressure ratios [equation (3.100)] by the pressure ratio [equation (3.169)]:

$$\frac{p_{01}}{p_{02}} = \left(\frac{p_2}{p_{02}}\right)\left(\frac{p_{01}}{p_1}\right)\left(\frac{p_1}{p_2}\right)$$

$$\frac{p_{01}}{p_{02}} = \left[1 + \frac{(k-1)}{2} M_2^2\right]^{k/(1-k)} \left[1 + \frac{(k-1)}{2} M_1^2\right]^{k/(k-1)}$$

$$\times \left[\frac{M_2}{M_1}\left(\frac{1 + \frac{(k-1)}{2} M_1^2}{1 + \frac{(k-1)}{2} M_2^2}\right)^{-1/2}\right]$$

which reduces to

$$\frac{p_{01}}{p_{02}} = \frac{M_2}{M_1}\left[\frac{1 + \frac{(k-1)}{2} M_1^2}{1 + \frac{(k-1)}{2} M_2^2}\right]^{(k+1)/2(k-1)} \tag{3.173}$$

Substituting p_0 for p_{01}, p_0^* for p_{02}, M for M_1, and 1 for M_2 in equation (3.173) results in the following:

$$\frac{p_{01}}{p_{02}} = \frac{M_2}{M_1}\left[\frac{1 + \frac{(k-1)}{2} M_1^2}{1 + \frac{(k-1)}{2} M_2^2}\right]^{(k+1)/2(k-1)}$$

$$= \frac{p_0}{p_0^*} = \frac{1}{M}\left[\frac{1 + \frac{(k-1}{2} M^2}{1 + \frac{(k-1)}{2} \times 1^2}\right]^{(k+1)/2(k-1)}$$

$$\frac{p_0}{p_0^*} = \frac{1}{M}\left[\frac{1 + \frac{(k-1)}{2} M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \tag{3.174}$$

The density ratio is obtained by writing the equation of state (1.37) $\rho = p/RT$ and substituting equation (3.169) for p_2/p_1 and equation (3.98) for T_1/T_2 as follows:

$$\frac{\rho_2}{\rho_1} = \frac{p_2/RT_2}{p_1/RT_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{T_1}{T_2}\right)$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_1^2}{1+\left(\frac{k-1}{2}\right)M_2^2}\right]^{1/2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_2^2}{1+\left(\frac{k-1}{2}\right)M_1^2}\right] \tag{3.175}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_2^2}{1+\left(\frac{k-1}{2}\right)M_1^2}\right]^{1/2}$$

Substituting in equation (3.175) for $p_1 = p$, $p_2 = p^*$, $M_1 = M$, and $M_2 = 1$ results in the following:

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_2^2}{1+\left(\frac{k-1}{2}\right)M_1^2}\right]^{1/2} = \frac{\rho^*}{\rho} = \frac{M}{1}\left[\frac{1+\left(\frac{k-1}{2}\right)\times 1^2}{1+\left(\frac{k-1}{2}\right)M^2}\right]^{1/2}$$

$$\frac{\rho^*}{\rho} = M\sqrt{\frac{\frac{k+1}{2}}{1+\left(\frac{k-1}{2}\right)M^2}} \tag{3.176}$$

From the continuity equation (3.6), the following can be written:

$$\dot{m} = \rho A v = \rho^* A v^* \qquad \text{or} \qquad \frac{v}{v^*} = \frac{\rho^*}{\rho} \tag{3.177}$$

Tabulated Values of Fanno Flow Functions

As in the case of isentropic flow and normal shock functions, it has been found useful to compute and tabulate certain standard Fanno flow functions. These functions are all dimensionless ratios and are functions of the inlet Mach number M. Table 3.4 contains the following ratios:

Table 3.4 Fanno Line Functions

			k = 1			
M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
0.00	1.000E+00	∞	∞	0.000E+00	∞	∞
0.01	1.000E+00	1.000E+02	6.066E+01	1.000E-02	9.990E+03	4.105E+00
0.02	1.000E+00	5.000E+01	3.033E+01	2.000E-02	2.491E+03	3.412E+00
0.03	1.000E+00	3.333E+01	2.023E+01	3.000E-02	1.103E+03	3.007E+00
0.04	1.000E+00	2.500E+01	1.518E+01	4.000E-02	6.176E+02	2.720E+00
0.05	1.000E+00	2.000E+01	1.215E+01	5.000E-02	3.930E+02	2.497E+00
0.06	1.000E+00	1.667E+01	1.013E+01	6.000E-02	2.712E+02	2.315E+00
0.07	1.000E+00	1.429E+01	8.686E+00	7.000E-02	1.978E+02	2.162E+00
0.08	1.000E+00	1.250E+01	7.606E+00	8.000E-02	1.502E+02	2.029E+00
0.09	1.000E+00	1.111E+01	6.767E+00	9.000E-02	1.176E+02	1.912E+00
0.10	1.000E+00	1.000E+01	6.096E+00	1.000E-01	9.439E+01	1.808E+00
0.15	1.000E+00	6.667E+00	4.089E+00	1.500E-01	3.965E+01	1.408E+00
0.20	1.000E+00	5.000E+00	3.094E+00	2.000E-01	2.078E+01	1.129E+00
0.25	1.000E+00	4.000E+00	2.503E+00	2.500E-01	1.223E+01	9.175E-01
0.30	1.000E+00	3.333E+00	2.115E+00	3.000E-01	7.703E+00	7.490E-01
0.35	1.000E+00	2.857E+00	1.842E+00	3.500E-01	5.064E+00	6.111E-01
0.40	1.000E+00	2.500E+00	1.643E+00	4.000E-01	3.417E+00	4.963E-01
0.45	1.000E+00	2.222E+00	1.491E+00	4.500E-01	2.341E+00	3.998E-01
0.50	1.000E+00	2.000E+00	1.375E+00	5.000E-01	1.614E+00	3.181E-01
0.60	1.000E+00	1.667E+00	1.210E+00	6.000E-01	7.561E-01	1.908E-01
0.70	1.000E+00	1.429E+00	1.107E+00	7.000E-01	3.275E-01	1.017E-01
0.80	1.000E+00	1.250E+00	1.044E+00	8.000E-01	1.162E-01	4.314E-02
0.90	1.000E+00	1.111E+00	1.010E+00	9.000E-01	2.385E-02	1.036E-02
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	1.000E+00	9.091E-01	1.010E+00	1.100E+00	1.707E-02	9.690E-03
1.20	1.000E+00	8.333E-01	1.038E+00	1.200E+00	5.909E-02	3.768E-02
1.30	1.000E+00	7.692E-01	1.086E+00	1.300E+00	1.164E-01	8.264E-02
1.40	1.000E+00	7.143E-01	1.154E+00	1.400E+00	1.831E-01	1.435E-01
1.50	1.000E+00	6.667E-01	1.245E+00	1.500E+00	2.554E-01	2.195E-01
1.60	1.000E+00	6.250E-01	1.363E+00	1.600E+00	3.306E-01	3.100E-01
1.70	1.000E+00	5.882E-01	1.513E+00	1.700E+00	4.073E-01	4.144E-01
1.80	1.000E+00	5.556E-01	1.703E+00	1.800E+00	4.842E-01	5.322E-01
1.90	1.000E+00	5.263E-01	1.941E+00	1.900E+00	5.607E-01	6.631E-01

Table 3.4 (*continued*) Fanno Line Functions

			k = 1			
M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
2.00	1.000E+00	5.000E-01	2.241E+00	2.000E+00	6.363E-01	8.069E-01
2.10	1.000E+00	4.762E-01	2.620E+00	2.100E+00	7.106E-01	9.631E-01
2.20	1.000E+00	4.545E-01	3.100E+00	2.200E+00	7.835E-01	1.132E+00
2.30	1.000E+00	4.348E-01	3.714E+00	2.300E+00	8.549E-01	1.312E+00
2.40	1.000E+00	4.167E-01	4.502E+00	2.400E+00	9.245E-01	1.505E+00
2.50	1.000E+00	4.000E-01	5.522E+00	2.500E+00	9.926E-01	1.709E+00
2.60	1.000E+00	3.846E-01	6.852E+00	2.600E+00	1.059E+00	1.924E+00
2.70	1.000E+00	3.704E-01	8.600E+00	2.700E+00	1.124E+00	2.152E+00
2.80	1.000E+00	3.571E-01	1.092E+01	2.800E+00	1.187E+00	2.390E+00
2.90	1.000E+00	3.448E-01	1.402E+01	2.900E+00	1.248E+00	2.640E+00
3.00	1.000E+00	3.333E-01	1.820E+01	3.000E+00	1.308E+00	2.901E+00
3.10	1.000E+00	3.226E-01	2.389E+01	3.100E+00	1.367E+00	3.174E+00
3.20	1.000E+00	3.125E-01	3.172E+01	3.200E+00	1.424E+00	3.457E+00
3.30	1.000E+00	3.030E-01	4.257E+01	3.300E+00	1.480E+00	3.751E+00
3.40	1.000E+00	2.941E-01	5.776E+01	3.400E+00	1.534E+00	4.056E+00
3.50	1.000E+00	2.857E-01	7.922E+01	3.500E+00	1.587E+00	4.372E+00
3.60	1.000E+00	2.778E-01	1.098E+02	3.600E+00	1.639E+00	4.699E+00
3.70	1.000E+00	2.703E-01	1.540E+02	3.700E+00	1.690E+00	5.037E+00
3.80	1.000E+00	2.632E-01	2.181E+02	3.800E+00	1.739E+00	5.385E+00
3.90	1.000E+00	2.564E-01	3.123E+02	3.900E+00	1.788E+00	5.744E+00
4.00	1.000E+00	2.500E-01	4.520E+02	4.000E+00	1.835E+00	6.114E+00
4.50	1.000E+00	2.222E-01	3.364E+03	4.500E+00	2.058E+00	8.121E+00
5.00	1.000E+00	2.000E-01	3.255E+04	5.000E+00	2.259E+00	1.039E+01
5.50	1.000E+00	1.818E-01	4.085E+05	5.500E+00	2.443E+00	1.292E+01
6.00	1.000E+00	1.667E-01	6.637E+06	6.000E+00	2.611E+00	1.571E+01
6.50	1.000E+00	1.538E-01	1.394E+08	6.500E+00	2.767E+00	1.875E+01
7.00	1.000E+00	1.429E-01	3.784E+09	7.000E+00	2.912E+00	2.205E+01
7.50	1.000E+00	1.333E-01	1.325E+11	7.500E+00	3.048E+00	2.561E+01
8.00	1.000E+00	1.250E-01	5.987E+12	8.000E+00	3.175E+00	2.942E+01
9.00	1.000E+00	1.111E-01	2.615E+16	9.000E+00	3.407E+00	3.780E+01
10	1.000E+00	1.000E-01	3.145E+20	1.000E+01	3.615E+00	4.720E+01
20	1.000E+00	5.000E-02	2.191E+85	2.000E+01	4.994E+00	1.965E+02
30	1.000E+00	3.333E-02	5.468E+193	3.000E+01	5.804E+00	4.461E+02

k = 1.1

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
0.00	1.050E+00	∞	∞	0.000E+00	∞	∞
0.01	1.050E+00	1.025E+02	5.991E+01	1.025E-02	9.081E+03	4.093E+00
0.02	1.050E+00	5.123E+01	2.996E+01	2.049E-02	2.264E+03	3.400E+00
0.03	1.050E+00	3.416E+01	1.998E+01	3.074E-02	1.003E+03	2.995E+00
0.04	1.050E+00	2.562E+01	1.499E+01	4.099E-02	5.612E+02	2.707E+00
0.05	1.050E+00	2.049E+01	1.200E+01	5.123E-02	3.571E+02	2.485E+00
0.06	1.050E+00	1.708E+01	1.000E+01	6.148E-02	2.463E+02	2.303E+00
0.07	1.050E+00	1.464E+01	8.581E+00	7.172E-02	1.796E+02	2.150E+00
0.08	1.050E+00	1.281E+01	7.514E+00	8.196E-02	1.364E+02	2.017E+00
0.09	1.050E+00	1.138E+01	6.685E+00	9.220E-02	1.068E+02	1.900E+00
0.10	1.049E+00	1.024E+01	6.023E+00	1.024E-01	8.565E+01	1.796E+00
0.15	1.049E+00	6.827E+00	4.042E+00	1.536E-01	3.592E+01	1.397E+00
0.20	1.048E+00	5.118E+00	3.059E+00	2.047E-01	1.879E+01	1.118E+00
0.25	1.047E+00	4.092E+00	2.476E+00	2.558E-01	1.103E+01	9.068E-01
0.30	1.045E+00	3.408E+00	2.093E+00	3.067E-01	6.936E+00	7.388E-01
0.35	1.044E+00	2.919E+00	1.825E+00	3.575E-01	4.549E+00	6.016E-01
0.40	1.042E+00	2.552E+00	1.629E+00	4.082E-01	3.062E+00	4.877E-01
0.45	1.039E+00	2.266E+00	1.480E+00	4.588E-01	2.093E+00	3.920E-01
0.50	1.037E+00	2.037E+00	1.365E+00	5.092E-01	1.439E+00	3.113E-01
0.60	1.031E+00	1.693E+00	1.204E+00	6.094E-01	6.705E-01	1.858E-01
0.70	1.025E+00	1.446E+00	1.104E+00	7.087E-01	2.887E-01	9.853E-02
0.80	1.017E+00	1.261E+00	1.042E+00	8.069E-01	1.019E-01	4.158E-02
0.90	1.009E+00	1.116E+00	1.010E+00	9.041E-01	2.078E-02	9.928E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.901E-01	9.046E-01	1.009E+00	1.095E+00	1.468E-02	9.168E-03
1.20	9.795E-01	8.247E-01	1.036E+00	1.188E+00	5.050E-02	3.541E-02
1.30	9.682E-01	7.569E-01	1.080E+00	1.279E+00	9.885E-02	7.709E-02
1.40	9.563E-01	6.985E-01	1.142E+00	1.369E+00	1.544E-01	1.329E-01
1.50	9.438E-01	6.477E-01	1.223E+00	1.457E+00	2.138E-01	2.016E-01
1.60	9.309E-01	6.030E-01	1.326E+00	1.544E+00	2.749E-01	2.824E-01
1.70	9.174E-01	5.634E-01	1.454E+00	1.628E+00	3.362E-01	3.742E-01
1.80	9.036E-01	5.281E-01	1.610E+00	1.711E+00	3.969E-01	4.764E-01
1.90	8.895E-01	4.964E-01	1.801E+00	1.792E+00	4.563E-01	5.882E-01

Table 3.4 (*continued*) Fanno Line Functions

k = 1.1

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
2.00	8.750E-01	4.677E-01	2.032E+00	1.871E+00	5.140E-01	7.089E-01
2.10	8.603E-01	4.417E-01	2.312E+00	1.948E+00	5.698E-01	8.380E-01
2.20	8.454E-01	4.179E-01	2.651E+00	2.023E+00	6.237E-01	9.748E-01
2.30	8.304E-01	3.962E-01	3.061E+00	2.096E+00	6.754E-01	1.119E+00
2.40	8.152E-01	3.762E-01	3.560E+00	2.167E+00	7.251E-01	1.270E+00
2.50	8.000E-01	3.578E-01	4.165E+00	2.236E+00	7.726E-01	1.427E+00
2.60	7.848E-01	3.407E-01	4.901E+00	2.303E+00	8.182E-01	1.590E+00
2.70	7.695E-01	3.249E-01	5.799E+00	2.368E+00	8.617E-01	1.758E+00
2.80	7.543E-01	3.102E-01	6.896E+00	2.432E+00	9.034E-01	1.931E+00
2.90	7.392E-01	2.965E-01	8.237E+00	2.493E+00	9.432E-01	2.109E+00
3.00	7.241E-01	2.837E-01	9.880E+00	2.553E+00	9.812E-01	2.291E+00
3.10	7.092E-01	2.717E-01	1.190E+01	2.611E+00	1.017E+00	2.476E+00
3.20	6.944E-01	2.604E-01	1.438E+01	2.667E+00	1.052E+00	2.666E+00
3.30	6.798E-01	2.499E-01	1.743E+01	2.721E+00	1.085E+00	2.858E+00
3.40	6.654E-01	2.399E-01	2.119E+01	2.773E+00	1.117E+00	3.054E+00
3.50	6.512E-01	2.306E-01	2.583E+01	2.824E+00	1.147E+00	3.252E+00
3.60	6.371E-01	2.217E-01	3.157E+01	2.874E+00	1.176E+00	3.452E+00
3.70	6.233E-01	2.134E-01	3.866E+01	2.921E+00	1.204E+00	3.655E+00
3.80	6.098E-01	2.055E-01	4.743E+01	2.967E+00	1.230E+00	3.859E+00
3.90	5.964E-01	1.980E-01	5.829E+01	3.012E+00	1.256E+00	4.066E+00
4.00	5.833E-01	1.909E-01	7.175E+01	3.055E+00	1.280E+00	4.273E+00
4.50	5.217E-01	1.605E-01	2.058E+02	3.250E+00	1.386E+00	5.327E+00
5.00	4.667E-01	1.366E-01	5.977E+02	3.416E+00	1.472E+00	6.393E+00
5.50	4.179E-01	1.175E-01	1.731E+03	3.556E+00	1.543E+00	7.456E+00
6.00	3.750E-01	1.021E-01	4.949E+03	3.674E+00	1.601E+00	8.507E+00
6.50	3.373E-01	8.936E-02	1.388E+04	3.775E+00	1.649E+00	9.538E+00
7.00	3.043E-01	7.881E-02	3.798E+04	3.862E+00	1.689E+00	1.054E+01
7.50	2.754E-01	6.997E-02	1.012E+05	3.936E+00	1.723E+00	1.152E+01
8.00	2.500E-01	6.250E-02	2.621E+05	4.000E+00	1.752E+00	1.248E+01
9.00	2.079E-01	5.066E-02	1.614E+06	4.104E+00	1.798E+00	1.429E+01
10	1.750E-01	4.183E-02	8.874E+06	4.183E+00	1.832E+00	1.600E+01
20	5.000E-02	1.118E-02	2.290E+12	4.472E+00	1.953E+00	2.846E+01
30	2.283E-02	5.036E-03	5.746E+15	4.532E+00	1.977E+00	3.629E+01

k = 1.2						
M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
0.00	1.100E+00	∞	∞	0.000E+00	∞	∞
0.01	1.100E+00	1.049E+02	5.921E+01	1.049E-02	8.324E+03	4.081E+00
0.02	1.100E+00	5.244E+01	2.961E+01	2.098E-02	2.075E+03	3.388E+00
0.03	1.100E+00	3.496E+01	1.974E+01	3.146E-02	9.188E+02	2.983E+00
0.04	1.100E+00	2.622E+01	1.481E+01	4.195E-02	5.142E+02	2.696E+00
0.05	1.100E+00	2.097E+01	1.186E+01	5.243E-02	3.271E+02	2.473E+00
0.06	1.100E+00	1.748E+01	9.887E+00	6.292E-02	2.256E+02	2.291E+00
0.07	1.099E+00	1.498E+01	8.480E+00	7.340E-02	1.644E+02	2.138E+00
0.08	1.099E+00	1.311E+01	7.426E+00	8.388E-02	1.248E+02	2.005E+00
0.09	1.099E+00	1.165E+01	6.607E+00	9.435E-02	9.772E+01	1.888E+00
0.10	1.099E+00	1.048E+01	5.953E+00	1.048E-01	7.837E+01	1.784E+00
0.15	1.098E+00	6.984E+00	3.996E+00	1.571E-01	3.281E+01	1.385E+00
0.20	1.096E+00	5.234E+00	3.026E+00	2.093E-01	1.713E+01	1.107E+00
0.25	1.093E+00	4.182E+00	2.451E+00	2.614E-01	1.004E+01	8.964E-01
0.30	1.090E+00	3.480E+00	2.073E+00	3.132E-01	6.298E+00	7.290E-01
0.35	1.087E+00	2.978E+00	1.809E+00	3.649E-01	4.121E+00	5.926E-01
0.40	1.083E+00	2.601E+00	1.615E+00	4.162E-01	2.768E+00	4.794E-01
0.45	1.078E+00	2.307E+00	1.469E+00	4.673E-01	1.887E+00	3.846E-01
0.50	1.073E+00	2.072E+00	1.356E+00	5.180E-01	1.294E+00	3.048E-01
0.60	1.062E+00	1.717E+00	1.199E+00	6.183E-01	5.999E-01	1.811E-01
0.70	1.049E+00	1.463E+00	1.100E+00	7.168E-01	2.570E-01	9.557E-02
0.80	1.034E+00	1.271E+00	1.041E+00	8.134E-01	9.016E-02	4.013E-02
0.90	1.018E+00	1.121E+00	1.010E+00	9.079E-01	1.828E-02	9.530E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.813E-01	9.005E-01	1.009E+00	1.090E+00	1.277E-02	8.700E-03
1.20	9.615E-01	8.172E-01	1.034E+00	1.177E+00	4.367E-02	3.339E-02
1.30	9.410E-01	7.462E-01	1.075E+00	1.261E+00	8.500E-02	7.225E-02
1.40	9.197E-01	6.850E-01	1.132E+00	1.343E+00	1.320E-01	1.237E-01
1.50	8.980E-01	6.317E-01	1.205E+00	1.421E+00	1.817E-01	1.865E-01
1.60	8.758E-01	5.849E-01	1.296E+00	1.497E+00	2.323E-01	2.594E-01
1.70	8.534E-01	5.434E-01	1.407E+00	1.570E+00	2.825E-01	3.414E-01
1.80	8.308E-01	5.064E-01	1.540E+00	1.641E+00	3.316E-01	4.316E-01
1.90	8.082E-01	4.732E-01	1.697E+00	1.708E+00	3.791E-01	5.291E-01

Table 3.4 (*continued*) Fanno Line Functions

			k = 1.2			
M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
2.00	7.857E-01	4.432E-01	1.884E+00	1.773E+00	4.247E-01	6.332E-01
2.10	7.634E-01	4.160E-01	2.103E+00	1.835E+00	4.683E-01	7.432E-01
2.20	7.412E-01	3.913E-01	2.359E+00	1.894E+00	5.099E-01	8.584E-01
2.30	7.194E-01	3.688E-01	2.660E+00	1.951E+00	5.493E-01	9.783E-01
2.40	6.980E-01	3.481E-01	3.011E+00	2.005E+00	5.868E-01	1.102E+00
2.50	6.769E-01	3.291E-01	3.421E+00	2.057E+00	6.222E-01	1.230E+00
2.60	6.563E-01	3.116E-01	3.898E+00	2.106E+00	6.557E-01	1.361E+00
2.70	6.362E-01	2.954E-01	4.455E+00	2.154E+00	6.874E-01	1.494E+00
2.80	6.166E-01	2.804E-01	5.103E+00	2.199E+00	7.173E-01	1.630E+00
2.90	5.975E-01	2.665E-01	5.858E+00	2.242E+00	7.456E-01	1.768E+00
3.00	5.789E-01	2.536E-01	6.735E+00	2.283E+00	7.724E-01	1.907E+00
3.10	5.609E-01	2.416E-01	7.755E+00	2.322E+00	7.977E-01	2.048E+00
3.20	5.435E-01	2.304E-01	8.940E+00	2.359E+00	8.215E-01	2.191E+00
3.30	5.266E-01	2.199E-01	1.032E+01	2.395E+00	8.441E-01	2.334E+00
3.40	5.102E-01	2.101E-01	1.191E+01	2.429E+00	8.655E-01	2.477E+00
3.50	4.944E-01	2.009E-01	1.376E+01	2.461E+00	8.857E-01	2.622E+00
3.60	4.791E-01	1.923E-01	1.590E+01	2.492E+00	9.048E-01	2.766E+00
3.70	4.643E-01	1.842E-01	1.838E+01	2.521E+00	9.229E-01	2.911E+00
3.80	4.501E-01	1.765E-01	2.124E+01	2.549E+00	9.401E-01	3.056E+00
3.90	4.363E-01	1.694E-01	2.454E+01	2.576E+00	9.563E-01	3.200E+00
4.00	4.231E-01	1.626E-01	2.836E+01	2.602E+00	9.718E-01	3.345E+00
4.50	3.636E-01	1.340E-01	5.796E+01	2.714E+00	1.038E+00	4.060E+00
5.00	3.143E-01	1.121E-01	1.163E+02	2.803E+00	1.090E+00	4.757E+00
5.50	2.733E-01	9.505E-02	2.281E+02	2.875E+00	1.130E+00	5.430E+00
6.00	2.391E-01	8.150E-02	4.359E+02	2.934E+00	1.163E+00	6.077E+00
6.50	2.105E-01	7.059E-02	8.108E+02	2.982E+00	1.190E+00	6.698E+00
7.00	1.864E-01	6.168E-02	1.469E+03	3.023E+00	1.212E+00	7.292E+00
7.50	1.660E-01	5.433E-02	2.593E+03	3.056E+00	1.230E+00	7.861E+00
8.00	1.486E-01	4.819E-02	4.467E+03	3.084E+00	1.245E+00	8.404E+00
9.00	1.209E-01	3.863E-02	1.238E+04	3.129E+00	1.268E+00	9.424E+00
10	1.000E-01	3.162E-02	3.162E+04	3.162E+00	1.286E+00	1.036E+01
20	2.683E-02	8.190E-03	2.196E+07	3.276E+00	1.344E+00	1.690E+01
30	1.209E-02	3.665E-03	1.175E+09	3.298E+00	1.356E+00	2.088E+01

k = 1.3

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
0.00	1.150E+00	∞	∞	0.000E+00	∞	∞
0.01	1.150E+00	1.072E+02	5.853E+01	1.072E-02	7.684E+03	4.069E+00
0.02	1.150E+00	5.362E+01	2.927E+01	2.145E-02	1.916E+03	3.376E+00
0.03	1.150E+00	3.574E+01	1.952E+01	3.217E-02	8.479E+02	2.971E+00
0.04	1.150E+00	2.681E+01	1.464E+01	4.289E-02	4.744E+02	2.684E+00
0.05	1.150E+00	2.144E+01	1.172E+01	5.361E-02	3.017E+02	2.461E+00
0.06	1.149E+00	1.787E+01	9.774E+00	6.433E-02	2.081E+02	2.280E+00
0.07	1.149E+00	1.531E+01	8.384E+00	7.504E-02	1.516E+02	2.126E+00
0.08	1.149E+00	1.340E+01	7.342E+00	8.575E-02	1.151E+02	1.994E+00
0.09	1.149E+00	1.191E+01	6.533E+00	9.646E-02	9.006E+01	1.877E+00
0.10	1.148E+00	1.072E+01	5.886E+00	1.072E-01	7.220E+01	1.773E+00
0.15	1.146E+00	7.137E+00	3.952E+00	1.606E-01	3.018E+01	1.374E+00
0.20	1.143E+00	5.346E+00	2.994E+00	2.138E-01	1.573E+01	1.097E+00
0.25	1.139E+00	4.270E+00	2.426E+00	2.668E-01	9.201E+00	8.863E-01
0.30	1.135E+00	3.551E+00	2.054E+00	3.196E-01	5.759E+00	7.196E-01
0.35	1.129E+00	3.036E+00	1.793E+00	3.719E-01	3.760E+00	5.839E-01
0.40	1.123E+00	2.649E+00	1.602E+00	4.239E-01	2.520E+00	4.714E-01
0.45	1.116E+00	2.348E+00	1.459E+00	4.754E-01	1.714E+00	3.775E-01
0.50	1.108E+00	2.106E+00	1.348E+00	5.264E-01	1.172E+00	2.985E-01
0.60	1.091E+00	1.741E+00	1.193E+00	6.267E-01	5.409E-01	1.767E-01
0.70	1.071E+00	1.479E+00	1.097E+00	7.245E-01	2.305E-01	9.280E-02
0.80	1.049E+00	1.280E+00	1.040E+00	8.195E-01	8.045E-02	3.878E-02
0.90	1.025E+00	1.125E+00	1.009E+00	9.114E-01	1.623E-02	9.164E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.733E-01	8.969E-01	1.008E+00	1.085E+00	1.122E-02	8.278E-03
1.20	9.457E-01	8.104E-01	1.032E+00	1.167E+00	3.816E-02	3.160E-02
1.30	9.174E-01	7.368E-01	1.070E+00	1.245E+00	7.388E-02	6.798E-02
1.40	8.887E-01	6.734E-01	1.123E+00	1.320E+00	1.142E-01	1.158E-01
1.50	8.598E-01	6.182E-01	1.189E+00	1.391E+00	1.564E-01	1.735E-01
1.60	8.309E-01	5.697E-01	1.271E+00	1.458E+00	1.990E-01	2.400E-01
1.70	8.022E-01	5.269E-01	1.369E+00	1.523E+00	2.408E-01	3.141E-01
1.80	7.739E-01	4.887E-01	1.484E+00	1.583E+00	2.814E-01	3.948E-01
1.90	7.460E-01	4.546E-01	1.618E+00	1.641E+00	3.203E-01	4.813E-01

Table 3.4 (*continued*) Fanno Line Functions

k = 1.3

M	T/T^*	ρ/ρ^*	p_o/p_o^*	$v/v^* = \rho^*/\rho$	fL^*/D	s^*/R
2.00	7.188E-01	4.239E-01	1.773E+00	1.696E+00	3.573E-01	5.728E-01
2.10	6.921E-01	3.962E-01	1.951E+00	1.747E+00	3.924E-01	6.686E-01
2.20	6.663E-01	3.710E-01	2.156E+00	1.796E+00	4.255E-01	7.680E-01
2.30	6.412E-01	3.482E-01	2.388E+00	1.842E+00	4.567E-01	8.707E-01
2.40	6.170E-01	3.273E-01	2.654E+00	1.885E+00	4.860E-01	9.759E-01
2.50	5.935E-01	3.082E-01	2.954E+00	1.926E+00	5.135E-01	1.083E+00
2.60	5.710E-01	2.906E-01	3.295E+00	1.965E+00	5.394E-01	1.193E+00
2.70	5.493E-01	2.745E-01	3.681E+00	2.001E+00	5.636E-01	1.303E+00
2.80	5.285E-01	2.596E-01	4.116E+00	2.036E+00	5.864E-01	1.415E+00
2.90	5.085E-01	2.459E-01	4.607E+00	2.068E+00	6.077E-01	1.528E+00
3.00	4.894E-01	2.332E-01	5.160E+00	2.099E+00	6.277E-01	1.641E+00
3.10	4.710E-01	2.214E-01	5.781E+00	2.128E+00	6.465E-01	1.755E+00
3.20	4.535E-01	2.104E-01	6.478E+00	2.155E+00	6.642E-01	1.868E+00
3.30	4.367E-01	2.002E-01	7.259E+00	2.181E+00	6.808E-01	1.982E+00
3.40	4.206E-01	1.908E-01	8.133E+00	2.205E+00	6.964E-01	2.096E+00
3.50	4.053E-01	1.819E-01	9.110E+00	2.228E+00	7.110E-01	2.209E+00
3.60	3.906E-01	1.736E-01	1.020E+01	2.250E+00	7.248E-01	2.322E+00
3.70	3.766E-01	1.659E-01	1.142E+01	2.271E+00	7.379E-01	2.435E+00
3.80	3.632E-01	1.586E-01	1.277E+01	2.290E+00	7.501E-01	2.547E+00
3.90	3.504E-01	1.518E-01	1.427E+01	2.309E+00	7.617E-01	2.658E+00
4.00	3.382E-01	1.454E-01	1.594E+01	2.326E+00	7.726E-01	2.769E+00
4.50	2.848E-01	1.186E-01	2.739E+01	2.402E+00	8.189E-01	3.310E+00
5.00	2.421E-01	9.841E-02	4.596E+01	2.460E+00	8.543E-01	3.828E+00
5.50	2.077E-01	8.286E-02	7.522E+01	2.506E+00	8.819E-01	4.320E+00
6.00	1.797E-01	7.065E-02	1.201E+02	2.543E+00	9.037E-01	4.788E+00
6.50	1.567E-01	6.091E-02	1.872E+02	2.573E+00	9.212E-01	5.232E+00
7.00	1.377E-01	5.302E-02	2.853E+02	2.598E+00	9.355E-01	5.654E+00
7.50	1.219E-01	4.654E-02	4.258E+02	2.618E+00	9.472E-01	6.054E+00
8.00	1.085E-01	4.117E-02	6.231E+02	2.635E+00	9.570E-01	6.435E+00
9.00	8.745E-02	3.286E-02	1.266E+03	2.662E+00	9.722E-01	7.143E+00
10	7.188E-02	2.681E-02	2.416E+03	2.681E+00	9.832E-01	7.790E+00
20	1.885E-02	6.865E-03	2.042E+05	2.746E+00	1.020E+00	1.223E+0
30	8.456E-03	3.065E-03	2.943E+06	2.759E+00	1.027E+00	1.489E+0

k = 1.4						
M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	$\bar{f}$L*/D	s*/R
0.00	1.200E+00	∞	∞	0.000E+00	∞	∞
0.01	1.200E+00	1.095E+02	5.787E+01	1.095E-02	7.134E+03	4.058E+00
0.02	1.200E+00	5.477E+01	2.894E+01	2.191E-02	1.778E+03	3.365E+00
0.03	1.200E+00	3.651E+01	1.930E+01	3.286E-02	7.871E+02	2.960E+00
0.04	1.200E+00	2.738E+01	1.448E+01	4.381E-02	4.404E+02	2.673E+00
0.05	1.199E+00	2.190E+01	1.159E+01	5.476E-02	2.800E+02	2.450E+00
0.06	1.199E+00	1.825E+01	9.666E+00	6.570E-02	1.930E+02	2.269E+00
0.07	1.199E+00	1.564E+01	8.292E+00	7.664E-02	1.407E+02	2.115E+00
0.08	1.198E+00	1.368E+01	7.262E+00	8.758E-02	1.067E+02	1.983E+00
0.09	1.198E+00	1.216E+01	6.461E+00	9.851E-02	8.350E+01	1.866E+00
0.10	1.198E+00	1.094E+01	5.822E+00	1.094E-01	6.692E+01	1.762E+00
0.15	1.195E+00	7.287E+00	3.910E+00	1.639E-01	2.793E+01	1.364E+00
0.20	1.190E+00	5.455E+00	2.964E+00	2.182E-01	1.453E+01	1.086E+00
0.25	1.185E+00	4.355E+00	2.403E+00	2.722E-01	8.483E+00	8.766E-01
0.30	1.179E+00	3.619E+00	2.035E+00	3.257E-01	5.299E+00	7.105E-01
0.35	1.171E+00	3.092E+00	1.778E+00	3.788E-01	3.452E+00	5.755E-01
0.40	1.163E+00	2.696E+00	1.590E+00	4.313E-01	2.308E+00	4.638E-01
0.45	1.153E+00	2.386E+00	1.449E+00	4.833E-01	1.566E+00	3.706E-01
0.50	1.143E+00	2.138E+00	1.340E+00	5.345E-01	1.069E+00	2.926E-01
0.60	1.119E+00	1.763E+00	1.188E+00	6.348E-01	4.908E-01	1.724E-01
0.70	1.093E+00	1.493E+00	1.094E+00	7.318E-01	2.081E-01	9.018E-02
0.80	1.064E+00	1.289E+00	1.038E+00	8.251E-01	7.229E-02	3.752E-02
0.90	1.033E+00	1.129E+00	1.009E+00	9.146E-01	1.451E-02	8.824E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.662E-01	8.936E-01	1.008E+00	1.081E+00	9.935E-03	7.894E-03
1.20	9.317E-01	8.044E-01	1.030E+00	1.158E+00	3.364E-02	2.999E-02
1.30	8.969E-01	7.285E-01	1.066E+00	1.231E+00	6.483E-02	6.420E-02
1.40	8.621E-01	6.632E-01	1.115E+00	1.300E+00	9.974E-02	1.088E-01
1.50	8.276E-01	6.065E-01	1.176E+00	1.365E+00	1.361E-01	1.623E-01
1.60	7.937E-01	5.568E-01	1.250E+00	1.425E+00	1.724E-01	2.233E-01
1.70	7.605E-01	5.130E-01	1.338E+00	1.482E+00	2.078E-01	2.909E-01
1.80	7.282E-01	4.741E-01	1.439E+00	1.536E+00	2.419E-01	3.639E-01
1.90	6.969E-01	4.394E-01	1.555E+00	1.586E+00	2.743E-01	4.416E-01

Table 3.4 (*continued*) Fanno Line Functions

k = 1.4

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	*f*L*/D	s*/R
2.00	6.667E-01	4.082E-01	1.688E+00	1.633E+00	3.050E-01	5.232E-01
2.10	6.376E-01	3.802E-01	1.837E+00	1.677E+00	3.339E-01	6.081E-01
2.20	6.098E-01	3.549E-01	2.005E+00	1.718E+00	3.609E-01	6.956E-01
2.30	5.831E-01	3.320E-01	2.193E+00	1.756E+00	3.862E-01	7.853E-01
2.40	5.576E-01	3.111E-01	2.403E+00	1.792E+00	4.099E-01	8.768E-01
2.50	5.333E-01	2.921E-01	2.637E+00	1.826E+00	4.320E-01	9.695E-01
2.60	5.102E-01	2.747E-01	2.896E+00	1.857E+00	4.526E-01	1.063E+00
2.70	4.882E-01	2.588E-01	3.183E+00	1.887E+00	4.718E-01	1.158E+00
2.80	4.673E-01	2.441E-01	3.500E+00	1.914E+00	4.898E-01	1.253E+00
2.90	4.474E-01	2.307E-01	3.850E+00	1.940E+00	5.065E-01	1.348E+00
3.00	4.286E-01	2.182E-01	4.235E+00	1.964E+00	5.222E-01	1.443E+00
3.10	4.107E-01	2.067E-01	4.657E+00	1.987E+00	5.368E-01	1.538E+00
3.20	3.937E-01	1.961E-01	5.121E+00	2.008E+00	5.504E-01	1.633E+00
3.30	3.776E-01	1.862E-01	5.629E+00	2.028E+00	5.632E-01	1.728E+00
3.40	3.623E-01	1.770E-01	6.184E+00	2.047E+00	5.752E-01	1.822E+00
3.50	3.478E-01	1.685E-01	6.790E+00	2.064E+00	5.864E-01	1.915E+00
3.60	3.341E-01	1.606E-01	7.450E+00	2.081E+00	5.970E-01	2.008E+00
3.70	3.210E-01	1.531E-01	8.169E+00	2.096E+00	6.068E-01	2.100E+00
3.80	3.086E-01	1.462E-01	8.951E+00	2.111E+00	6.161E-01	2.192E+00
3.90	2.969E-01	1.397E-01	9.799E+00	2.125E+00	6.248E-01	2.282E+00
4.00	2.857E-01	1.336E-01	1.072E+01	2.138E+00	6.331E-01	2.372E+00
4.50	2.376E-01	1.083E-01	1.656E+01	2.194E+00	6.676E-01	2.807E+00
5.00	2.000E-01	8.944E-02	2.500E+01	2.236E+00	6.938E-01	3.219E+00
5.50	1.702E-01	7.501E-02	3.687E+01	2.269E+00	7.140E-01	3.607E+00
6.00	1.463E-01	6.376E-02	5.318E+01	2.295E+00	7.299E-01	3.974E+00
6.50	1.270E-01	5.482E-02	7.513E+01	2.316E+00	7.425E-01	4.319E+00
7.00	1.111E-01	4.762E-02	1.041E+02	2.333E+00	7.528E-01	4.646E+00
7.50	9.796E-02	4.173E-02	1.418E+02	2.347E+00	7.612E-01	4.955E+00
8.00	8.696E-02	3.686E-02	1.901E+02	2.359E+00	7.682E-01	5.248E+00
9.00	6.977E-02	2.935E-02	3.272E+02	2.377E+00	7.790E-01	5.791E+00
10	5.714E-02	2.390E-02	5.359E+02	2.390E+00	7.868E-01	6.284E+00
20	1.481E-02	6.086E-03	1.538E+04	2.434E+00	8.126E-01	9.641E+00
30	6.630E-03	2.714E-03	1.144E+05	2.443E+00	8.176E-01	1.165E+01

k = 1.5

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	ƒL*/D	s*/R
0.00	1.250E+00	∞	∞	0.000E+00	∞	∞
0.01	1.250E+00	1.118E+02	5.725E+01	1.118E-02	6.659E+03	4.047E+00
0.02	1.250E+00	5.590E+01	2.863E+01	2.236E-02	1.660E+03	3.354E+00
0.03	1.250E+00	3.726E+01	1.909E+01	3.354E-02	7.344E+02	2.949E+00
0.04	1.250E+00	2.795E+01	1.433E+01	4.471E-02	4.108E+02	2.662E+00
0.05	1.249E+00	2.235E+01	1.147E+01	5.588E-02	2.612E+02	2.439E+00
0.06	1.249E+00	1.863E+01	9.562E+00	6.705E-02	1.800E+02	2.258E+00
0.07	1.248E+00	1.596E+01	8.203E+00	7.821E-02	1.311E+02	2.104E+00
0.08	1.248E+00	1.396E+01	7.184E+00	8.937E-02	9.948E+01	1.972E+00
0.09	1.247E+00	1.241E+01	6.393E+00	1.005E-01	7.781E+01	1.855E+00
0.10	1.247E+00	1.117E+01	5.760E+00	1.117E-01	6.235E+01	1.751E+00
0.15	1.243E+00	7.433E+00	3.870E+00	1.672E-01	2.598E+01	1.353E+00
0.20	1.238E+00	5.562E+00	2.934E+00	2.225E-01	1.350E+01	1.076E+00
0.25	1.231E+00	4.438E+00	2.380E+00	2.774E-01	7.863E+00	8.672E-01
0.30	1.222E+00	3.686E+00	2.017E+00	3.317E-01	4.902E+00	7.017E-01
0.35	1.213E+00	3.147E+00	1.764E+00	3.855E-01	3.187E+00	5.674E-01
0.40	1.202E+00	2.741E+00	1.579E+00	4.385E-01	2.126E+00	4.565E-01
0.45	1.190E+00	2.424E+00	1.439E+00	4.908E-01	1.439E+00	3.641E-01
0.50	1.176E+00	2.169E+00	1.332E+00	5.423E-01	9.802E-01	2.868E-01
0.60	1.147E+00	1.785E+00	1.183E+00	6.425E-01	4.479E-01	1.684E-01
0.70	1.114E+00	1.508E+00	1.092E+00	7.387E-01	1.891E-01	8.771E-02
0.80	1.078E+00	1.298E+00	1.037E+00	8.305E-01	6.536E-02	3.633E-02
0.90	1.040E+00	1.133E+00	1.009E+00	9.176E-01	1.306E-02	8.508E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.597E-01	8.906E-01	1.008E+00	1.078E+00	8.863E-03	7.545E-03
1.20	9.191E-01	7.989E-01	1.029E+00	1.150E+00	2.988E-02	2.853E-02
1.30	8.787E-01	7.211E-01	1.063E+00	1.219E+00	5.736E-02	6.082E-02
1.40	8.389E-01	6.542E-01	1.108E+00	1.282E+00	8.790E-02	1.026E-01
1.50	8.000E-01	5.963E-01	1.165E+00	1.342E+00	1.195E-01	1.524E-01
1.60	7.622E-01	5.456E-01	1.232E+00	1.397E+00	1.508E-01	2.089E-01
1.70	7.257E-01	5.011E-01	1.311E+00	1.448E+00	1.812E-01	2.710E-01
1.80	6.906E-01	4.617E-01	1.402E+00	1.496E+00	2.103E-01	3.377E-01
1.90	6.570E-01	4.266E-01	1.504E+00	1.540E+00	2.377E-01	4.082E-01

Table 3.4 (*continued*) Fanno Line Functions

k = 1.5

M	T/T*	ρ/ρ*	p_o/p_o*	v/v* = ρ*/ρ	fL*/D	s*/R
2.00	6.250E-01	3.953E-01	1.619E+00	1.581E+00	2.636E-01	4.819E-01
2.10	5.945E-01	3.672E-01	1.747E+00	1.619E+00	2.877E-01	5.580E-01
2.20	5.656E-01	3.419E-01	1.889E+00	1.655E+00	3.103E-01	6.362E-01
2.30	5.382E-01	3.190E-01	2.046E+00	1.687E+00	3.313E-01	7.158E-01
2.40	5.123E-01	2.982E-01	2.218E+00	1.718E+00	3.508E-01	7.967E-01
2.50	4.878E-01	2.794E-01	2.407E+00	1.746E+00	3.690E-01	8.783E-01
2.60	4.647E-01	2.622E-01	2.613E+00	1.772E+00	3.858E-01	9.605E-01
2.70	4.429E-01	2.465E-01	2.838E+00	1.797E+00	4.015E-01	1.043E+00
2.80	4.223E-01	2.321E-01	3.082E+00	1.820E+00	4.160E-01	1.125E+00
2.90	4.029E-01	2.189E-01	3.347E+00	1.841E+00	4.296E-01	1.208E+00
3.00	3.846E-01	2.067E-01	3.633E+00	1.861E+00	4.422E-01	1.290E+00
3.10	3.674E-01	1.955E-01	3.943E+00	1.879E+00	4.539E-01	1.372E+00
3.20	3.511E-01	1.852E-01	4.278E+00	1.896E+00	4.648E-01	1.453E+00
3.30	3.358E-01	1.756E-01	4.638E+00	1.912E+00	4.750E-01	1.534E+00
3.40	3.213E-01	1.667E-01	5.025E+00	1.927E+00	4.846E-01	1.614E+00
3.50	3.077E-01	1.585E-01	5.441E+00	1.941E+00	4.935E-01	1.694E+00
3.60	2.948E-01	1.508E-01	5.886E+00	1.955E+00	5.018E-01	1.773E+00
3.70	2.826E-01	1.437E-01	6.363E+00	1.967E+00	5.096E-01	1.851E+00
3.80	2.711E-01	1.370E-01	6.874E+00	1.979E+00	5.169E-01	1.928E+00
3.90	2.603E-01	1.308E-01	7.419E+00	1.990E+00	5.238E-01	2.004E+00
4.00	2.500E-01	1.250E-01	8.000E+00	2.000E+00	5.302E-01	2.079E+00
4.50	2.062E-01	1.009E-01	1.151E+01	2.043E+00	5.572E-01	2.443E+00
5.00	1.724E-01	8.305E-02	1.620E+01	2.076E+00	5.775E-01	2.785E+00
5.50	1.460E-01	6.947E-02	2.233E+01	2.101E+00	5.931E-01	3.106E+00
6.00	1.250E-01	5.893E-02	3.017E+01	2.121E+00	6.052E-01	3.407E+00
6.50	1.081E-01	5.058E-02	4.004E+01	2.137E+00	6.149E-01	3.690E+00
7.00	9.434E-02	4.388E-02	5.226E+01	2.150E+00	6.227E-01	3.956E+00
7.50	8.299E-02	3.841E-02	6.721E+01	2.161E+00	6.291E-01	4.208E+00
8.00	7.353E-02	3.390E-02	8.526E+01	2.169E+00	6.344E-01	4.446E+00
9.00	5.882E-02	2.695E-02	1.324E+02	2.183E+00	6.426E-01	4.886E+00
10	4.808E-02	2.193E-02	1.973E+02	2.193E+00	6.485E-01	5.285E+00
20	1.238E-02	5.562E-03	2.934E+03	2.225E+00	6.679E-01	7.984E+00
30	5.531E-03	2.479E-03	1.465E+04	2.231E+00	6.716E-01	9.592E+00

			k = 5/3			
M	T/T*	ρ/ρ*	p_o/p_o^*	v/v* = ρ*/ρ	fL*/D	s*/R
0.00	1.335E+00	∞	∞	0.000E+00	∞	∞
0.01	1.335E+00	1.155E+02	5.623E+01	1.155E-02	5.980E+03	4.030E+00
0.02	1.335E+00	5.777E+01	2.812E+01	2.311E-02	1.490E+03	3.337E+00
0.03	1.335E+00	3.851E+01	1.875E+01	3.466E-02	6.594E+02	2.931E+00
0.04	1.334E+00	2.888E+01	1.407E+01	4.620E-02	3.687E+02	2.644E+00
0.05	1.334E+00	2.310E+01	1.126E+01	5.775E-02	2.344E+02	2.422E+00
0.06	1.333E+00	1.925E+01	9.394E+00	6.928E-02	1.615E+02	2.240E+00
0.07	1.333E+00	1.649E+01	8.059E+00	8.081E-02	1.176E+02	2.087E+00
0.08	1.332E+00	1.443E+01	7.059E+00	9.233E-02	8.916E+01	1.954E+00
0.09	1.331E+00	1.282E+01	6.282E+00	1.038E-01	6.971E+01	1.838E+00
0.10	1.331E+00	1.153E+01	5.661E+00	1.153E-01	5.583E+01	1.734E+00
0.15	1.325E+00	7.674E+00	3.805E+00	1.727E-01	2.321E+01	1.336E+00
0.20	1.317E+00	5.739E+00	2.887E+00	2.296E-01	1.202E+01	1.060E+00
0.25	1.308E+00	4.574E+00	2.344E+00	2.859E-01	6.980E+00	8.519E-01
0.30	1.296E+00	3.795E+00	1.989E+00	3.415E-01	4.337E+00	6.875E-01
0.35	1.282E+00	3.235E+00	1.741E+00	3.963E-01	2.810E+00	5.543E-01
0.40	1.267E+00	2.814E+00	1.560E+00	4.503E-01	1.868E+00	4.446E-01
0.45	1.250E+00	2.485E+00	1.424E+00	5.032E-01	1.260E+00	3.536E-01
0.50	1.232E+00	2.220E+00	1.320E+00	5.549E-01	8.549E-01	2.777E-01
0.60	1.191E+00	1.819E+00	1.176E+00	6.549E-01	3.878E-01	1.620E-01
0.70	1.147E+00	1.530E+00	1.087E+00	7.496E-01	1.625E-01	8.382E-02
0.80	1.099E+00	1.311E+00	1.035E+00	8.388E-01	5.575E-02	3.449E-02
0.90	1.050E+00	1.139E+00	1.008E+00	9.223E-01	1.106E-02	8.021E-03
1.00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	0.000E+00	0.000E+00
1.10	9.499E-01	8.860E-01	1.007E+00	1.072E+00	7.404E-03	7.017E-03
1.20	9.006E-01	7.908E-01	1.027E+00	1.139E+00	2.481E-02	2.636E-02
1.30	8.524E-01	7.102E-01	1.057E+00	1.200E+00	4.733E-02	5.582E-02
1.40	8.059E-01	6.412E-01	1.098E+00	1.257E+00	7.212E-02	9.359E-02
1.50	7.612E-01	5.817E-01	1.148E+00	1.309E+00	9.749E-02	1.381E-01
1.60	7.187E-01	5.298E-01	1.207E+00	1.356E+00	1.225E-01	1.882E-01
1.70	6.783E-01	4.845E-01	1.275E+00	1.400E+00	1.465E-01	2.428E-01
1.80	6.402E-01	4.445E-01	1.351E+00	1.440E+00	1.692E-01	3.009E-01
1.90	6.043E-01	4.091E-01	1.436E+00	1.477E+00	1.906E-01	3.619E-01

Table 3.4 (*continued*) Fanno Line Functions

k = 5/3

M	T/T*	ρ/ρ*	po/po*	v/v* = ρ*/ρ	fL*/D	s*/R
2.00	5.705E-01	3.777E-01	1.530E+00	1.511E+00	2.105E-01	4.251E-01
2.10	5.389E-01	3.496E-01	1.632E+00	1.542E+00	2.290E-01	4.900E-01
2.20	5.093E-01	3.244E-01	1.744E+00	1.570E+00	2.461E-01	5.561E-01
2.30	4.816E-01	3.017E-01	1.865E+00	1.596E+00	2.619E-01	6.230E-01
2.40	4.557E-01	2.813E-01	1.995E+00	1.620E+00	2.766E-01	6.905E-01
2.50	4.315E-01	2.628E-01	2.135E+00	1.642E+00	2.901E-01	7.583E-01
2.60	4.089E-01	2.460E-01	2.285E+00	1.663E+00	3.026E-01	8.262E-01
2.70	3.878E-01	2.307E-01	2.445E+00	1.681E+00	3.142E-01	8.940E-01
2.80	3.681E-01	2.167E-01	2.616E+00	1.699E+00	3.249E-01	9.615E-01
2.90	3.497E-01	2.039E-01	2.797E+00	1.715E+00	3.348E-01	1.029E+00
3.00	3.325E-01	1.922E-01	2.990E+00	1.730E+00	3.440E-01	1.095E+00
3.10	3.164E-01	1.814E-01	3.195E+00	1.744E+00	3.525E-01	1.162E+00
3.20	3.013E-01	1.715E-01	3.411E+00	1.757E+00	3.604E-01	1.227E+00
3.30	2.872E-01	1.624E-01	3.639E+00	1.769E+00	3.677E-01	1.292E+00
3.40	2.740E-01	1.540E-01	3.880E+00	1.780E+00	3.746E-01	1.356E+00
3.50	2.616E-01	1.461E-01	4.134E+00	1.790E+00	3.810E-01	1.419E+00
3.60	2.499E-01	1.389E-01	4.401E+00	1.800E+00	3.869E-01	1.482E+00
3.70	2.390E-01	1.321E-01	4.682E+00	1.809E+00	3.925E-01	1.544E+00
3.80	2.287E-01	1.258E-01	4.976E+00	1.817E+00	3.977E-01	1.605E+00
3.90	2.190E-01	1.200E-01	5.285E+00	1.825E+00	4.025E-01	1.665E+00
4.00	2.099E-01	1.145E-01	5.608E+00	1.833E+00	4.071E-01	1.724E+00
4.50	1.715E-01	9.203E-02	7.456E+00	1.864E+00	4.261E-01	2.009E+00
5.00	1.424E-01	7.547E-02	9.721E+00	1.887E+00	4.402E-01	2.274E+00
5.50	1.199E-01	6.296E-02	1.245E+01	1.905E+00	4.510E-01	2.522E+00
6.00	1.022E-01	5.329E-02	1.568E+01	1.918E+00	4.594E-01	2.752E+00
6.50	8.810E-02	4.566E-02	1.947E+01	1.929E+00	4.660E-01	2.969E+00
7.00	7.666E-02	3.955E-02	2.385E+01	1.938E+00	4.714E-01	3.172E+00
7.50	6.728E-02	3.458E-02	2.887E+01	1.945E+00	4.757E-01	3.363E+00
8.00	5.949E-02	3.049E-02	3.458E+01	1.951E+00	4.793E-01	3.543E+00
9.00	4.745E-02	2.420E-02	4.824E+01	1.960E+00	4.849E-01	3.876E+00
10	3.870E-02	1.967E-02	6.518E+01	1.967E+00	4.889E-01	4.177E+00
20	9.889E-03	4.972E-03	4.940E+02	1.989E+00	5.020E-01	6.203E+00
30	4.413E-03	2.214E-03	1.644E+03	1.993E+00	5.044E-01	7.405E+00

Function	*Equation(s)*
$\frac{T}{T^*}$	3.167
$\frac{p}{p^*}$	3.170
$\frac{p_0}{p_0{}^*}$	3.174
$\frac{v}{v^*} = \frac{\rho^*}{\rho}$	3.176, 3.177
$\frac{fL^*}{D}$	3.166
$\frac{s^*}{R}$	3.172

In using Table 3.4 it should be again noted, as in Tables 3.1 and 3.3, that all data are based on the assumption that the gas is ideal and that the molecular weight, specific heats, and ratios of specific heats are constant. Table D.2 gives values of k for ideal gases as a function of temperature. When the temperature range is known before calculations, the average value of k should be used. If one of the temperatures is not known, use the k value for the known temperature, and check for variation after the other is computed.

Application of Fanno Flow Functions

Figure 3.17 shows a real pipe and two imaginary pipes. The first imaginary pipe has the length required to pass the gas from a Mach number of M_1 (inlet Mach number of the real pipe) to sonic velocity M^*. The second imaginary pipe has the length required to pass the gas from a Mach number of M_2 (exit Mach number of the real pipe) to sonic velocity M^*.

The length of the pipe L may be calculated as follows: The maximum length of an imaginary pipe $fL_1{}^*/D$ is obtained from Table 3.4 [or computed from equation (3.166)] for a Mach number of M_1. In like manner, the maximum length $fL_2{}^*/D$ is obtained for a Mach number of M_2 for the second imaginary pipe. The length L of the real pipe is then calculated from the following:

$$L = \frac{f}{D}\left[\left(\frac{fL_1^*}{D}\right) - \left(\frac{fL_2^*}{D}\right)\right] = L_1^* - L_2^* \qquad (3.178)$$

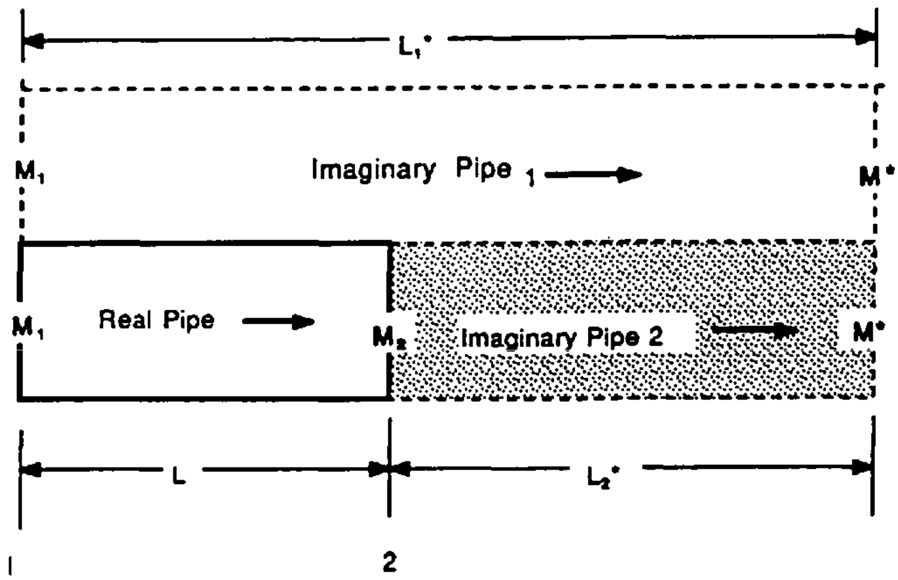


Figure 3.17 Notation for equation (3.178).

Isothermal Flow in Constant-Area Ducts with Friction

As demonstrated in this section, the limiting Mach number for isothermal flow is $1/k^{1/2}$. For Mach numbers less than this value, the pipe must be heated to maintain isothermal flow; for Mach numbers greater than $1/k^{1/2}$, the pipe must be cooled to maintain isothermal flow. Flow in gas transmission pipelines is essentially isothermal. These lines are uninsulated and their flowing temperature is very close to ambient temperature. Flows in these lines are at low Mach numbers, significantly less than $1/k^{1/2}$. Figure 3.18 shows relations for isothermal flow.

General Considerations

Isothermal compressible flow of an ideal gas with friction in a constant-area duct must satisfy the following requirements:

1. *The ideal gas law*: The equation of state for an ideal gas is equation (1.37):

$$pv = RT \tag{1.37}$$

2. *The process relationship*: For an ideal gas undergoing an isothermal process, this relationship is given by equation (1.31); the numerical value of the process exponent is $n = 1$:

$$pv^n = pv^1 = p_1v_1 = p_2v_2 = RT \tag{3.179}$$

3. *Constant-area duct*: The flow area must be the same at all sections:

$$A = A_1 = A_2 = \cdots = A_n$$

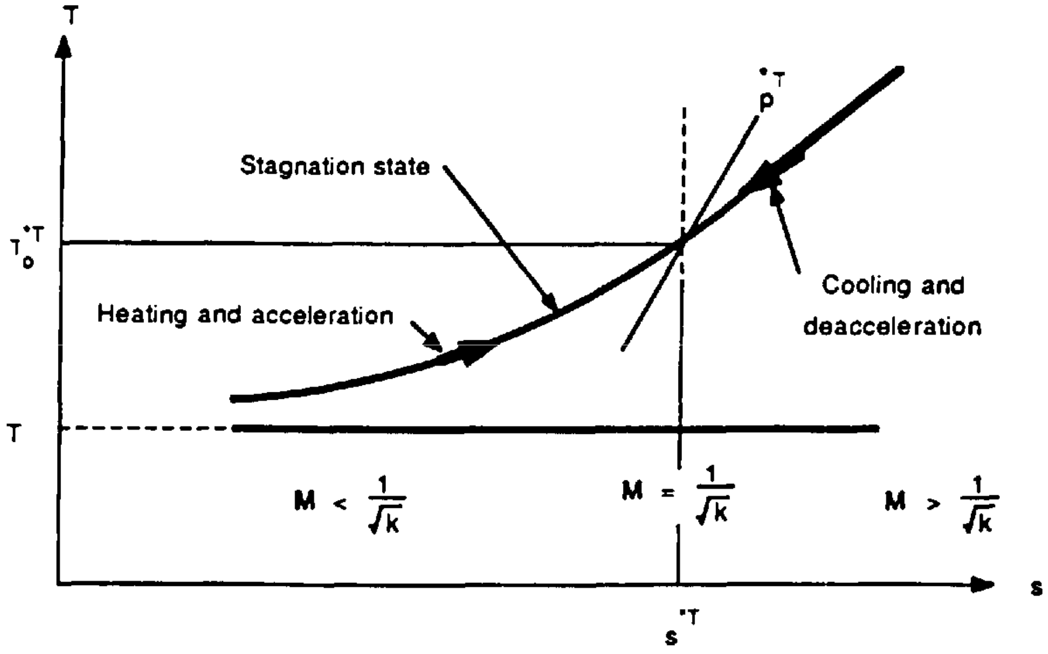


Figure 3.18 Notation for isothermal flow.

4. *Conservation of mass*: The continuity equation may be expressed as follows:

$$\dot{m} = \frac{Av}{\nu} = \frac{Av_1}{\nu_1} = \frac{Av_2}{\nu_2} \tag{3.6}$$

5. *Equation of motion*:

$$\frac{v\,dv}{g_c} + \nu\,dp + \frac{f}{D}\frac{v^2}{2g_c}\,dL = 0 \tag{3.150}$$

6. *Heat transfer*: The general energy equation for an ideal gas (3.86) in the absence of flow work, for a horizontal pipe, and for an isothermal process becomes the following:

$$q = \frac{v_2^2 - v_1^2}{2g_c} \tag{3.180}$$

7. *Stagnation properties*: Flowing fluid properties at Mach number M are assumed to achieve the stagnation state ($M = 0$) by an isentropic process, so the relations developed in Section 3.4 may be applied:

$$T_0 = T\left[1 + \left(\frac{k-1}{2}\right)M^2\right] \tag{3.99}$$

$$p_0 = p\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/(k-1)} \tag{3.100}$$

$$\rho_0 = \rho\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{1/(k-1)} \tag{3.101}$$

Pressure Loss

An equation for the calculation of pressure loss for isothermal flow may be developed as follows. Multiplying equation (3.150) by $2g_c/v^2$ results in the following:

$$\frac{v\,dv}{g_c} + v\,dp + \frac{f}{D}\frac{v^2}{2g_c}\,dL = 0 = \left(\frac{2g_c}{v^2}\right)\frac{v\,dv}{g_c} + \left(\frac{2g_c}{v^2}\right)v\,dp + \left(\frac{2g_c}{v^2}\right)\frac{f}{D}\frac{v^2}{2g_c}dL$$

$$\frac{2}{v}\,dv + \frac{2g_c v}{v^2}\,dp + \frac{f}{D}\,dL = 0 \tag{3.181}$$

Substituting from the ideal gas continuity equation ($v^2 = (\dot{m}RT/Ap)^2$ and $v = RT/p$ from the equation of state (1.37) in equation (3.181) yields the following:

$$\frac{2}{v}\,dv + \frac{2g_c(RT/p)}{(\dot{m}RT/Ap)^2}dp + \frac{f}{D}\,dL = 0 = \frac{2}{v}\,dv + \frac{2g_c p}{(\dot{m}/A)^2 RT}\,dp + \frac{f}{D}\,dL \tag{3.182}$$

Integrating equation (3.182), yields the following:

$$\begin{aligned} &2\int_1^2 \frac{dv}{v} + \frac{2g_c}{(\dot{m}/A)^2 RT}\int_1^2 p\,dp + \frac{f}{D}\int_0^L dL = 0 \\ &2\log_e\left(\frac{v_2}{v_1}\right) + \frac{g_c}{(\dot{m}/A)^2 RT}(p_2^2 - p_1^2) + \frac{fL}{D} = 0 \end{aligned} \tag{3.183}$$

Noting again from the continuity equation for isothermal flow that $v_2/v_1 = (\dot{m}RT/Ap_2)/(\dot{m}RT/Ap_1) = p_1/p_2$, substituting in equation (3.183), and solving for p_2^2 results in the following:

$$p_2^2 = p_1^2 - \frac{(\dot{m}/A)^2}{g_c}RT\left[2\log_e\left(\frac{p_1}{p_2}\right) + \frac{fL}{D}\right] \tag{3.184}$$

Examination of equation (3.184) indicates that a reiterative solution is necessary to compute p_2. In most cases, the term $2\log_e(p_1/p_2)$ is small compared with fL/D and may be ignored for a first trial solution of p_2.

Limiting Mach Number

In the derivation of equations for acoustic velocity in Section 1.6, the velocity of a pressure wave was developed as the following:

$$c = \sqrt{\frac{Eg_c}{\rho}} \tag{1.57}$$

The value of E depends on the process. In Section 1.6 it was shown in equation (1.48) that, for an ideal gas, the value of the isothermal bulk modulus is $E_T = p$. Substituting p for E and $\rho = p/RT$ from the equation of state (1.37) in equation (1.57) results in the following:

$$v_{\max T} = c_T = \sqrt{\frac{pg_c}{p/RT}} = \sqrt{g_c RT} \tag{3.185}$$

The limiting Mach number M^{*T} is obtained by dividing equation (3.185) by the acoustic velocity of an ideal gas $c = (kg_c RT)^{1/2}$ from equation (1.59):

$$M^{*T} = \frac{v_{\max)T}}{c} = \frac{\sqrt{g_c RT}}{\sqrt{kg_c RT}} = \sqrt{\frac{1}{k}} \tag{3.186}$$

Maximum Length

Application of equation (3.160) for constant temperature ($dT = 0$) results in the following:

$$\frac{dp}{p} = \frac{dT}{T} - \frac{dv}{v} = 0 - \frac{dv}{v} \qquad \text{or} \qquad \frac{dp}{p} = -\frac{dv}{v} \tag{3.187}$$

Substituting in the equation of motion (3.181) $v = RT/p$ from the equation of state (1.37), $dp/p = -dv/v$ from equation (3.187), noting that $dv/v = dM/M = dM^2/2M^2$, and solving for $f_d L/D$ results in the following:

$$\frac{f}{D}\,dL = -2\,\frac{dv}{v} - \frac{2g_c RT}{v^2}\,\frac{dp}{p} = \left(-2 + \frac{2g_c RT}{v^2}\right)\frac{dv}{v}$$

$$\frac{f}{D}\,dL = 2\left(\frac{g_c RT}{kg_c RTM^2} - 1\right)\frac{dM}{M} = \left(\frac{1}{kM^2} - 1\right)\frac{dM^2}{M^2} = \frac{1 - kM^2}{kM^4}\,dM^2 \tag{3.188}$$

Integrating equation (3.188) between the limits of $L = 0$ and $L = L^{*T}$ and M and $M^{*T} = 1/\sqrt{k}$ results in the following:

$$\frac{f}{D}\int_0^{L^{*T}} dL = \frac{1}{k}\int_M^{1/\sqrt{k}} \frac{1 - kM^2}{M^4}\, dM^2 = \frac{fL^{*T}}{D} = \frac{1 - kM^2}{M^2} + \log_e(kM^2) \tag{3.189}$$

Entropy Change

Expressing equation (3.144) in dimensionless form and noting for an isothermal process $dT = 0$, the change in entropy can be written as follows:

$$\frac{ds}{R} = \frac{k}{k-1}\frac{dT}{T} - \frac{dp}{p} = \frac{k}{k-1}\frac{0}{T} - \frac{dp}{p} = -\frac{dp}{p} \tag{3.190}$$

From equation (3.187) $dv/v = -dp/p = dM/M$; substituting in equation (3.190) results in the following:

$$\frac{ds}{R} = -\frac{dp}{p} = \frac{dv}{v} = \frac{dM}{M} \tag{3.191}$$

Integrating equation (3.191) between the limits of 1 and 2 yields the following:

$$\frac{1}{R}\int_1^2 ds = \int_1^2 \frac{dM}{M} = \frac{s_2 - s_1}{R} = \log_e\left(\frac{M_2}{M_1}\right) \tag{3.192}$$

Substituting $s_2 - s_1$ for s^{*T} and $M_1 = M$, and $M_2 = 1/\sqrt{k}$ in equation (3.192) results in the following:

$$\frac{s^{*T}}{R} = -\log_e(\sqrt{k}M) \tag{3.193}$$

Heat Transfer

Examination of equation (3.180) indicates that the heat transfer needed to maintain isothermal flow is the change in kinetic energy. Substitution of $v^2 = kg_c RTM^2$ from equation (3.93) in equation (3.180) results in the following:

$$q = \frac{kg_c RTM_2^2 - kg_c RTM_1^2}{2g_c} = \frac{kRT}{2}(M_2^2 - M_1^2) \tag{3.194}$$

Substituting q^{*T} for q and $M_1 = M$ and $M_2 = 1/\sqrt{k}$ in equation (3.194) yields the following:

$$q^{*T} = \frac{kRT}{2}\left(\left(\frac{1}{\sqrt{k}}\right)^2 - M^2\right) = \frac{RT}{2}(1 - kM^2) \tag{3.195}$$

Note that if M is greater than $1/\sqrt{k}$, the pipe must be cooled to maintain isothermal flow, if M is less, the pipe must be heated.

4

Flow in Pipes

4.1 INTRODUCTION

The approach in this chapter is a combination of the theory developed in the first three chapters to produce solutions to engineering problems. *The examples in this chapter are solved in USCS units only.*

4.2 BACKGROUND

The current approach to pipe friction originates from a paper published in 1944 by Lewis F. Moody (1944). Moody combined the results of Johann Nikuradse on artificially roughened pipe published in 1933 and the analysis of C. F. Colebrook in 1938 that resulted in the equation that bears his name. Colebrook's analysis, combined with the boundary layer theory developed by L. Prandtl and T. von Karman, led to the adoption of the Colebrook equation by the major American technical societies and practicing engineers. This was followed in 1950 by R. J. S. Piggot with a comparable analysis on pressure losses in tubing, pipe, and fittings (see references provided at the conclusion of this chapter if further development is desired).

4.3 PHYSICAL ANALYSIS FOR INCOMPRESSIBLE FLOW

In Section 3.3 the energy "lost" due to friction for an *incompressible* fluid was shown to be as follows:

$$\frac{g}{g_c}(z_2 - z_1) + \frac{v_2^2 - v_1^2}{2g_c} + v(p_2 - p_1) + H_f = 0 \tag{3.22}$$

For a horizontal pipe ($\Delta z = 0$) with uniform cross section ($\Delta v = 0$), equation (3.22) reduces to equation (4.1) after substitution of the definition of specific volume (equation (1.15), $v = 1/\rho$).

$$H_f = \frac{p_1 - p_2}{\rho} = \frac{\Delta p}{\rho} \tag{4.1}$$

Experimental results and dimensional analysis of a flowing system show that the pressure drop is related to density and velocity as follows:

$$\Delta p = \frac{K\rho v^2}{2g_c} \tag{4.2}$$

In equation (4.2), K is a proportionality constant, also known as the resistance coefficient. Substitution of equation (4.2) into equation (4.1) yields the following expression for H_f:

$$H_f = \frac{Kv^2}{2g_c} \tag{4.3}$$

To calculate friction losses in pipes, it is common engineering practice to use the friction factor, f. Through experimental analysis, the following definition of f has been established:

$$H_f = \frac{f}{D}\frac{v^2}{2g_c}\int_0^L dL = \frac{fL}{D}\frac{v^2}{2g_c} \tag{4.4}$$

The friction factor, f, defined in equation (4.4) is also known as the D'Arcy Weisbach friction factor.

The relation of the resistance coefficient K to the friction factor f is obtained by equating equations (4.3) and (4.4):

$$H_f = \frac{Kv^2}{2g_c} = \frac{fL}{D}\frac{v^2}{2g_c} \quad \text{or} \quad K = \frac{fL}{D} \tag{4.5}$$

The relation of friction factor f to the wall shear stress τ is obtained by first rewriting the relationship between pressure drop (Δp) and shear stress as follows (see Section 4.6):

$$\Delta p A = \tau A_s \quad \text{or} \quad \Delta p = \frac{\tau A_s}{A} \tag{4.6}$$

In equation (4.6), A is the cross-sectional area of a cylindrical pipe ($= \pi D^2/4$) and A_s is the shear area ($= \pi DL$). Substituting these area relationships into equation (4.6) yields equation (4.7):

$$\Delta p = \frac{\tau A_s}{A} = \frac{\tau(\pi DL)}{(\pi D^2/4)} = \frac{4\tau L}{D} \tag{4.7}$$

Substituting for Δp from equation (4.1) yields the following:

$$\frac{\Delta p}{\rho} = H_f = \frac{4\tau L}{D\rho} \tag{4.8}$$

Equating equations (4.4) and (4.8) and solving for τ yields the following relationship:

$$H_f = \frac{fL}{D}\frac{v^2}{2g_c}$$
$$= \frac{4\tau L}{D\rho}$$

or

$$\tau = \frac{f\rho v^2}{8g_c} \tag{4.9}$$

4.4 DYNAMIC SIMILARITY FOR INCOMPRESSIBLE FLOW

Consideration of the forces involved in incompressible flow (inertia, viscosity, gravity, buoyancy, and pressure) indicates that the gravitational forces will be balanced by the buoyant forces. If the inertial and viscous forces are specified, then the pressure force is also specified for equilibrium. The Reynolds number is the ratio of the inertial to the viscous forces, for a given ratio of absolute surface roughness to diameter (ε/D); the types of flow possible are shown in Table 4.1.

Experiments are necessary to establish numerical values of the Reynolds number for the types of flow shown previously. Experimental friction factor data are usually plotted in the form of a Moody diagram, as shown in Figure 4.1.

Table 4.1 Force, Reynolds Number, and Flow Type Relationships

Predominant force	Reynolds number value	Type of flow
Viscosity	Low	Laminar
Neither	Intermediate	Critical transition
Inertia	High	Turbulent

4.5 REYNOLDS NUMBER EQUATIONS

Flow equations involving the Reynolds number are derived in this section to facilitate design calculations. The Reynolds number is defined for pipe flow with a characteristic length of D as follows:

$$\mathrm{Re} = \frac{\rho D v}{\mu g_c} = \frac{D v}{\nu} \tag{4.10}$$

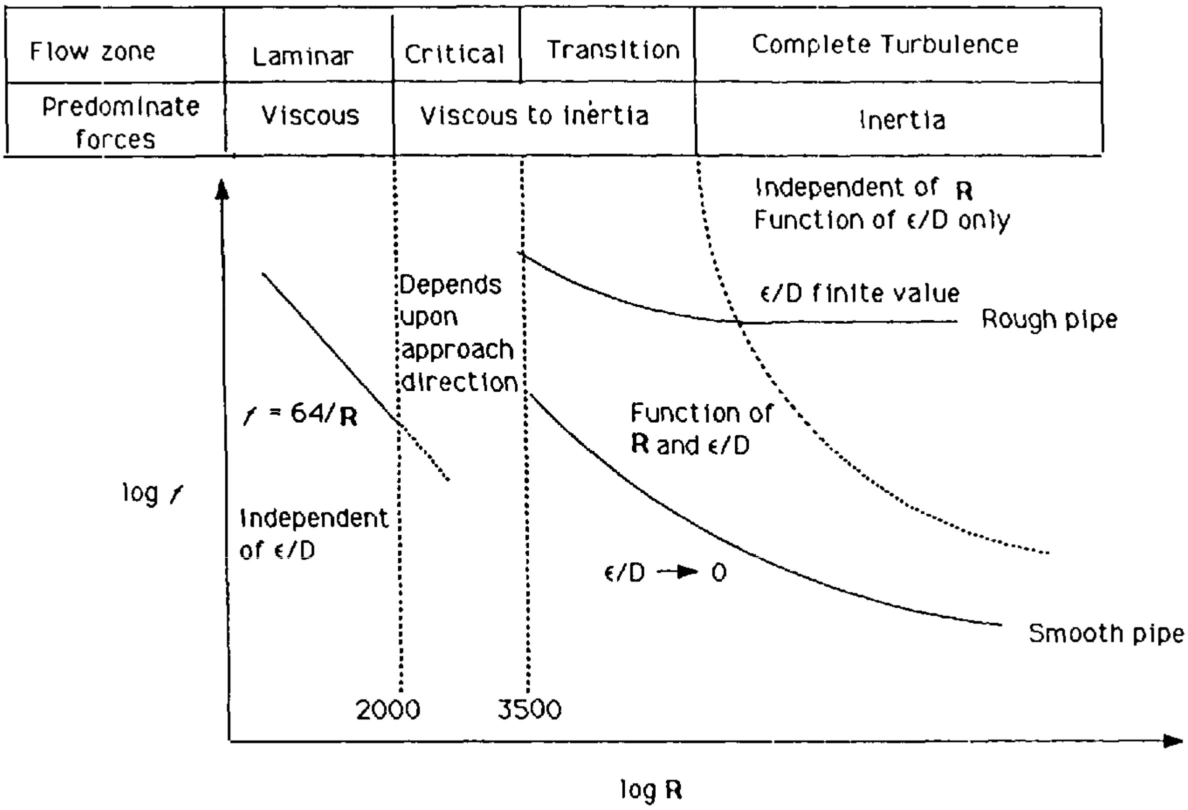


Figure 4.1 Moody diagram for pipe friction.

From the continuity equation [equation (3.6)], velocity can be expressed in terms of mass flow rate as follows:

$$v = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{4\dot{m}}{\pi\rho D^2} \quad (4.11)$$

The Reynolds number can then be expressed in terms of mass flow rate by substituting equation (4.11) into equation (4.10) as follows:

$$\mathrm{Re} = \frac{\rho D v}{\mu g_c} = \frac{\rho D(4\dot{m}/\pi\rho D^2)}{\mu g_c} = \frac{4\dot{m}}{\pi D\mu g_c} \quad (4.12)$$

In like manner, the Reynolds number can be expressed in terms of volumetric flow rate. Equation (3.2) expresses average flow velocity in terms of volumetric flow rate:

$$v = \frac{Q}{A} = \frac{Q}{\pi D^2/4} \quad (3.2)$$

Substitution of equation (3.2) into equation (4.10) results in the following expression for the Reynolds number:

$$\mathrm{Re} = \frac{\rho D v}{\mu g_c} = \frac{\rho D(4Q/\pi D^2)}{\mu g_c} = \frac{4\rho Q}{\pi D\mu g_c} \quad (4.13)$$

4.6 LAMINAR FLOW IN PIPES

Consider the three-dimensional laminar flow shown in Figure 4.2. The change in pressure is $p_2 - p_1$ for a distance L along the pipe. For equilibrium of a "free body," a force balance gives the following:

$$\sum F = 0 = (p_2 - p_1)A = \tau A_s \quad (4.14)$$

For a cylindrical pipe of radius r, the cross-sectional area $A = \pi r^2$ and the shear area $A_s = 2\pi rL$. When solving for τ, the equation (4.14) reduces to

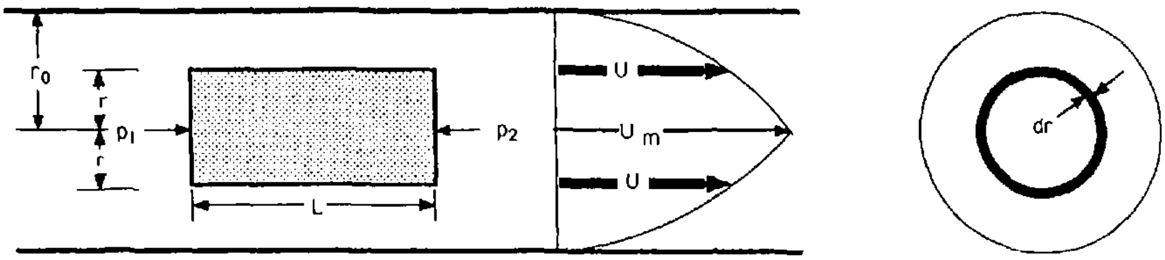


Figure 4.2 Notation for laminar flow analysis.

equation (4.15):

$$\tau = \frac{(p_2 - p_1)r}{2L} \tag{4.15}$$

If the flow is laminar, only viscous forces are acting, so equation (1.2) [$\tau = \mu(dU/dy)$] applies. Noting that $dy = dr$ for the case of the pipe, substitution of equation (4.15) for τ yields the following:

$$\tau = \frac{(p_2 - p_1)r}{2L} = \mu\left(\frac{dU}{dr}\right)$$

or

$$dU = \left(\frac{p_2 - p_1}{2L\mu}\right) r \; dr \tag{4.16}$$

Integrating equation (4.16) over limits of the pipe radius (0 to r_0) and the velocity (U_m to 0), yields the following equation for the centerline velocity:

$$\int_{U_m}^{0} dU = \frac{p_2 - p_1}{2L\mu}\int_0^{r_0} r \; dr$$

$$-U_m = \frac{(p_2 - p_1)r_0^2}{4L\mu}$$

or

$$U_m = \left(\frac{\Delta p}{4L\mu}\right) r_0^2 \tag{4.17}$$

Note that U_m varies as r_0^2, so the velocity profile is parabolic. It can be shown that the centerline velocity is related to the average flow velocity as follows:

$$U_m = 2v \tag{4.18}$$

Substituting from equation (4.18) into equation (4.17) yields the following expression for the pressure drop:

$$v = \frac{U_m}{2} = \left(\frac{\Delta p}{8L\mu}\right) r_0^2$$

or

$$\Delta p = \frac{8L\mu v}{r_0^2} \tag{4.19}$$

As previously developed, $\Delta p = K\rho v^2/2g_c$ [equation (4.2)] and $K = fL/D$ [equation (4.5)]. Substituting these expressions into equation (4.19), and noting that $r_0 = D/2$, yields the following expression for the friction factor:

$$\Delta p = \frac{8L\mu v}{(D/2)^2} = \frac{(fL/D)\rho v^2}{2g_c}$$

or

$$f = \frac{64\mu g_c}{Dv\rho} \tag{4.20}$$

Substituting the definition of the Reynolds number, $\text{Re} = Dv\rho/\mu g_c$ [equation (4.10)], equation (4.20) simplifies to the following form:

$$f = \frac{64}{\text{Re}} \tag{4.21}$$

It should be noted that equation (4.21) is valid *for laminar flow only*.

From the derivation of equation (4.21), the following form of the pressure drop was obtained:

$$\Delta p = \frac{(fL/D)\rho v^2}{2g_c} \tag{4.22}$$

Substitution of equation (3.6), $v = \dot{m}/\rho A$, into equation (4.22) yields the following expression for pressure drop:

$$\Delta p = \frac{(fL/D)\rho v^2}{2g_c} = \left(\frac{fL}{D}\right)\frac{\rho(\dot{m}/\rho A)^2}{2g_c} = \left(\frac{fL}{D}\right)\frac{(\dot{m}/A)^2}{2\rho g_c} \tag{4.23}$$

Critical Reynolds Numbers

Experiments show that when the Reynolds number is 2,000 or less, the flow is normally laminar. If the flow is initially turbulent, it cannot be maintained indefinitely. For *stable* flow, the Reynolds number of 2,000 is the lower limit. It is possible to maintain a laminar flow to very high Reynolds numbers if care is taken to increase the flow gradually; normally, however, the slightest disturbance will destroy the laminar flow pattern when the Reynolds number is greater than 3,000. For stable flow, the Reynolds number of 3,000 is the upper limit. Between 2,000 and 3,000, the flow depends upon many factors, such as external vibration, direction of flow (increasing or decreasing), and internal upstream obstructions. Flow between Reynolds numbers of 2,000 to 3,000 is generally unstable, and designers of piping systems must account for this.

Example 4.1: *Pressure Loss in Laminar Flow.* An oil flows in a 1-in. schedule 40* horizontal steel pipe, 20 ft long at a rate of 324 lbm/hr. The oil has a density of 59 lbm/ft^3 and a viscosity of 20×10^{-3} $lbf\ s/ft^2$. Compute the pressure loss.

Solution

This example is solved by determining the type of flow and employing the proper equations to compute the pressure loss.

(a) Obtain the pipe dimensions.
Table C.3 for 1-in. schedule 40 steel pipe gives: $D = 0.08742$ ft, $A = 0.006002$ ft^2.

(b) Determine the type of flow by computing the Reynolds number

$$\mathrm{Re} = \frac{4\dot{m}}{\pi D \mu g_c} \tag{4.12}$$

$$\mathrm{Re} = \frac{4(324\,\mathrm{lbm/hr})(1\,\mathrm{hr}/3600\,\mathrm{s})}{\pi(0.08742\,\mathrm{ft})(20 \times 10^{-3}\,\mathrm{lbf \cdot s/ft^2})(32.174\,\mathrm{lbm\,ft/lbf \cdot s^2})}$$

$$\mathrm{Re} = 2.037$$

The flow is laminar, because the Reynolds number is less than 2,000.

(c) Compute the friction factor.
For laminar flow, equation (4.21) should be used:

$$f = \frac{64}{\mathrm{Re}} \tag{4.21}$$

$$f = \frac{64}{2.037}$$

$$f = 31.42$$

(d) Compute the pressure loss using equation (4.23).

$$\Delta p = \left(\frac{fL}{D}\right)\frac{(\dot{m}/A)^2}{2\rho g_c} \tag{4.23}$$

$$\Delta p = \left(\frac{31.42 \times 20\,\mathrm{ft}}{0.08742\,\mathrm{ft}}\right)\frac{(324\,\mathrm{lbm/hr}/0.006002\,\mathrm{ft^2} \times 1\,\mathrm{hr}/3600\,\mathrm{s})^2}{2(59\,\mathrm{lbm/ft^3})(32.174\,\mathrm{lbm \cdot ft/lbf \cdot s^2})}$$

$$\Delta p = 425.7\,\mathrm{lbf/ft^2} \times 1.44\,\mathrm{in.^2}/1\,\mathrm{ft^2}$$

$$\Delta p = 2.96\,\mathrm{psi}$$

*Pipe schedules are given in Appendix B. The wall thickness of popular pipe schedules are tabulated in Appendix C.

4.7 TURBULENT FLOW IN PIPES

By dimensional, physical, and similarity analyses, it has been determined that the friction factor is some function of the Reynolds number and relative roughness. Experimental data indicate that at Reynolds numbers below 2,000, the flow is laminar and independent of relative roughness. For Reynolds numbers above 3,000, the friction factor is some function of both the Reynolds number and relative roughness. Conventional engineering practice is to use the Colebrook equation, which was the result of an analysis of a large quantity of experimental data. This equation is shown here as equation (4.24):

$$\frac{1}{\sqrt{f}} = -2\ \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{4.24}$$

Examination of equation (4.24) indicates that if the value of surface roughness is small compared with the pipe diameter ($\varepsilon/D \rightarrow 0$), then the friction factor is a function of the Reynolds number only. A *smooth pipe* is one in which the ratio $\varepsilon/3.7D$ is small compared with $2.51/\mathrm{Re}\sqrt{f}$. On the other hand, as the Reynolds number increases so that $2.51/\mathrm{Re}\sqrt{f} \rightarrow 0$, then the friction factor becomes purely a function of relative roughness, and the pipe is called a *rough pipe*. Thus, for purposes of analysis, the same pipe may be considered smooth under one flow condition and rough under another.

Equation (4.25) requires a trial-and-error solution for the friction factor. For rough and smooth pipes, A. K. Jain (1975) derived the following approximation:

$$f = \left[1.14 - 2\ \log_{10}\left(\frac{\varepsilon}{D} + \frac{21.25}{\mathrm{Re}^{0.9}}\right)\right]^{-2} \tag{4.25}$$

Table 4.2 illustrates the valid ranges of relative roughness and Reynolds number for equation (4.25) and the maximum deviation of calculated friction factor from equation (4.24). Table 4.2 illustrates that equation (4.24) may be used with engineering accuracy for all design calculations for turbulent flow.

Examination of Figure 4.3 shows that the zone of complete turbulence (rough pipes) is separated from the transition zone by a dotted line. This line is defined by the following equation:

$$\mathrm{Re}_t = \frac{3200}{\varepsilon/D} \tag{4.26}$$

where

Table 4.2 Equation (4.25): Valid Ranges of Relative Roughness and Reynolds Number and the Maximum Deviation of f from Equation (4.24)

Range	Relative Roughness Minimum	Relative Roughness Maximum	Reynolds Number Minimum	Reynolds Number Maximum	Maximum deviation
Extreme	0	1×10^{-2}	3×10^{3}	1×10^{10}	3.5 %
Useful	1×10^{-6}	1×10^{-3}	5×10^{3}	1×10^{8}	1.5 %
Practical	0	1×10^{-2}	1×10^{4}	1×10^{7}	−0.8 %

Re_t = minimum Reynolds number for complete turbulence, dimensionless

ε/D = relative roughness of the pipe surface, dimensionless

Values of Re_t and ε/D for wrought steel and stainless steel pipes are given in Table C.3. Table 4.3 shows recommended values of absolute roughness of new, clean commercial pipes.

Solving equation (4.24) for the friction factor at a Reynolds number of infinity results in equation (4.27):

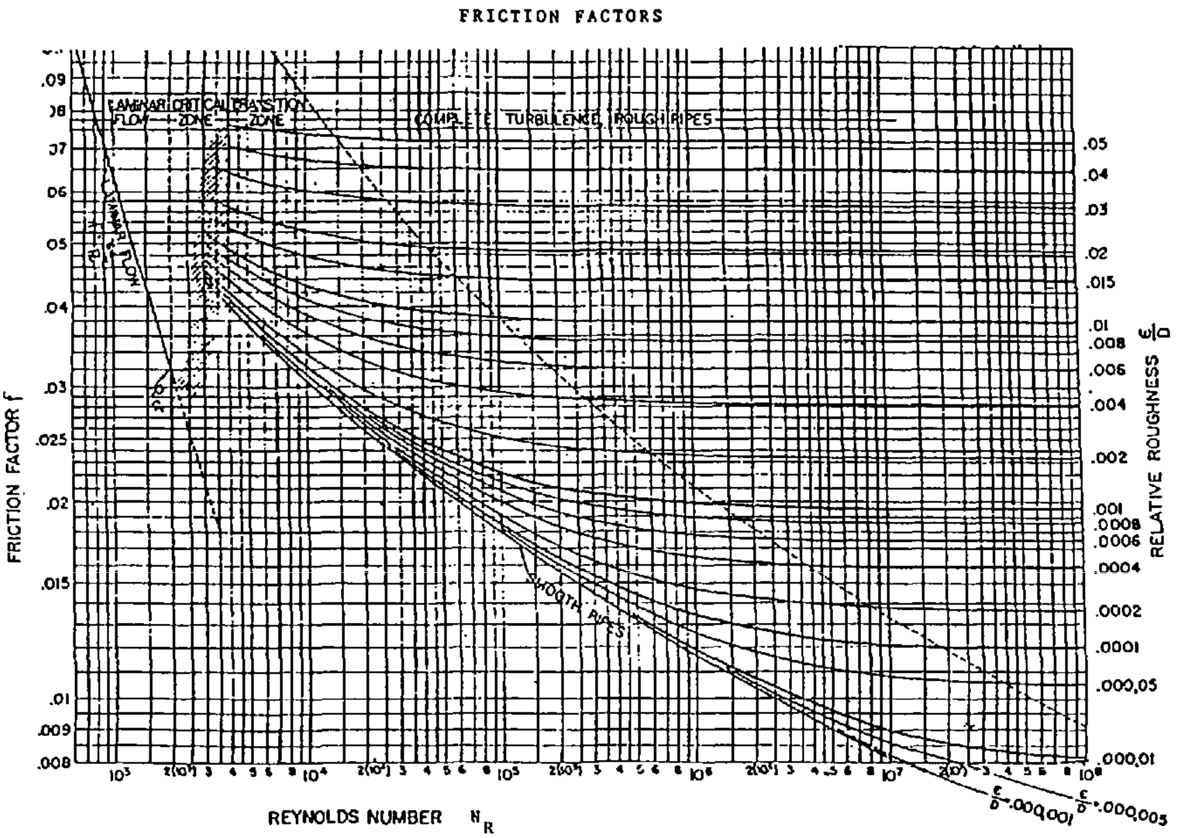


Figure 4.3 Friction factor as a function of Reynolds number.

Table 4.3 Design Values of Absolute Roughness for New, Clean Commercial Pipes

	Absolute roughness (ε) × 10^6		
Pipe	Range (ft)	Design value (ft)	Design value (m)
Asphalted cast iron	400	400	122
Brass and copper	5	5	1.5
Concrete	1,000–10,000	4,000	1,219
Cast iron	850	850	259
Galvanized iron	500	500	152
Wrought iron and steel	150	150	46
Riveted steel	3,000–30,000	6,000	1,829
Wood stave	600–3,000	2,000	610

$$f_\infty = \frac{0.25}{\left[\log_{10}\left(\frac{\varepsilon/D}{3.7}\right)\right]^2} \tag{4.27}$$

where

f = friction factor at a Reynolds number of infinity, dimensionless

Values of f_∞ for wrought steel and stainless steel pipe are given in Table C.3.

4.8 ENERGY LOSSES IN PIPING SYSTEMS

In long pipelines, the effect of bends, valves, and fittings is usually negligible; but in systems where there is little straight pipe, it is the controlling factor. Underdesign will result in the failure of the system to deliver the required capacity. Overdesign will result in inefficient operation, because it will be necessary to "throttle" one or more of the valves.

The following energy losses must be considered in computing friction losses in piping systems:

1. loss due to fluid entering the system
2. loss due to pipe friction
3. loss due to valves and fittings
4. loss due to bends
5. loss due to changes in flow areas
6. loss due to fluid leaving the system

Entrance Loss

The energy loss due to fluid entering a system depends on the geometry of the entrance. With complete turbulence assumed at the inlet, resistance is independent of Reynolds number. Inlet energy loss (H_i) can be calculated with the following expression:

$$H_i = K_i \frac{v_i^2}{2g_c} \tag{4.28}$$

Typical values of the inlet resistance coefficient K_i are shown in Table 4.4 .

Pipe Friction Losses

The pipe friction energy loss H_{fp}, may be computed using the following:

$$H_{fp} = \sum_i^e \left(\frac{fL}{D} \frac{v^2}{2g_c} \right)_{fp} \tag{4.29}$$

Values of the individual friction factors f may be calculated using equation (4.21) for laminar flow and equation (4.25) for turbulent flow.

Valve and Fitting Losses

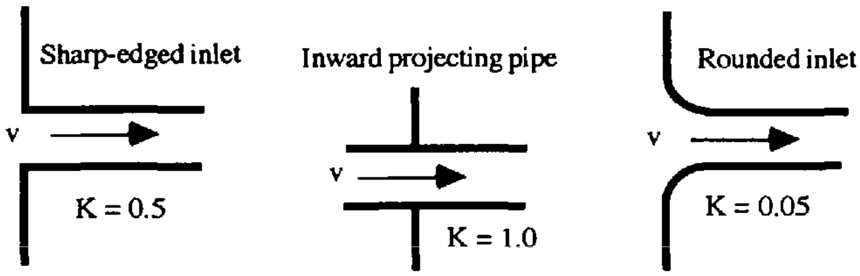
The flow of a fluid through valves and fittings is generally considered to be one of complete turbulence and independent of Reynolds number. Valves and fittings are treated as straight pipes with a very large surface roughness ε. One common engineering practice is to describe the frictional resistance in terms of equivalent lengths (L_e) of straight pipe. The resistance coefficients K_v for valves and K_F for fittings are then calculated using the following equations as appropriate:

$$K_v = f_\infty \left(\frac{L_e}{D} \right)_v \tag{4.30}$$

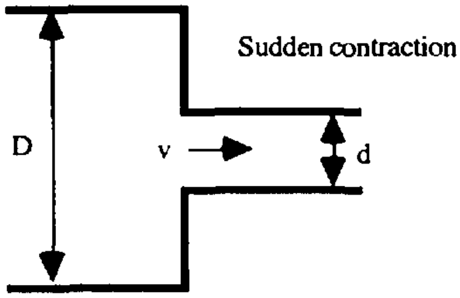
$$K_F = f_\infty \left(\frac{L_e}{D} \right)_F \tag{4.31}$$

There are two widely used sources of data for piping components: the *Hydraulic Institute Engineering Data Book* and *Crane Company Technical Paper 410*. The Hydraulic Institute method uses the resistance coefficient K and distinguishes between screwed and flanged fittings. The Crane paper uses a modified equivalent-length method to determine the resistance coefficient using equation (4.30) for valves and equation (4.31) for fittings.

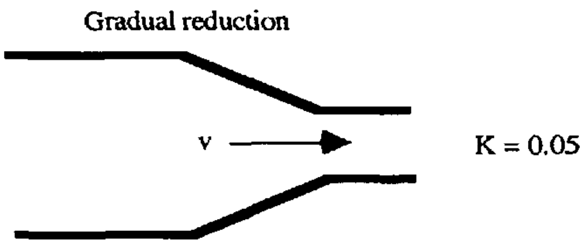
Table 4.4 Resistance Coefficients for Inlets, Exits, and Area Changes



All types of exits: K = 1.0



D/d	1.5	2.0	2.5	3.0	3.5	4.0
K	0.28	0.36	0.40	0.42	0.44	0.46

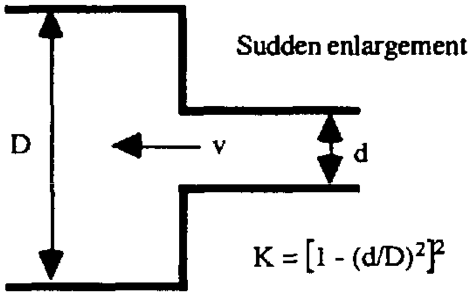


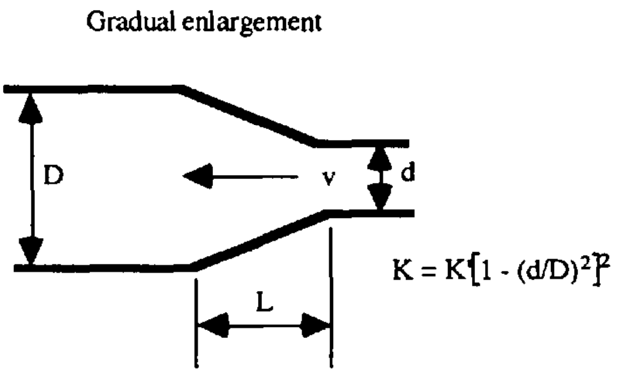
Examination of Tables 4.5 and 4.6 shows significant differences between these two sources.

Valves

It is valve industry practice to determine valve capacity and pressure loss using a valve flow coefficient C_v, defined as the flow in gallons per minute (GPM) of water at 60°F with a pressure drop of 1 pound per square inch (psi):

Table 4.4 (*continued*) Resistance Coefficients for Inlets, Exits, and Area changes





(D-d)/2L	0.05	0.10	0.20	0.30	0.40	0.50	0.60
K'	0.14	0.20	0.47	0.76	0.95	1.05	1.10

$$C_v = \text{GPM}\sqrt{\frac{1}{\text{psi}}} \tag{4.32}$$

Substitution of appropriate conversion factors in the energy equation results in:

$$K_v = \frac{18.5 \times 10^6 D^4}{C_v^2} \tag{4.33}$$

Most of the energy losses in process and power piping systems are due to valves rather than straight pipe. Whenever possible, the designer should use C_v values obtained from the valve manufacturer based on actual test data for the specific valve type. See Table 4.7.

The energy loss H_{vF} due to valves and fittings may be calculated using the following equations:

Table 4.5 Resistance Coefficients (K) for Valves and Fittings

Nominal Pipe Size inches	Hydraulic Institute Engineering Data Book		Crane Co. T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410
	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both
	Globe Valve			**Gate Valve**			**Angle Valve**			**Swing Check Valve**		
3/8	19.50		9.4[illegible]	0.38		0.22	13.50		4.15	8.03		2.77
1/2	14.90		8.81	0.33		0.21	9.80		3.89	5.52		2.59
3/4	9.41		8.15	0.28		0.19	6.24		3.60	3.74		2.40
1	8.98	13.09	7.65	0.24	0.70	0.18	4.53		3.37	3.00	2.00	2.25
1 1/4	8.00	11.56	7.13	0.22	0.55	0.17	3.53	3.60	3.15	2.56	2.00	2.10
1 1/2	7.63	10.44	6.86	0.20	0.45	0.16	2.88	3.00	3.03	2.51	2.00	2.02
2	7.08	8.90	6.46	0.17	0.33	0.15	2.09	2.25	2.85	2.33	2.00	1.90
2 1/2	6.68	7.86	6.19	0.16	0.26	0.15	1.63	2.20	2.73	2.15	2.00	1.82
3	6.37	7.10	5.89	0.14	0.22	0.14	1.33	2.20	2.60	2.05	2.00	1.73
4	5.91	6.45	5.54	0.12	0.16	0.13	0.97	2.20	2.44	2.00	2.00	1.63
6		6.00	5.07		0.10	0.12		2.10	2.24		2.00	1.49
8		5.70	4.78		0.08	0.11		2.10	2.11		2.00	1.41
10		5.70	4.57		0.06	0.11		2.10	2.01		2.00	1.34
12		5.70	4.41		0.05	0.10		2.10	1.94		2.00	1.30
14		5.70	4.32		0.04	0.10		2.10	1.91		2.00	1.27
16		5.70	4.21		0.04	0.10		2.10	1.86		2.00	1.24
18		5.70	4.11		0.03	0.10		2.10	1.81		2.00	1.21
20		5.70	4.02		0.03	0.09		2.10	1.77		2.00	1.18

Nominal Pipe Size inches	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both
	90° Standard Elbow			**90° Long Radius Elbow**			**45° Standard Elbow**			**45° Long Radius Elbow**		
3/8	2.58		0.83	1.64		0.44	0.38		0.44			
1/2	2.19		0.78	1.31		0.41	0.37		0.41			
3/4	1.75		0.72	0.96		0.38	0.35		0.38			
1	1.49	0.43	0.67	0.77	0.42	0.36	0.34		0.36		0.23	
1 1/4	1.32	0.41	0.63	0.64	0.38	0.34	0.33		0.34		0.22	
1 1/2	1.19	0.39	0.61	0.56	0.35	0.32	0.33		0.32		0.21	
2	1.01	0.36	0.57	0.45	0.30	0.30	0.32		0.30		0.20	
2 1/2	0.89	0.34	0.55	0.38	0.27	0.29	0.31		0.29		0.19	
3	0.81	0.33	0.52	0.33	0.25	0.28	0.30		0.28		0.19	
4	0.69	0.30	0.49	0.26	0.22	0.26	0.29		0.26		0.18	
6		0.27	0.45		0.18	0.24			0.24		0.17	
8		0.26	0.42		0.16	0.23			0.23		0.16	
10		0.24	0.40		0.14	0.21			0.21		0.16	
12		0.23	0.39		0.13	0.21			0.21		0.15	
14		0.22	0.38		0.12	0.20			0.20		0.15	
16		0.22	0.37		0.11	0.20			0.20		0.15	
18		0.21	0.36		0.11	0.19			0.19		0.14	
20		0.20	0.35		0.10	0.19			0.19		0.14	

Nominal Pipe Size inches	Hydraulic Insutute Engineering Data Book Screwed	Hydraulic Insutute Engineering Data Book Flanged	Crane Co T.P 410 Both	Hydraulic Insutute Engineering Data Book Screwed	Hydraulic Insutute Engineering Data Book Flanged	Crane Co T.P 410 Both	Hydraulic Insutute Engineering Data Book Screwed	Hydraulic Insutute Engineering Data Book Flanged	Crane Co T.P 410 Both	Hydraulic Insutute Engineering Data Book Screwed	Hydraulic Insutute Engineering Data Book Flanged	Crane Co T.P 410 Both
	Close Pattern Return Bend			**Long Radius Return Bend**			**Tee - Line Flow**			**Tee - Branch Flow**		
3/8	2.65		1.38				0.90		0.55	2.63		1.66
1/2	2.24		1.30				0.90		0.52	2.37		1.55
3/4	1.76		1.20				0.90		0.48	2.05		1.44
1	1.49	0.43	1.12		0.43		0.90	0.26	0.45	1.85	1.00	1.35
1 1/4	1.31	0.41	1.05		0.39		0.90	0.23	0.42	1.71	0.95	1.26
1 1/2	1.17	0.39	1.01		0.36		0.90	0.21	0.40	1.60	0.90	1.21
2	0.99	0.36	0.95		0.31		0.90	0.19	0.38	1.44	0.84	1.14
2 1/2	0.87	0.34	0.91		0.28		0.90	0.17	0.36	1.33	0.80	1.09
3	0.78	0.32	0.87		0.26		0.90	0.15	0.35	1.25	0.76	1.04
4	0.66	0.30	0.81		0.23		0.90	0.13	0.33	1.13	0.71	0.98
6		0.27	0.75		0.19			0.11	0.30		0.64	0.89
8		0.25	0.70		0.17			0.10	0.28		0.59	0.84
10		0.24	0.67		0.15			0.09	0.27		0.56	0.81
12		0.23	0.65		0.14			0.08	0.26		0.54	0.78
14		0.22	0.64		0.13			0.07	0.25		0.52	0.76
16		0.21	0.62		0.12			0.07	0.25		0.50	0.74
18		0.20	0.60		0.12			0.07	0.24		0.49	0.72
20		0.20	0.59		0.11			0.06	0.24		0.47	0.71

Note: Crane resistance coefficients were calculated on the basis of Standard Weight pipe

Table 4.6 Equivalent Lengths (L_e/D) for Valves and Fittings

Nominal Pipe Size inches	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co T.P. 410
	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both
	Globe Valve			**Gate Valve**			**Angle Valve**			**Swing Check Valve**		
3/8	706			14			488			290		
1/2	575			13			378			213		
3/4	392			11			260			156		
1	399	582		11	31		201			133	89	
1 1/4	381	551		10	26		169	160		122	95	
1 1/2	378	518		10	23		143	143		124	99	
2	373	469		9	18		110	111		123	105	
2 1/2	367	431		9	14		90	116		118	110	
3	368	410	340	8	13	8	77	121	150	118	116	100
4	363	396	All	8	10	All	59	127	All	129	123	All
6		403	Sizes		7	Sizes		141	Sizes		134	Sizes
8		405			5			149			142	
10		424			4			156			149	
12		440			4			162			154	
14		448			3			165			157	
16		461			3			170			162	
18		472			3			174			166	
20		482			2			178			169	
	90° Standard Elbow			**90° Long Radius Elbow**			**45° Standard Elbow**			**45° Long Radius Elbow**		
3/8	93			59			14					
1/2	85			51			14					
3/4	73			40			15					
1	66	19		34	19		15				10	
1 1/4	63	19		31	18		16				10	
1 1/2	59	19		28	17		16				10	
2	53	19		23	16		17				11	
2 1/2	49	19		21	15		17				11	
3	47	19	30	19	14	16	17		16		11	
4	42	19	All	16	13	All	18		All		11	
6		18	Sizes		12	Sizes			Sizes		11	
8		18			11						12	
10		18			11						12	
12		18			10						12	
14		17			9						12	
16		17			9						12	
18		17			9						12	
20		17			9						12	

Nominal Pipe Size inches	Hydraulic Institute Engineering Data Book		Crane Co. T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co. T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co. T.P. 410	Hydraulic Institute Engineering Data Book		Crane Co. T.P. 410
	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both	Screwed	Flanged	Both
	Close Pattern Return Bend			**Long Radius Return Bend**			**Tee - Line Flow**			**Tee - Branch Flow**		
3/8	96						33			95		
1/2	86						35			92		
3/4	74						38			86		
1	66	19			19		40	12		82	44	
1 1/4	62	19			19		43	11		81	45	
1 1/2	58	19			18		45	11		79	45	
2	52	19			17		47	10		76	44	
2 1/2	48	19			16		49	9		73	44	
3	45	19	50		15		52	9	20	72	44	60
4	40	18	All		14		55	8	All	69	43	All
6		18	Sizes		13			7	Sizes		43	Sizes
8		18			12			7			42	
10		18			11			6			42	
12		17			11			6			41	
14		17			10			6			41	
16		17			10			6			40	
18		17			10			5			40	
20		17			9			5			40	

Note: Hydraulic Institute equivalent lengths were calculated on the basis of Standard Weight pipe.

Table 4.7 Resistance Coefficients (C_r) for Globe Valves

Nominal pipe size	Class 300 schd. 40	Class 600 schd. 80	Class 900 schd. 160	Class 1500 schd. 160	Class 2500 XXS
2-1/2		3.7		3.9	3.8
3	11.2	4.3		4.1	5.4
4	16.1	4.1	3.8	4.0	7.1
5	12.5	4.3	4.2	4.7	7.8
6	7.5	2.8	2.9	3.0	5.4
8	10.7	3.0	2.9	3.1	7.1
10	9.0	3.2	3.0	3.1	7.3
12	7.9	3.0	3.0	3.1	8.3
14		3.1	3.0	3.2	

$$H_{rF} = \sum_i^e \left(K_r \frac{v^2}{2g_c}\right)_r + \sum_i^e \left(K_F \frac{v^2}{2g_c}\right)_F$$

$$H_{rF} = \sum_i^e \left(\frac{f_\infty L_e}{D} \frac{v^2}{2g_c}\right)_r + \sum_i^e \left(\frac{f_\infty L_e}{D} \frac{v^2}{2g_c}\right)_F \tag{4.34}$$

Values of K_r and K_F may be taken from Table 4.5 or calculated from equation (4.30), (4.31), or (4.33). Values of L_e/D may be taken directly from Table 4.6.

Bend Losses

Energy losses in bends arise via two primary sources. The first is the secondary flows that result from the fluid rotation as the fluid flows through the bend. This loss does not take place in the bend itself, since the dissipation of the secondary flows occurs over a length of about 50 pipe diameters downstream of the bend. The second source is the frictional loss in the actual length of bend.

The first effect is completely turbulent and independent of Reynolds number. The second is dependent on both relative roughness and Reynolds number. Pigott's (1950) analysis of bend flow resulted in the following equation for the bend resistance coefficient K_b:

$$K_b = 0.106\left(\frac{D}{r}\right)^{2.5} + 2000 f^{2.5} \tag{4.35}$$

In this equation, r is the radius of the bend, D is the pipe diameter, and f is the friction factor calculated from equation (4.25). The bend energy losses (H_b) are then calculated as follows:

$$H_b = \sum_i^e \left(K_b \frac{v^2}{2g_c} \right)_b \tag{4.36}$$

Area Change Losses

Resistance coefficients (K_a) for area changes due to enlargement or reduction of flow passages are given in Table 4.4 for common situations. Once again the flow is assumed to be completely turbulent and independent of Reynolds number. The resultant energy loss H_a may be calculated using the following equation:

$$H_a = \sum_i^e \left(K_a \frac{v^2}{2g_c} \right)_a \tag{4.37}$$

Exit Loss

The exit energy loss (H_e) is one velocity head (i.e., $K_e = 1$). Equation (4.38) is used to calculate the exit energy loss:

$$H_e = \frac{v_e^2}{2g_c} \tag{4.38}$$

Total System Losses

The system energy loss is the sum of the individual losses, or:

$$H_{fs} = H_i + F_{fp} + H_{vF} + H_b + H_a + H_e \tag{4.39}$$

Substitution of H_{fs} in equation (4.39) for H_f of equation (3.22) and solving for system pressure loss results in the following:

$$\frac{p_i - p_e}{\rho} = \frac{\Delta p_{\text{system}}}{\rho} = \frac{g}{g_c}(z_e - z_i) + \frac{v_e^2 - v_i^2}{2g_c} + H_{fs} \tag{4.40}$$

4.9 DESIGN VELOCITIES

It is always desirable to design piping systems for the highest possible velocity in order to reduce the size of the pipe and its components. The maximum design velocity for a given piping system is limited by the following considerations:

The maximum design velocity must be economically justifiable in terms of pumping power and/or fuel costs.
The maximum design velocity must not cause physical problems such as erosion, corrosion, vibration, noise, and cavitation.

In general, design velocities have been established as the result of experience and experimentation. This section examines some of the current practices.

Process Piping Systems

Preliminary Sizing

In the early stages of design, a study layout is drawn that requires the internal diameter of the pipe to be determined before the actual lengths are known. Kent (1980) proposed the following equations for sizing internal diameters. Kent's data suggest that 80% of the sizes computed by his equations remained unchanged after layouts were completed and more precise checks were made.

Maximum Velocities for Liquids (v, ft/s; ρ, lbm/ft^3)

Clear liquids:

$$v = \frac{48}{\rho^{1/3}} \tag{4.41}$$

Erosive and/or corrosive liquids:

$$v = \frac{24}{\rho^{1/3}} \tag{4.42}$$

Diameters Based on Typical Velocities (D, ft; Q, ft^3/s; ρ, lbm/ft^3; $\dot{m}$, lbm/s):

Liquids, except suction, drains, or vent lines:

$$D = 0.379Q^{0.434} \tag{4.43}$$

Liquid suction, drain, or vent lines:

$$D = 0.512Q^{0.434} \tag{4.44}$$

Gases, except suction and vent lines:

$$D = 0.150 \frac{\dot{m}^{0.408}}{\rho^{0.343}} \tag{4.45}$$

Gases—suction and vent lines:

$$D = 0.198 \frac{\dot{m}^{0.408}}{\rho^{0.343}} \tag{4.46}$$

Central Station Systems

Over a period of years, velocities in main water and steam piping have increased beyond the limiting values given in handbooks. Soldan and Seigel (1964) described Public Service Electric and Gas Company's experiences with design velocities from 1937 to 1963. During this period, water velocities were increased from 8 to 40 ft/s and steam from 100 to 415 ft/s. A curve fit of their data resulted in the following relations (v, ft/s; D, ft):

Maximum velocity for miscellaneous water service:

$$v = 0.5458 + 57.88D - 24.31D^2 \tag{4.47}$$

Maximum velocity for boiler feedwater flow:

$$v = 5.551 + 58.31D - 23.46D^2 \tag{4.48}$$

Maximum velocity for main steam flow:

$$v = 120.35 + 257.95D \tag{4.49}$$

The foregoing maximum velocities will yield a pressure loss of about 10 psi per 100 feet for miscellaneous water service, about 15 psi per 100 feet for boiler feedwater flow, and about 60 psi per 100 feet for main steam velocities, depending on steam conditions.

Typical Design Velocities

Table 4.8 presents typical design velocities for general use in the absence of more specific data for a given industry.

Table 4.8 Typical Design Velocities

Service	Design velocity Minimum (ft/s)	Maximum (ft/s)
Bleed steam lines	65	100
Boiler and turbine leads	100	200
Branch steam lines	100	250
Compressed air lines	25	35
Compressor discharge lines	100	250
Compressor suction lines	75	200
Crude oil lines (6 in. to 30 in.)	1	6
Drain lines (water)	3	5
Drain lines (hydrocarbon liquids)	3	5
Economizer tubes (water)	2	5
Exhaust and low-pressure steam lines	100	250
Feed water lines	4	15
Natural gas lines (cross country)	15	25
Pump discharge lines (fuel oils)	1	6
Pump discharge lines (hydrocarbon liquids)	3	7
Pump discharge lines (water)	2	7
Pump suction lines (hydrocarbon liquids)	2	6
Pump suction lines (fuel oils)	3	6
Pump suction lines (water)	2	5
Service water mains	2	5
Steam headers	100	125
Steam superheater tubes	35	85
Two-phase flow lines	35	75
Vacuum steam lines	300	600
Ventilating ducts	15	50

4.10 ENGINEERING DESIGN CALCULATIONS

Engineering pipe flow computations usually fall into one of two classes: preliminary design or design verification.

Preliminary Design

In this stage the flow rate Q and the length of the pipe L are known. The pipe diameter D and the wall thickness are then selected on the basis of one of the following criteria:

1. The maximum allowable velocity v to prevent erosion, vibration, or excessive energy loss. Values of maximum velocities may be calculated using equations given in Section 4.9 or Table 4.8, as appropriate.
2. The maximum allowable pressure loss Δp that can be tolerated for proper functioning of equipment or to prevent excessive energy loss. This type of loss is usually expressed in terms of pressure drop per foot $(\Delta p/L)$. For turbulent flow of fluids in smooth pipes the following equation may be used to estimate the internal pipe diameter:

$$D = 0.9235\left[\frac{\dot{m}^{1.843}\mu^{0.157}}{\rho(\Delta p/L)}\right]^{0.2065} \tag{4.50}$$

Design Verification

In this stage the piping components have been selected, the diameter D, the length of pipe L, and the flow rate Q are known, and the pressure loss Δp is to be verified.

4.11 RECOMMENDED PROCEDURES FOR DESIGN CALCULATIONS

The following steps are recommended to facilitate preliminary and final pipe sizing and pressure losses in piping systems.

Step A: Data Reduction

Obtain fluid and pipe data and convert as necessary to the following units:

densities into pound-mass per cubic foot
diameters and lengths into feet
mass flow rates into pound-mass per second
pressures into pound-force per square foot
velocities into feet per second
viscosities into pound-force-seconds per square foot
volume flow rates into cubic feet per second

Step B: Select Pipe Schedule

Estimate the minimum pipe schedule using the approximate equation (B.3). *Note: All stress calculations must conform with the American National Standard ANSI B.31, Code for Pressure Piping. The verification of stress* *calculations is beyond the scope of this book.)* Equation (B.3) will always give a conservative value.

Step C: Estimate Internal Pipe Diameter

Depending upon the circumstances, use either the maximum velocity method or the pressure loss per length method. Select pipe from piping tables. If the loss per unit length method is used, verify that the velocity does not exceed the maximum allowable.

Step D: Verify the Preliminary Design

Once a tentative diameter has been selected, check to see if maximum velocities have been exceeded. If some minimum energy loss is a design requirement, it should be calculated using the appropriate equations from Section 4.8. If either or both of these requirements have not been met, repeat as necessary using the next larger standard size. The following examples illustrate this procedure.

Example 4.2: *Preliminary Design of a Boiler Feed Water System.* The boiler feed water system shown in Figure 4.4 is for a small cogeneration steam generator. The boiler feed pump is to deliver 300 GPM of water at 300 F and 1,500 psia to the steam drum. The allowable stress in the piping is 12,500 psi. Select the size of pipe to be used. The pressure drop due to friction may not exceed 10 pounds per square inch. Also estimate the required pump discharge pressure for a steam drum pressure of 1,500 psia.

Solution

This example is solved by estimating the pipe internal diameter from equations given in Section 4.9 and verifying by calculating friction losses from Section 4.8.

Step A: Data Reduction

1. *Fluid properties* (Table D.3 at 1,500 psia and 300 °F):

$$\rho = 57.64\,\text{lbm/ft}^3, \qquad \mu = 3.838 \times 10^{-6}\,\text{lbf} \cdot \text{s/ft}^2$$

2. *Unit conversions*:

$$Q = 300\,\text{GPM} \times 1.3368 \times 10^{-1}\,\text{ft}^3/\text{gal} \times 1\,\text{min}/60\,\text{s} = 0.6684\,\text{ft}^3/\text{s}$$

$$\dot{m} = Q\rho = 0.6684\,\text{ft}^3/\text{s} \times 57.64\,\text{lbm/ft}^3 = 38.52\,\text{lbm/s}$$

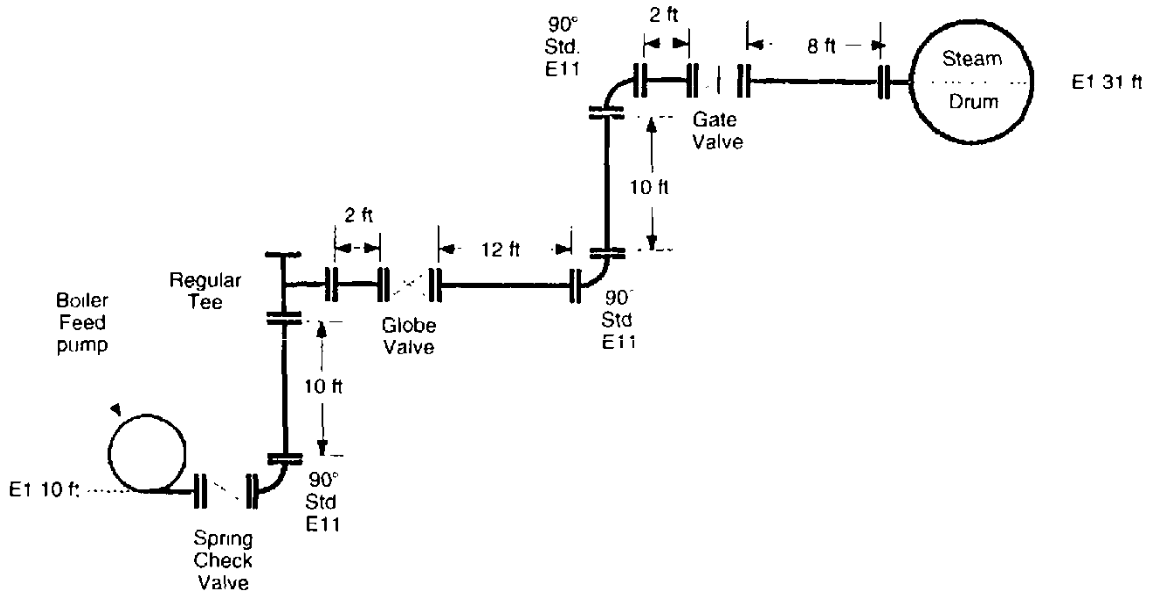


Figure 4.4 Notation for Example 4.2.

Step B: Select Pipe Schedule
From equation (B.3):

$$N_s = 1000p/S_a = 1000 \times (1500 - 14.7)\text{psi}/12{,}500\,\text{psi} = 119$$

Use schedule 120 (next higher size).

Step C: Estimate Internal Pipe Diameter
For the first trial, use the simplest equation, equation (4.41):

$$v = \frac{48}{\rho^{1/3}} \tag{4.41}$$

$$v = \frac{48}{57.64^{1/3}}$$

$$v = 12.42\,\text{ft/s}$$

From the continuity equation:

$$D = \sqrt{\frac{4Q}{\pi v}}$$

$$D = \sqrt{\frac{4 \times 0.6684\,\text{ft}^3/\text{s}}{\pi \times 12.42\,\text{ft/s}}}$$

$$D = 0.2618\,\text{ft}$$

From Table C.3, the nearest normally used larger pipe size is 4-in. schedule 120 ($D = 0.3020$ ft).

Step D: Verify Preliminary Design
Check velocity:

$$v = \frac{4Q}{\pi D^2}$$

$$v = \frac{4 \times 0.6684\,\text{ft}^3/\text{s}}{\pi \times (0.3020\,\text{ft})^2}$$

$$v = 9.33\,\text{ft/s}$$

From Table 4.8, design velocities for feed water lines range from range 4 ft/s to 15 ft/s. Therefore, design velocity is satisfactory.

Verify pressure loss: To verify system loss, use the equations and methods of Section 4.8.

1. *Loss due to the fluid entering the system*: In this system the pump discharge has the same area as the swing check valve inlet, so there is no entrance loss:

$$H_i = 0$$

2. *Loss due to pipe friction*: The Reynolds number is calculated using the following equation:

$$\text{Re} = \frac{\rho D v}{\mu g_c} \tag{4.10}$$

$$\text{Re} = \frac{57.64\,\text{lbm/ft}^3 \times 9.33\,\text{ft/s} \times 0.302\,\text{ft}}{3.838\,\text{lbf s/ft}^2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$\text{Re} = 1{,}315{,}000\,\text{(turbulent flow)}$$

Table C.3 gives the relative roughness (ε/D) as 4.967×10^{-4}. The friction factor is then calculated using equation (4.25):

$$f = \left[1.14 - 2\log_{10}\left(\frac{\varepsilon}{D} + \frac{21.25}{\text{Re}^{0.9}}\right)\right]^{-2} \tag{4.25}$$

$$f = \left[1.14 - 2\log_{10}\left(4.967 \times 10^{-4} + \frac{21.25}{(1{,}315{,}000)^{0.9}}\right)\right]^{-2}$$

$$f = 0.01714$$

The total length of straight pipe is $10 + 2 + 12 + 10 + 2 + 8 = 44\,\text{ft}$.

$$H_{fp} = \sum_i^e \left(\frac{fL}{D} \frac{v^2}{2g_c} \right)_{fp} \tag{4.29}$$

$$H_{fp} = \frac{0.01714 \times 44\,\text{ft}}{0.302\,\text{ft}} \times \frac{(9.33\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{fp} = 3.38\,\text{ft} \cdot \text{lbf/lbm}$$

3. *Loss due to valves and fittings*
Valves (from Table 4.5)

	K_v (Hydraulic Institute)	K_v (Crane TP410)
1 swing check	2.00	1.63
1 globe	6.45	5.54
1 gate	0.16	0.13
ΣK_v	8.61	4.30

The most conservative design would be to use the Hydraulic Institute value of $\Sigma K_v = 8.61$.

Fittings (from Table 4.5)

	K_F (Hydraulic Institute)	K_F (Crane TP410)
3 90° standard elbows	$3 \times 0.30 = 0.90$	$3 \times 0.49 = 1.47$
1 regular tee—branch flow	$1 \times 0.71 = 0.71$	$1 \times 0.98 = 0.98$
ΣK_F	1.61	2.45

Again, take the most conservative $\Sigma K_F = 2.45$ (from Crane TP410). The valve and fitting loss is calculated using equation (4.34).

$$H_{vF} = \sum_i^e \left(K_v \frac{v^2}{2g_c} \right)_v + \sum_i^e \left(K_F \frac{v^2}{2g_c} \right)_F$$

$$H_{vF} = (8.61 + 2.45) \frac{(9.33\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{vF} = 14.96\,\text{ft} \cdot \text{lbf/lbm}$$

4. *Loss due to bends*: There are no bends, so $H_b = 0$.

5. *Loss due to changes in flow areas*: There are no area changes, so $H_a = 0$.

6. *Loss due to fluid leaving the system*: In this system the drum inlet has the same area as the pipe, so there is no exit loss, and $H_e = 0$.

The system friction loss is the sum of the component pressure losses:

$$H_{fs} = H_i + H_{fp} + H_{vF} + H_b + H_a + H_e \tag{4.39}$$

$$H_{fs} = 0 + 3.38 + 14.96 + 0 + 0 + 0$$

$$H_{fs} = 18.34\ \text{ft} \cdot \text{lbf/lbm}$$

The pressure loss due to friction is calculated as follows:

$$\Delta p = H_{fs}\rho \tag{4.8}$$

$$\Delta p = 18.34\ \text{ft} \cdot \text{lbf/lbm} \times 57.64\ \text{lbm/ft}^3$$

$$\Delta p = 18.34\ \text{lbf/ft}^2 \times 1\ \text{ft}^2/144\ \text{in.}^2$$

$$\Delta p = 7.34\ \text{psi}$$

The design is verified because the maximum pressure loss due to friction is not to exceed 10 psi.

In order to calculate the pump outlet pressure, the total pressure loss of the system must first be calculated (*note*: inlet velocity = outlet velocity because the pipe has a uniform cross section):

$$\Delta p_{\text{system}} = \rho\left[\frac{g}{g_c}(z_e - z_i) + \frac{v_e^2 - v_i^2}{2g_c} + H_{fs}\right] \tag{4.40}$$

$$\Delta p_{\text{system}} = 57.64\ \text{lbm/ft}^3\left[\frac{32.17\ \text{ft/s}^2}{32.17\ \text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}(31.2\ \text{ft} - 10\ \text{ft}) + 0 + 18.34\ \text{ft} \cdot \text{lbf/lbm}\right]$$

$$\Delta p_{\text{system}} = 2{,}279\ \text{lbf/ft}^2 \times 1\ \text{ft}^2/144\ \text{in.}^2$$

$$\Delta p_{\text{system}} = 16\ \text{psi}$$

The pump outlet pressure is the system pressure drop added to the steam drum pressure. Therefore, the pump outlet pressure is the following:

pump outlet pressure = 1500 + 16 = 1516 psia

Example 4.3: *Preliminary Design of a Fuel Oil Piping System.* A preliminary design and an estimate of the pressure loss for the fuel oil piping system shown in Figure 4.5 is required. Each pump delivers 50 GPM of fuel oil to the system at 600 psia and 100 F. At this condition the viscosity of the oil is 4,000 SSU and the gravity is equivalent to 11.5 API at 60 F/60 F. The maximum allowable stress of the piping material is 15,000 psi.

Solution

In this system there is parallel flow from the pumps to the tee and then series flow to the heater. The example is solved by breaking the system into two parts: Part I– flow from pumps to tee, and Part II– flow from tee to heater. The pipe diameters are estimated from equations given in Section 4.9 and the friction loss from Section 4.8.

Step A: Data Reduction

1. *Fluid properties*: $S = 11.5$ API at 60 F/60 F, $\mu = 4{,}000$ SSU. The specific gravity of the oil is calculated using equation (1.20):

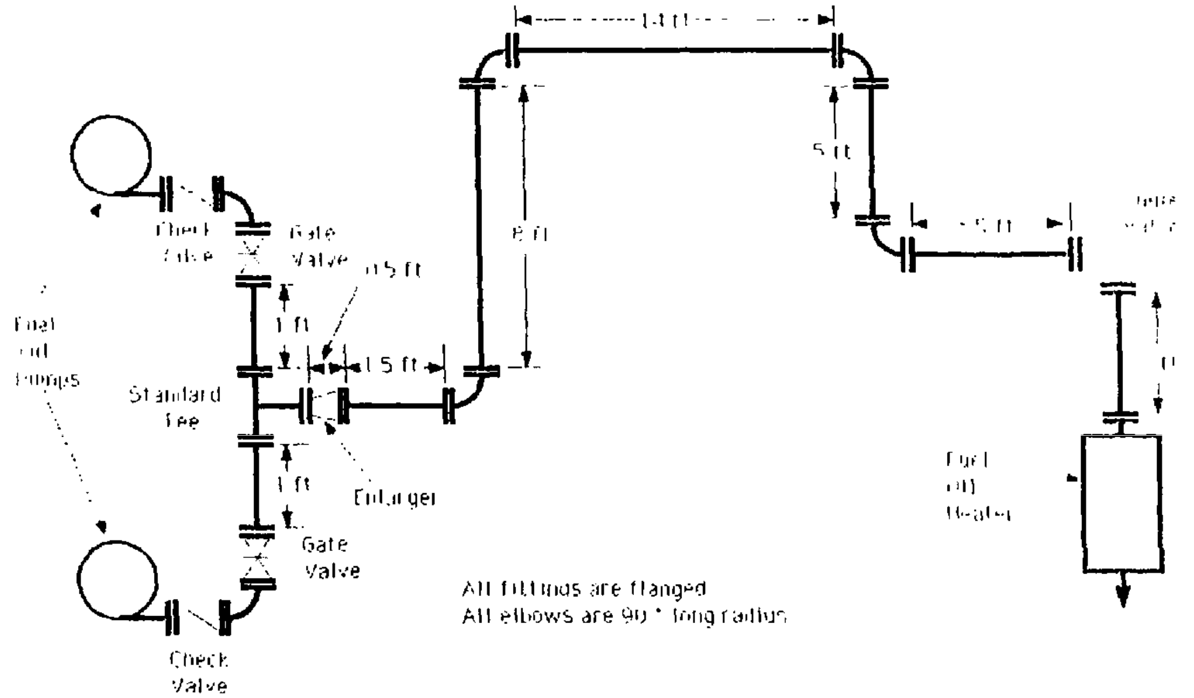


Figure 4.5 Notation for Example 4.3.

$$S_{60/60\,F} = \frac{141.5}{131.5 + \text{API}} \tag{1.20}$$

$$S_{60/60\,F} = \frac{141.5}{131.5 + 11.5}$$

$$S_{60/60\,F} = 0.9895$$

The density of water at 60 °F is 62.37 lbm/ft^3 (Section 1.4). The oil density is therefore calculated as follows:

$$\rho = S_{60/60\,F} \times \rho_{w,60\,F} \tag{1.17}$$

$$\rho = 0.9895 \times 62.37$$

$$\rho = 61.72\,\text{lbm/ft}^3$$

The kinematic viscosity is calculated using equation (1.28)

$$\nu = 0.220(\text{SSU}) - \frac{135}{\text{SSU}} \tag{1.28}$$

$$\nu = 0.220(4000) - \frac{135}{4000}$$

$$\nu = 880\,\text{centistokes}$$

The kinematic viscosity is converted to dynamic viscosity using equation (1.22):

$$\mu = \frac{\rho\nu}{g_c} \tag{1.22}$$

$$\mu = \frac{61.72\,\text{lbm/ft}^2 \left(880\,\text{centistokes} \times \dfrac{10.76 \times 10^{-6}\,\text{ft}^2/\text{s}}{\text{centistoke}}\right)}{32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$\mu = 18.17 \times 10^{-3}\,\text{lbf} \cdot \text{s/ft}^2$$

Part I – From Pumps to Tee:

2. *Unit conversions:*

$$Q_I = 50\,\text{GPM} \times 1.3368 \times 10^{-1}\,\text{ft}^3/\text{gal} \times 1\,\text{min}/60\,\text{s} = 0.1114\,\text{ft}^3/\text{s}$$

$$\dot{m}_I = Q_I \rho = 0.1114\,\text{ft}^3/\text{s} \times 61.72\,\text{lbm/ft}^3 = 6.876\,\text{lbm/s}$$

Step B: Select pipe schedule
From equation (B.3):

$$N_s = 1000p/S_a = 1{,}000 \times (600 - 14.7)\,\text{psi}/15{,}000\,\text{psi} = 39$$

Use schedule 40 (next higher size).

Step C: Estimate internal pipe diameter
Use equation (4.43) for pump discharge lines:

$$D_l = 0.379 Q_l^{0.434} \tag{4.43}$$

$$D_l = 0.379(0.1114)^{0.434}$$

$$D_l = 0.1462\,\text{ft}$$

From Table C.3, the nearest larger pipe size is 2-in, schedule 40 ($D_l = 0.1723\,\text{ft}$, $A_l = 0.02330\,\text{ft}^2$). From the continuity equation, $v_l = Q_l/A_l = 0.1114\,\text{ft}^3/\text{s}/0.02330\,\text{ft}^2 = 4.78\,\text{ft/s}$.

Step D: Estimate energy loss

1. *Loss due to fluid entering the system*: In this system each pump discharge has the same area as the tee inlet, so there is no entrance loss:

$$H_{l_e} = 0$$

2. *Loss due to pipe friction*: The Reynolds number is calculated using the following equation:

$$\text{Re}_l = \frac{\rho D_l v_l}{\mu g_c} \tag{4.10}$$

$$\text{Re}_l = \frac{61.72\,\text{lbm/ft}^3 \times 4.78\,\text{ft/s} \times 0.1732\,\text{ft}}{18.17 \times 10^{-3}\,\text{lbf}\cdot\text{s/ft}^2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}$$

$$\text{Re}_l = 87.42\,(\text{laminar flow})$$

Since Re_l is less than 3,000, the flow is laminar; the friction factor is calculated using equation (4.21):

$$f_l = 64/\text{Re}_l = 64/87.42 = 0.7321$$

The total length of straight pipe is 1 foot:

$$H_{lfp} = \sum_{i}^{e} \left(\frac{fL}{D} \frac{v^2}{2g_c} \right)_{ip} \tag{4.29}$$

$$H_{lfp} = \frac{0.7321 \times 1\,\text{ft}}{0.1732\,\text{ft}} \times \frac{(4.78\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}$$

$$H_{lfp} = 1.50\,\text{ft}\cdot\text{lbf/lbm}$$

3. *Loss due to valves and fittings*:

Valves (from Table 4.5 for 2-in. pipe, flanged)

	K_v (Hydraulic Institute)	K_v (Crane TP410)
1 swing check	2.00	1.90
1 gate	0.33	0.15
ΣK_v	2.33	2.05

The most conservative design would be to use the Hydraulic Institute value of $\Sigma K_v = 2.33$.

Fittings (from Table 4.5 for 2-in. pipe, flanged)

	K_F (Hydraulic Institute)	K_F (Crane TP410)
3 90 long-radius elbows	0.30	0.30

The valve and fitting loss is calculated using equation (4.34):

$$H_{lvF} = \sum_i^v \left(K_v \frac{v^2}{2g_c} \right)_v + \sum_i^v \left(K_F \frac{v^2}{2g_c} \right)_F$$

$$H_{lvF} = (2.33 + 0.30) \frac{(4.78\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{lvF} = 0.93\,\text{ft} \cdot \text{lbf/lbm}$$

4. *Loss due to bends*: There are no bends, so $H_{lb} = 0$.

5. *Loss due to changes in flow areas*: There are no area changes up to the inlet of the tee, so $H_{la} = 0$.

6. *Loss due to fluid leaving the system*: In this system the tee inlet has the same area as the pipe, so there is no exit loss. Hence, $H_{le} = 0$.

The system friction loss for Part I is the sum of the component pressure losses:

$$H_{lfs} = H_{li} + H_{lfp} + H_{lvF} + H_{lb} + H_{la} + H_{le} \tag{4.39}$$

$$H_{lfs} = 0 + 1.50 + 0.93 + 0 + 0 + 0$$

$$H_{lfs} = 2.43\,\text{ft} \cdot \text{lbf/lbm}$$

Part II –From Tee to Heater:

2. *Unit conversions:*

$$Q_{II} = 100\,\text{GPM} \times 1.3368 \times 10^{-1}\,\text{ft}^3/\text{gal} \times 1\,\text{min}/60\,\text{s} = 0.2228\,\text{ft}^3/\text{s}$$
$$\dot{m}_{II} = Q_{II}\rho = 0.2228\,\text{ft}^3/\text{s} \times 61.72\,\text{lbm}/\text{ft}^3 = 13.75\,\text{lbm}/\text{s}$$

Step B: Select pipe schedule
From equation (B.3):

$$N_s = 1000p/S_a = 1{,}000 \times (600 - 14.7)\,\text{psi}/15{,}000\,\text{psi} = 39$$

Use schedule 40 (next higher size).

Step C: Estimate internal pipe diameter
Use equation (4.43) for pump discharge lines:

$$D_{II} = 0.379 Q_{II}^{0.434} \tag{4.43}$$
$$D_{II} = 0.379(0.2228)^{0.434}$$
$$D_{II} = 0.1975\,\text{ft}$$

From Table C.3, the nearest larger pipe size is 2-in. schedule 40 ($D_{II} = 0.2058\,\text{ft}$, $A_{II} = 0.03325\,\text{ft}^2$). From the continuity equation, $v_{II} = Q_{II}/A_{II} = 0.2228\,\text{ft}^3/\text{s}/0.03325\,\text{ft}^2 = 6.7\,\text{ft/s}$. However, the calculated velocity of 6.7 ft/s is over the upper limit of 6 ft/s (see Table 4.8) recommended for fuel oil discharge lines. From Table C.3, the next larger pipe size is 3-in. schedule 40 ($D_{II} = 0.2557\,\text{ft}$, $A_{II} = 0.05134\,\text{ft}^2$). From the continuity equation, $v_{II} = Q_{II}/A_{II} = 0.2228\,\text{ft}^3/\text{s}/\ 0.05134\,\text{ft}^2 = 4.34\,\text{ft/s}$. Therefore, the 3-in. size should be selected.

Step D: Estimate energy loss

1. *Loss due to fluid entering the system:* In this system the tee outlet is the same size as the heater inlet, so there is no entrance loss. Hence $H_{II_i} = 0$.

2. *Loss due to pipe friction:* The Reynolds number is calculated using the following equation:

$$\text{Re}_{II} = \frac{\rho D_{II} v_{II}}{\mu g_c} \tag{4.10}$$
$$\text{Re}_{II} = \frac{61.72\,\text{lbm}/\text{ft}^3 \times 4.34\,\text{ft/s} \times 0.2557\,\text{ft}}{18.17 \times 10^{-3}\,\text{lbf}\cdot\text{s}/\text{ft}^2 \times 32.17\,\text{lbm}\cdot\text{ft}/\text{lbf}\cdot\text{s}^2}$$
$$\text{Re}_{II} = 117.2 \text{ (laminar flow)}$$

Since Re_{II} is less than 3,000, the flow is laminar, and the friction factor is calculated using equation (4.21):

$$f_{II} = \frac{64}{\text{Re}_{II}} = \frac{64}{117.2} = 0.5461$$

The total length of straight pipe is $1.5 + 8 + 14 + 5 + 5 + 5 = 38.5$ ft.

$$H_{IIfp} = \sum_i^e \left(\frac{fL}{D} \frac{v^2}{2g_c} \right)_{fp} \tag{4.29}$$

$$H_{IIfp} = \frac{0.5461 \times 38.5\,\text{ft}}{0.2557\,\text{ft}} \times \frac{(4.34\,\text{ft/s})^2}{2. \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{IIfp} = 24.07\,\text{ft} \cdot \text{lbf/lbm}$$

3. *Loss due to valves and fittings*:

Valves (from Table 4.5 for flanged pipe)

	K_v (Hydraulic Institute)	K_v (Crane TP410)
1 angle	2.20	2.60

The most conservative design would be to use the Crane value of $K_v = 2.60$.
The loss due to valves is calculated as follows:

$$H_{IIv} = \sum_i^e \left(K_v \frac{v_{II}^2}{2g_c} \right)_v$$

$$H_{IIv} = (2.60) \frac{(4.34\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{IIv} = 0.76\,\text{ft} \cdot \text{lbf/lbm}$$

Fittings (from Table 4.5)

	K_f (Hydraulic Institute)	K_f (Crane TP410)
4 3-in. 90 long-radius elbows	$4 \times 0.25 = 1.00$	$4 \times 0.28 = 1.12$
1 2-in. tee branch flow	0.84	1.14

The branch outlet of the tee has the flow of both pumps; therefore its velocity is $2V_I$. Using the most conservative values, the fitting loss is calculated as follows:

$$H_{IIF} = K_{\text{elbow}} \frac{v_{II}^2}{2g_c} + K_{\text{tee}} \frac{(2v_I)^2}{2g_c}$$

$$H_{IIF} = 1.12 \frac{(4.34\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2} + 1.14 \frac{(2 \times 4.78)^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{IIF} = 1.95\,\text{ft} \cdot \text{lbf/lbm}$$

4. *Loss due to bends*: There are no bends, therefore $H_{IIb} = 0$.

5. *Loss due to changes in flow areas*: The enlarger gradually increases the pipe diameter from 0.1723 ft to 0.2557 ft in a distance of 0.5 ft. From Table 4.4, K_a is calculated as follows for a gradual enlargement:

$$\frac{D - d}{2L} = \frac{0.2557 - 0.1723}{2 \times 0.5} = 0.083$$

Therefore, $K' = 0.18$.

$$K_a = K'\left[1 - \left(\frac{d}{D}\right)^2\right]^2$$

$$K_a = 0.18\left[1 - \left(\frac{0.1723}{0.2557}\right)^2\right]^2$$

$$K_a = 0.05$$

The loss due to the area change is therefore calculated as follows:

$$H_{IIa} = \sum_i^e \left(K_a \frac{(2v_I)^2}{2g_c}\right)_a$$

$$H_{IIa} = 0.05 \frac{(2 \times 4.78\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{IIa} = 0.07\,\text{ft} \cdot \text{lbf/lbm}$$

6. *Loss due to the fluid leaving the system*: In this system the enlarger outlet has the same area as the heater inlet, so there is no exit loss. Hence $H_{IIe} = 0$.

The system friction loss is:

$$H_{IIs} = H_{IIi} + H_{IIfp} + H_{IIeF} + H_{IIb} + H_{IIa} + H_{IIe} \tag{4.39}$$

$$H_{IIs} = 0 + 24.07 + 2.71 + 0 + 0.07 + 0$$

$$H_{IIs} = 26.85\,\text{ft} \cdot \text{lbf/lbm}$$

Part III– Total System

$$H_{fs} + H_{Ifs} + H_{IIfs} = 2.43 + 26.85 = 29.28\,\text{ft}\cdot\text{lbf/lbm}$$

The total system pressure drop is therefore calculated as follows:

$$\Delta p_{\text{system}} = \rho\left[\frac{g}{g_c}(z_e - z_i) + \frac{v_e^2 - v_i^2}{2g_c} + H_{fs}\right] \tag{4.40}$$

$$\Delta p_{\text{system}} = 62.37\,\text{lbm/ft}^3\left[0 + \frac{(4.78\,\text{ft/s})^2 - (4.34\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2} + 29.28\,\text{ft}\cdot\text{lbf/lbm}\right]$$

$$\Delta p_{\text{system}} = 1{,}830\,\text{lbf/ft}^2 \times 1\,\text{ft}^2/144\,\text{in.}^2$$

$$\Delta p_{\text{system}} = 12.17\,\text{psi}$$

Example 4.4: *Power Required to Pump Water.* It is required to pump water at 100°F from Tank 1 to Tank 2 at the rate of 825 GPM with the piping system shown in Figure 4.6. All connecting piping is 6-in. schedule 40 steel pipe. All valves and fittings are 6-in. size with welded connections. The total length of straight pipe is 590 ft. Estimate the power that the pump must add to the fluid.

Solution

This example is solved by calculating friction losses as in Section 4.8 (the required pump power must overcome this friction).

Step A: Data reduction

1. *Fluid properties* (Table D.1 at 14.696 psia and 100°F):

$$\rho = 62.00\,\text{lbm/ft}^3, \qquad \mu = 1.432 \times 10^{-5}\,\text{lbf}\cdot\text{s/ft}^2$$

2. *Unit conversions*:

$$Q = 825\,\text{GPM} \times 1.3368 \times 10^{-1}\,\text{ft}^3/\text{gal} \times 1\,\text{min}/60\,\text{s} = 1.838\,\text{ft}^3/\text{s}$$

$$\dot{m} = Q\rho = 1.838\,\text{ft}^3/\text{s} \times 61.00\,\text{lbm/ft}^3 = 114\,\text{lbm/s}$$

Step B: Pipe schedule
The pipe schedule is given as schedule 40 in the problem statement.

Step C: Internal pipe diameter
The pipe size is given as 6-in. schedule 40 in the problem statement.

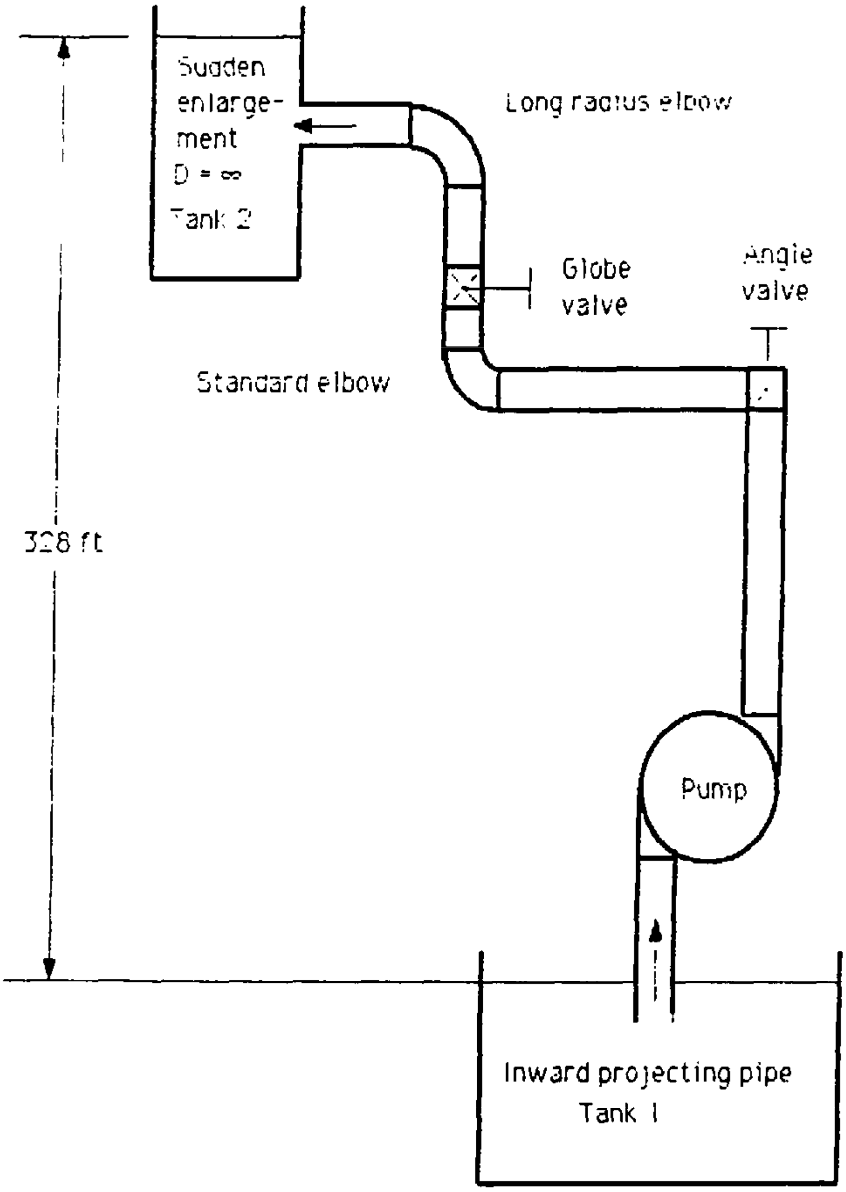


Figure 4.6 Notation for Example 4.4.

From Table C.3:

$$D = 0.5054\,\text{ft}, \qquad A = 0.2006\,\text{ft}^2, \qquad \frac{\varepsilon}{d} = 2.968 \times 10^{-4}$$

From the continuity equation, $v = Q/A = 1.838\,\text{ft}^3/\text{s}/0.2006\,\text{ft}^2 = 9.16\,\text{ft/s}$.

Step D: Compute pumping power
To calculate system loss, use the equations and methods of Section 4.8.

1. *Loss due to fluid entering the system*: From Table 4.4 for a projecting pipe inlet, $K_i = 1$. The inlet loss is calculated using equation (4.28):

$$H_i = K_i \frac{v_i^2}{2g_c} \tag{4.28}$$

$$H_i = 1 \frac{(9.16\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_i = 1.30\,\text{ft} \cdot \text{lbf/lbm}$$

2. *Loss due to pipe friction*: The Reynolds number is calculated using the following equation:

$$\text{Re} = \frac{\rho D v}{\mu g_c} \tag{4.10}$$

$$\text{Re} = \frac{62.00\,\text{lbm/ft}^3 \times 9.16\,\text{ft/s} \times 0.5054\,\text{ft}}{1.432 \times 10^{-5}\,\text{lbf} \cdot \text{s/ft}^2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$\text{Re} = 623{,}058\,(\text{turbulent flow})$$

The friction factor is then calculated using equation (4.25):

$$f = \left[1.14 - 2\log_{10}\left(\frac{\varepsilon}{D} + \frac{21.25}{\text{Re}^{0.9}}\right)\right]^{-2} \tag{4.25}$$

$$f = \left[1.14 - 2\log_{10}\left(2.968 \times 10^{-4} + \frac{21.25}{(623058)^{0.9}}\right)\right]^{-2}$$

$$f = 0.0161$$

The total length of straight pipe is 590 ft.

$$H_{fp} = \sum_i^e \left(\frac{fL}{D} \frac{v^2}{2g_c}\right)_{fp} \tag{4.29}$$

$$H_{fp} = \frac{0.0161 \times 590\,\text{ft}}{0.5054\,\text{ft}} \times \frac{(9.16\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{fp} = 24.51\,\text{ft} \cdot \text{lbf/lbm}$$

3. *Loss due to valves and fittings*:

Valves (from Table 4.5 for 6-in. schedule 40 pipe)

	K_v (Hydraulic Institute)	K_v (Crane TP410)
1 angle valve	2.10	2.24
1 globe valve	6.00	5.07
ΣK_i	8.10	7.31

The most conservative design would be to use the Hydraulic Institute value of $\Sigma K_e = 8.10$.

Fittings (from Table 4.5 for 6-in. schedule 40 pipe)

	K_F (Hydraulic Institute)	K_F (Crane TP410)
1 90 standard elbow	0.27	0.45
1 90 long-radius elbow	0.18	0.24
ΣK_F	0.45	0.69

Again we take the most conservative $\Sigma K_F = 0.69$ (from Crane TP410).

The valve and fitting loss is calculated using equation (4.34):

$$H_{vF} = \sum_i^e \left(K_v \frac{v^2}{2g_c} \right)_v + \sum_i^e \left(K_f \frac{v^2}{2g_c} \right)_F$$

$$H_{vF} = (8.10 + 0.69) \frac{(9.16\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_{vF} = 11.46\,\text{ft} \cdot \text{lbf/lbm}$$

4. *Loss due to bends*: There are no bends; $H_b = 0$.
5. *Loss due to changes in flow areas*: There are no area changes; $H_a = 0$.
6. *Loss due to fluid leaving the system*: From Table 4.4, $K_e = 1.0$. The loss due to the exit is therefore calculated from equation (4.38):

$$H_e = \frac{v_e^2}{2g_c} \tag{4.38}$$

$$H_e = \frac{(9.16\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$H_e = 1.30\,\text{ft} \cdot \text{lbf/lbm}$$

The system friction loss is:

$$H_{fs} = H_i + H_{fp} + H_{vF} + H_b + H_a + H_e \tag{4.39}$$

$$H_{fs} = 1.30 + 24.51 + 11.46 + 0 + 0 + 1.30$$

$$H_{fs} = 38.57\,\text{ft} \cdot \text{lbf/lbm}$$

The pump must supply the energy required to balance equation (3.22):

$$W_p = \frac{g}{g_c}(z_e - z_i) + \frac{v_e^2 - v_i^2}{2g_c} + \frac{(p_2 - p_1)}{\rho} + H_{fs}$$

If the water surface in Tank 1 is taken as the inlet, then $V_i = 0$. In like manner, if the surface in Tank 2 is taken as the exit, then $V_e = 0$. Using this logic, $P_e = P_i$ = atmospheric pressure. Substitution into the preceding equation yields the following:

$$W_p = \frac{32.17\,\text{ft/s}^2}{32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}(328\,\text{ft} - 0) + 0 + 0 + 38.57\,\text{ft}\cdot\text{lbf/lbm}$$

$$W_p = 366.57\,\text{ft}\cdot\text{lbf/lbm}$$

The pump power is the calculated energy multiplied by the mass flow rate:

$$P_p = W_p\dot{m}$$

$$P_p = 366.57\,\text{ft}\cdot\text{lbf/lbm} \times 114\,\text{lbm/s} \times \frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}$$

$$P_p = 76\,\text{hp}$$

4.12 COMPRESSIBLE FLOW IN PIPES

It is recommended that Section 3.4, "Gas Dynamics," be reviewed before proceeding with this section.

In most engineering applications, the cost of large energy losses limits the pressure drops in piping systems to less than one-tenth of the inlet pressure. For all practical purposes, the use of the incompressible equations for compressible fluids is satisfactory, provided that $\Delta p/p_1 < 0.10$.

Some real situations where incompressible flow equations will not give satisfactory results are:

Flow of gases in insulated ducts where $\Delta p/p_1 > 0.1$. The Fanno line function (Table 3.4) should be used in such applications (see Example 4.5).

Flow of steam at high velocities. In applications such as the flow through safety valve vents, start-up lines to a condenser, and flow through temporary blowout lines to the atmosphere, satisfactory results may be obtained by assuming that steam follows a Fanno line function as an ideal gas with an isentropic exponent of 1.3 (see Example 4.6).

Long natural gas pipe lines. In this application the flow may be assumed to be isothermal and the equations of Section 3.4 may be used (See Example 4.7). If equation (3.184) is solved for mass flow rate, the following results:

$$\dot{m} = A\sqrt{\frac{g_c(p_1^2 - p_2^2)}{RT\left[2\log_e\left(\frac{p_1}{p_2}\right) + \frac{fL}{D}\right]}} \tag{4.51}$$

Example 4.5: *Air at high velocity.* Air at 122°F and 200 psia enters a 10-in. schedule 40 pipe with a mass flow rate of 757,000 lbm/hr. The insulated horizontal pipe is 117 ft, 3 in. long. Determine (a) pressure loss for incompressible flow *and* (b) T_2, p_2, and v_2 for Fanno flow.

Solution

In this example only the pressure loss in straight pipe is involved, so equation (4.22) may be used for incompressible flow and Table 3.4 for Fanno flow.

(a) Data acquisition

1. *Fluid properties*:

Table D.2 Air, $M = 28.97\,\text{lbm/lbmol}$.

$$R = \frac{R_u}{M} = 1545\,\text{ft}\cdot\text{lbf/(lbmol)(R)}/28.97\,\text{lbm/lbmole}$$
$$= 53.33\,\text{ft}\cdot\text{lbf/(lbm}\cdot\text{R)}$$

Table D.2 Air at 122 F, $\mu = 4.10 \times 10^{-7}\,\text{lbf}\cdot\text{s/ft}^2$, $k = 1.401 \approx 1.4$.

2. *Pipe properties (Table C.3, 10-in. Schedule 40)*:

$$D = 0.8350\,\text{ft}, \qquad A = 0.5476\,\text{ft}^2$$

For complete turbulence

$$f_\infty = 0.01343; \qquad \text{Re for } f_\infty = 17{,}810{,}000$$

3. *Unit conversions*:

$$\dot{m} = 757{,}000\ \text{lbm/hr}/3600\,\text{s/hr} = 210.28\,\text{lbm/s}$$
$$L = 117 + \tfrac{3}{12} = 117.25\,\text{ft}, \qquad T_i = 122 + 460 = 582\ \text{R}$$

4. *Friction factor*: The Reynolds number is calculated using the following equation:

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu g_c} \tag{4.12}$$

$$\text{Re} = \frac{4 \times 210.28\,\text{lbm/s}}{\pi \times 0.8350\,\text{ft} \times 4.10 \times 10^{-7}\,\text{lbf}\cdot\text{s/ft}^2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}$$

$$\text{Re} = 24{,}310{,}000$$

Since the Reynolds number is greater than Re for f_∞, the friction factor is independent of Reynolds number and $f = f_\infty = 0.01343$.

5. *Inlet velocity*: The inlet density is calculated using equation (1.39):

$$\rho_1 = \frac{p_1}{RT_1} \tag{1.39}$$

$$\rho_1 = \frac{200\,\text{lbf/in.}^2 \times 144\,\text{in.}^2/\text{ft}^2}{533.33\,\text{ft} \cdot \text{lbf/(lbm} \cdot \text{R)} \times 582 \cdot \text{R}}$$

$$\rho_1 = 0.9279\,\text{lbm/ft}^3$$

The inlet velocity is calculated from the continuity equation as follows:

$$v_1 = \frac{\dot{m}}{\rho_1 A}$$

$$v_1 = \frac{210.28\,\text{lbm/s}}{0.9279\,\text{lbm/ft}^3 \times 0.5476\,\text{ft}^2}$$

$$v_1 = 413.84\,\text{ft/s}$$

Part (a)— incompressible pressure loss:

$$\Delta p = \left(\frac{fL}{D}\right)\frac{\rho_1 v_1^2}{2g_c} \tag{4.22}$$

$$\Delta p = \left(\frac{0.01343 \times 117.25\,\text{ft}}{0.8350\,\text{ft}}\right)\frac{0.9279\,\text{lbm/ft}^3 \times (413.83\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$\Delta p = 4657.65\,\text{lbf/ft}^2 \times 1\,\text{ft}^2/144\,\text{in.}^2$$

$$\Delta p = 32.34\,\text{psi}$$

$$\frac{\Delta p}{p_i} = \frac{32.24}{200} = 0.1617$$

Part (b)—Fanno flow:

1. *Inlet Mach number*: The inlet acoustic velocity is calculated using equation (1.59):

$$c_1 = \sqrt{kg_c RT_1} \tag{1.59}$$

$$c_1 = \sqrt{1.4 \times 32.17\,(\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2 \times 53.33\,\text{ft} \cdot \text{lbf/(lbm} \cdot \text{R)} \times 582\,\text{R}}$$

The inlet Mach number is calculated from equation (3.94):

$$M_1 = \frac{v_1}{c_1} \tag{3.94}$$

$$M_1 = \frac{413.84\,\text{ft/s}}{1182.32\,\text{ft/s}}$$

$$M_1 = 0.35$$

2. *From Table 3.4 (Fanno Line Functions)* at $k = 1.4$ at $M_1 = 0.35$: $T_1/T^* = 1.171, p_1/p^* = 3.092, v_1/v^* = 0.3788, fL_1^*/D = 3.452$. L_1^* is calculated as follows:

$$L_1^* = \left(\frac{fL_1^*}{D}\right)\frac{D}{f}$$

$$L_1^* = 3.452 \times \frac{0.8350\,\text{ft}}{0.01343}$$

$$L_1^* = 214.63\,\text{ft}$$

L_2^* is calculated as follows:

$$L_2^* = L_1^* - L = 214.63 - 117.25 = 97.38\,\text{ft}$$

Therefore, fL_2^*/D is calculated as follows:

$$\frac{fL}{D} = \frac{0.01343 \times 97.38\,\text{ft}}{0.8350\,\text{ft}}$$

$$\frac{fL_2^*}{D} = 1.566$$

3. *From Table 3.4 (Fanno Line Functions)* at $k = 1.4$ and $fL_2^* = 1.566$: $M_2 = 0.45$, $T_2/T^* = 1.153, p_2/p^* = 2.386, v_2/v^* = 0.4833$.

$$T_2 = T_1\left(\frac{T_2/T^*}{T_1/T^*}\right)$$

$$T_2 = 582\,\text{R}\left(\frac{1.153}{1.171}\right)$$

$$T_2 = 573\,\text{R} - 460$$

$$T_2 = 113\,\text{F}$$

$$p_2 = p_1\left(\frac{p_2/p^*}{p_1/p^*}\right)$$

$$p_2 = 200\,\text{psia}\left(\frac{2.386}{3.092}\right)$$

$$p_2 = 154.33\,\text{psia}$$

$$\Delta p = p_1 - p_2 = 200 - 154.33 = 45.67\,\text{psi}$$

$$\frac{\Delta p}{p_1} = \frac{45.67}{200} = 0.228$$

$$v_2 = v_1\left(\frac{v_2/v^*}{v_1/v^*}\right)$$

$$v_2 = 413.84\,\text{ft/s}\left(\frac{0.4833}{0.3788}\right)$$

$$v_2 = 528\,\text{ft/s}$$

Example 4.6: *Main Steam Blowout Line.* A main steam blowout line is 8-in. schedule 40 commercial steel pipe and has an equivalent length of 272.25 ft. Estimate the mass flow rate through the line and the exit conditions if the line is blown at a constant boiler pressure of 1500 psia (assume the steam is a saturated vapor).

Solution

This solution assumes that steam follows a Fanno line function as an ideal gas with an isentropic exponent of 1.3.

(a) Data acquisition

(1) *Fluid properties:*

Table D.1 –Water $M = 18.015\,\text{lbm/lb-mole}$:

$$R = \frac{R_u}{M} = 1545\,\text{ft}\cdot\text{lbf/(lb mol)(R)}/18.015\,\text{lbm/lb-mole}$$
$$= 85.76\,\text{ft}\cdot\text{lbf/(lbm}\cdot\text{ R)}$$

Table D.3 -Saturated water (vapor) at 1500 psia:

$T_{01} = T_{sat} = 596.90\,\text{F} = 596.90 + 460 = 1057\,\text{R}$ (*Note*: stagnation conditions exist in the steam drum.)

$$\mu_0 = 4.284 \times 10^{-7}\,\text{lbf}\cdot\text{s/ft}^2$$

(2) *Pipe properties* (Table C.3 for 8-in. schedule 40 pipe):

$$D = 0.6651\,\text{ft}, \qquad A = 0.3474\,\text{ft}^2$$

For complete turbulence:

$$f_\infty = 0.01407; \qquad \text{Re for } f_\infty = 14{,}190{,}000$$

(b) Inlet Mach number
Assume complete turbulence, and then check the assumption:

$$\left(\frac{fL_1^*}{D}\right) = \frac{0.01407 \times 272.25\,\text{ft}}{0.6651\,\text{ft}}$$

$$\left(\frac{fL_1^*}{D}\right) = 5.759$$

From Table 3.4 (Fanno Line Functions) at $k = 1.3$ and $(fL_1^*/D = 5.759$: $M_1 = 0.30$, $T_1/T^* = 1.135$, $p_1/p^* = 3.551$, $v_1/v^* = \rho^*/\rho_1 = 0.3196$.

From Table 3.1 (Isentropic Flow Functions) at $k = 1.3$ and $M_1 = 0.30$: $T_1/T_{01} = 0.9867$, $p_1/p_{01} = 0.9435$.

(c) Pressure and temperature
Stagnation conditions exist in the steam drum. Therefore, pressure and temperature must be converted to static conditions at the blowout pipe inlet as follows:

$$p_1 = p_{01}\left(\frac{p_1}{p_{01}}\right) = 1500\,\text{psia} \times 0.9435 = 1415\,\text{psia}$$

$$T_i = T_{01}\left(\frac{T_1}{T_{01}}\right) = 1057\,\text{R} \times 0.9867 = 1043\,\text{R}$$

$$\rho_1 = \frac{p_1}{RT_1} \tag{1.39}$$

$$\rho_1 = \frac{1415\,\text{lbf/in.}^2 \times 144\,\text{in.}^2/\text{ft}^2}{85.76\,\text{ft}\cdot\text{lbf/(lbm}\cdot\text{R)} \times 1043\,\text{R}}$$

$$\rho_1 = 2.278\,\text{lbm/ft}^3$$

(d) Inlet velocity
The inlet velocity is calculated using equation (3.94):

$$v_1 = M_1\sqrt{kg_cRT_1} \tag{3.94}$$

$$v_1 = 0.30\sqrt{1.3 \times 31.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2 \times 85.76\,\text{ft}\cdot\text{lbf/(lbm}\cdot\text{R)} \times 1043\,\text{R}}$$

$$v_1 = 580\,\text{ft/s}$$

(e) Exit conditions
Exit condition calculations are made by noting that sonic velocity is the maximum velocity that can exist in the pipe:

$$p_2 = p^* = \frac{p_1}{p_1/p^*} = 1415\,\text{psia}/3.551 = 398\,\text{psia}$$

$$T_2 = T^* = \frac{T_1}{T_1/T^*} = 1043\,\text{R}/1.135 = 919\,\text{R} - 460 = 459\,\text{F}$$

$$v_2 = v^* = \frac{v_1}{v_1/v^*} = 580\,\text{ft/s}/0.3196 = 1815\,\text{ft/s}$$

$$\rho_2 = \rho^* = \frac{\rho_1}{\rho_1/\rho^*} = 2.278\,\text{lbm/ft}^3 \times 0.3196 = 0.7280\,\text{lbm/ft}^3$$

(f) Mass flow rate
The mass flow rate is calculated using the continuity equation:

$$\dot{m} = \rho_1 A v_1 = 2.278\,\text{lbm/ft}^3 \times 0.3474\,\text{ft}^2 \times 580\,\text{ft/s} = 458\,\text{lbm/s}$$

(g) Assumption of complete turbulence
Check the assumption of complete turbulence:

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu g_c} \tag{4.12}$$

$$\text{Re} = \frac{4 \times 458\,\text{lbm/s}}{\pi \times 0.6651\,\text{ft} \times 4.284 \times 10^{-7}\,\text{lbf}\cdot\text{s/ft}^2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf/lbf}\cdot\text{s}^2}$$

$$\text{Re} = 63{,}600{,}000$$

Because the calculated Reynolds number is greater than the Reynolds number for f_∞ (14,190,000), the assumption of complete turbulence is valid.

Note: An energy analysis using actual steam properties resulted in the occurrence of choking flow at a pressure of 420 psia with a quality of 88.32%. At this condition the mass flow rate in the pipe was 670 lbm/s. Thus, if condensation had taken place, there would have been a margin of safety of 100(670/578) = 116%.

Example 4.7: *Capacity of a Long Natural Gas Pipeline.* Determine the carrying capacity in cubic feet of natural gas per hour at standard conditions (14.7 psia, 60 F) of a 60-mile-long, 8-in. schedule 30 pipe if the initial pressure is 315 psia, final pressure is 45 psia, flowing temperature is 60 F, specific gravity is 0.7, and viscosity is $0.225 \times 10^{-6}\,\text{ft}\cdot\text{lbf}\cdot\text{s/ft}^2$.

Solution

Approach: This example is solved assuming isothermal flow and applying equation (4.51).

(a) Data acquisition

(1) *Fluid properties*: From Table D.2 for air, $M_a = 28.97\,\text{lbm/lb-mole}$:

$$M_{ng} = SM_a = 0.7 \times 28.97\,\text{lbm/lb-mole} = 20.28\,\text{lbm/lb-mole}$$

$$R = \frac{R_u}{M_{ng}} = \frac{1545\,\text{ft} \cdot \text{lbf/(lb} \cdot \text{mol)(R)}}{20.28\,\text{lbm/lb-mole} = 76.18\,\text{ft} \cdot \text{lbf/lbm R}}$$

$$T = 60 + 460 = 520\,\text{R}$$

Standard density:

$$\rho_s = \frac{p_s}{RT_s} \tag{1.39}$$

$$\rho_s = \frac{14.7\,\text{lbf/in.}^2 \times 144\,\text{in.}^2/\text{ft}^2}{76.18\,\text{ft} \cdot \text{lbf/(lbm} \cdot \text{ R)} \times 520\,\text{R}}$$

$$\rho_s = 0.0534\,\text{lbm/ft}^3$$

(2) *Pipe properties*: From Table C.3 for 8-in. schedule 30 pipe:

$$D = 0.6726\,\text{ft}, \qquad A = 0.3553\,\text{ft}^2$$

$$f_\infty = 0.01404, \qquad \text{Re}_\infty = 14{,}350{,}000$$

$$\frac{\varepsilon}{D} = 2.23 \times 10^{-4}$$

First Trial—Assume complete turbulence and then check:

$$\dot{m} = A\sqrt{\frac{g_c(p_1^2 - p_2^2)}{RT\left[2\log_e\left(\frac{p_1}{p_2}\right) + \frac{fL}{D}\right]}} \tag{4.51}$$

$$\dot{m} = 0.3553\,\text{ft}^2$$

$$\sqrt{\frac{32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2[(315\,\text{lbf/in.}^2 \times 144\,\text{in.}^2/\text{ft}^2)^2 - (45\,\text{lbf/in.}^2 \times 144\,\text{in.}^2/\text{ft}^2)^2]}{(76.18\,\text{ft} \cdot \text{lbf/lbm} \cdot \text{ R})(520\,\text{R})\left[2\log_e\left(\frac{315\,\text{psia}}{45\,\text{psia}}\right) + \frac{(0.01404)(60\,\text{mi})(5280\,\text{ft/mi})}{0.6726\,\text{ft}}\right]}}$$

The first trial Reynolds number is calculated using equation (4.12):

$$\text{Re}_1 = \frac{4\dot{m}}{\pi D \mu g_c} \tag{4.12}$$

$$\text{Re}_1 = \frac{4 \times 5.588\,\text{lbm/s}}{\pi \times 0.6726\,\text{ft} \times 0.225 \times 10^{-6}\,\text{lbf} \cdot \text{s/ft}^2 \times 32.17\,\text{lbm} \cdot \text{ft/lbf} \cdot \text{s}^2}$$

$$\text{Re}_1 = 1{,}461{,}000$$

The first trial friction factor is calculated using equation (4.25).

$$f_1 = \left[1.14 - 2\log_{10}\left(\frac{\varepsilon}{D} + \frac{21.25}{\text{Re}^{0.9}}\right)\right]^{-2} \tag{4.25}$$

$$f_1 = \left[1.14 - 2\log_{10}\left(2.23 \times 10^{-4} + \frac{21.25}{1{,}461{,}000^{0.9}}\right)\right]^{-2}$$

$$f_1 = 0.01474$$

Second trial—The second trial, using the first trial friction factor substituted in equation (4.51), resulted in $\text{Re}_2 = 1{,}426{,}000$. Convergence to Re_∞ was achieved on the third trial with $\dot{m} = 5.4506\,\text{lbm/s}$. The volumetric flow rate at standard conditions is calculated from the continuity equation, using the standard density previously determined:

$$Q_s = \frac{\dot{m}}{\rho_s} = \frac{5.4506\,\text{lbm/s}}{0.0534\,\text{ft}^3/\text{lbm}} = 102\,\text{ft}^2/\text{s}$$

$$= 102\,\text{ft}^3/\text{s} \times 3600\,\text{s/hr} = 367{,}200\,\text{ft}^3/\text{hr}$$

4.13 NONCIRCULAR PIPES

The Colebrook equation may be used for noncircular pipes. To solve such problems, an equivalent (circular) diameter is obtained from the hydraulic radius and substituted for diameter in Reynolds number and friction factor equations. The hydraulic radius (which is also used to compute flow losses in conduits partially filled with liquids) is defined as follows:

$$\text{hydraulic radius} = \frac{\text{fluid flow area}}{\text{wetted perimeter}} = R_h = \frac{A}{P} \tag{4.52}$$

The wetted perimeter is the length of wall in contact with the flowing fluid at a given cross section. The hydraulic radius is related to the diameter of a circular pipe as follows:

$$R_h = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

Therefore, the equivalent diameter is expressed as follows:

$$D_e = 4R_h \tag{4.53}$$

Example 4.8 illustrates the calculation of friction loss in noncircular pipes and ducts.

Example 4.8: *Loss in a Rectangular Duct.* Air at 122 F and 14.7 psia flows in a rectangular duct 1 ft × 3 ft at a rate of 150 ft^3/s. The duct is horizontal and 1000 ft long and is made of galvanized iron. Estimate the pressure loss due to friction in this duct.

Solution

This problem is solved by substituting the equivalent diameter, as described earlier, and then computing pressure loss as if the duct were a circular pipe.

(a) Data acquisition

(1) *Fluid properties*: From Table D.2 for air at 122 F and atmospheric pressure:

$$\rho = 0.0684\,\text{lbm/ft}^3, \qquad \mu = 0.410 \times 10^{-6}\,\text{lbf}\cdot\text{s/ft}^2$$

(2) *Duct properties*: From Table 4.3 for galvanized iron, $\varepsilon = 500 \times 10^{-6}$ ft.

(3) *Equivalent diameter*: $A = 1\,\text{ft} \times 3\,\text{ft} = 3\,\text{ft}^2$

$$R_h = \frac{A}{P} = \frac{3\,\text{ft}^2}{(3+3+1+1)\,\text{ft}} = \frac{3}{8}\,\text{ft}$$

The equivalent diameter is:

$$D_e = 4R_h = 4 \times \frac{3}{8}\,\text{ft} = 1.5\,\text{ft}$$

(b) Velocity and mass flow rate
The velocity and the mass flow rate are computed from the continuity equation:

$$v = \frac{Q}{A} = \frac{150\,\text{ft}^3/\text{s}}{3\,\text{ft}^2} = 50\,\text{ft/s}$$

$$\dot{m} = \rho A v = \rho Q = 0.0684\,\text{lbm/ft}^3 \times 150\,\text{ft}^3/\text{s} = 10.26\,\text{lbm/s}$$

(c) Reynolds number
The Reynolds number is calculated with the help of equation (4.12):

$$\text{Re} = \frac{4\dot{m}}{\pi D_e \mu g_c} \tag{4.12}$$

$$\text{Re} = \frac{4 \times 10.26\,\text{lbm/s}}{\pi \times 1.5\,\text{ft} \times 0.410 \times 10^{-6}\,\text{lbf}\cdot\text{s/ft}^2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}$$

$$\text{Re} = 660{,}300$$

(d) Friction factor

The friction factor is calculated with equation (4.25):

$$f = \left[1.14 - 2\log_{10}\left(\frac{\varepsilon}{D_e} + \frac{21.25}{\text{Re}^{0.9}}\right)\right]^{-2} \tag{4.25}$$

$$f = \left[1.14 - 2\log_{10}\left(\frac{500 \times 10^{-6}\,\text{ft}}{1.5\,\text{ft}} + \frac{21.25}{660{,}300^{0.9}}\right)\right]^{-2}$$

$$f = 0.01635$$

(e) Pressure loss

The pressure loss is computed from equation (4.22), assuming incompressible flow:

$$\Delta p = \left(\frac{fL}{D_e}\right)\frac{\rho v^2}{2g_c} \tag{4.22}$$

$$\Delta p = \left(\frac{0.01635 \times 1000\,\text{ft}}{1.5\,\text{ft}}\right)\frac{0.0684\,\text{lbm/ft}^3 \times (50\,\text{ft/s})^2}{2 \times 32.17\,\text{lbm}\cdot\text{ft/lbf}\cdot\text{s}^2}$$

$$\Delta p = 28.97\,\text{lbf/ft}^2 \times \frac{1\,\text{ft}^2}{144\,\text{in.}^2} = 0.20\,\text{psi}$$

Verifying the assumption of incompressible flow:

$$\frac{\Delta p}{p_1} = \frac{0.20}{14.7} = 0.014 < 0.1 \qquad \text{(the assumption is correct)}$$

4.14 HAZEN–WILLIAMS EQUATION

For fifty years the Colebrook equation and the Moody chart have been universally accepted as a rational basis for calculating the flow of fluids in pipes. The empirical equation described in this section has not appeared in college fluid mechanics texts for over a quarter of a century. However, many practicing engineers engaged in design still use this equation to calculate the flow of water.

Hazen and Williams (1920) published the results of a study of all available experimental data on the flow of water in pipes. The pipes ranged in diameter from 1 in. to 180 in., and the materials included tin, lead, brass, wrought iron, cast iron, riveted steel, wood, cement, brick, and glass. They plotted the results on a log-log graph paper and proposed the following equation:

$$v = C_{HW} R_h^{0.63} S_Z^{0.54} 0.001^{-0.04} = 1.318 C_{HW} R_h^{0.63} S_Z^{0.54} \tag{4.54}$$

In equation (4.54), v is the velocity in ft/s, C_{HW} is the Hazen Williams coefficient, R_h is the hydraulic radius in ft, and S_Z is the hydraulic slope in ft/ft.

Commonly used values of C_{HW} are given in Table 4.9. Because this equation is still widely used to calculate flow for water supply systems, a lower limit is provided in Table 4.9 to account for the aging process.

The hydraulic slope S_Z is defined as follows:

$$S_Z = \frac{g_c H_f}{gL} = \frac{32.17 H_f}{32.17 L} = \frac{H_f}{L} \quad \text{(standard gravity)} \tag{4.55}$$

Table 4.9 Hazen Williams coefficients (C_{HW})

Type of pipe or tubing	Range		Average new pipe	Long-life design values
	High (best)	Low (poor)		
Brass	150	120	140	130
Cast iron	150	80	130	100
Cement	160	140	150	140
Cement lined steel			150	140
Concrete	152	85	120	100
Copper	150	120	140	130
Corrugated steel			60	60
Glass	150	120	140	130
Lead	150	120	140	130
Spiral-riveted steel			108	95
Tar-coated cast iron	145	80	130	100
Tin	150	120	140	130
Vitrified			110	100
Welded and seamless steel	150	80	140	100
Wood-stave	145	110	120	110
Wrought iron	150	80	130	100

For circular pipes, $R_h = D/4$, from equation (4.53). Substituting equations (4.53) and (4.55) in equation (4.54) results in the following expression:

$$v = 1.318C_{HW}\left(\frac{D}{4}\right)^{0.63}\left(\frac{H_f}{L}\right)^{0.54} = \frac{0.5503C_{HW}D^{0.63}H_f^{0.54}}{L^{0.54}} \tag{4.56}$$

The volumetric flow rate may be obtained by substituting v from equation (4.56) into the continuity equation ($Q = Av$):

$$Q = Av = \left(\frac{\pi D^2}{4}\right)\frac{0.5503C_{HW}D^{0.63}H_f^{0.54}}{L^{0.54}} = \frac{0.4322C_{HW}D^{2.63}H_f^{0.54}}{L^{0.54}} \tag{4.57}$$

Solving equation (4.57) for H_f results in the following:

$$H_f = \frac{4.782LQ^{1.852}}{D^{4.870}C_{HW}^{1.852}} \tag{4.58}$$

The relationship of the friction factor f to the Hazen-Williams coefficient C_{HW} may be established by solving equation (4.56) for H_f and equating it to equation (4.4):

$$H_f = \frac{3.0225Lv^{1.852}}{D^{1.167}C_{HW}^{1.852}} = \left(\frac{fL}{D}\right)\frac{v^2}{2g_c}$$

Solving for f yields the following:

$$f = \frac{194.46}{C_{HW}^{1.85}(vD)^{0.148}D^{0.019}} \tag{4.59}$$

If equation (4.59) is multiplied by $\nu^{0.148}/\nu^{0.148}$, and noting that the kinematic viscosity of water at 60F is 12.31×10^{-6} ft^2/s and that $\text{Re} = vD/\nu$ from equation (4.10), the results are as follows:

$$f = \frac{194.46}{C_{HW}^{1.852}\left(\frac{vD}{\nu}\right)^{0.148}D^{0.019}\nu^{0.148}} = \frac{194.46}{C_{HW}^{1.852}\text{Re}^{0.148}D^{0.019}(12.31 \times 10^{-6})^{0.148}}$$

$$f = \frac{1036}{C_{HW}^{1.852}\text{Re}^{0.148}D^{0.019}} \tag{4.60}$$

The friction factor obtained using equation (4.60) with C_{HW} for steel pipe of 140 compares favorably with the Colebrook equation (equation (4.24)) for pipe sizes 2 in. and greater and for velocities from 3 ft/s to 10 ft/s.

The Hazen Williams equation may be used with accuracy with fluids whose viscosity is close to that of water at 60°F. It will permit direct solutions to network problems. **This equation should not be used outside these limits.**

Example 4.9: *Use of the Hazen-Williams equation.* The water network system shown in Fig. 4.7 has an energy loss of 200 ft·lbf/lbm between sections A and D. The pipes are new 250 psi cast iron. Using the Hazen-Williams equation, estimate the volumetric flow rate.

Solution

This example is solved by noting that for the parallel flow between sections B and C, the energy loss is the same for the 12-in. and the 16-in. pipes. The series flow between sections A and D is the sum of the energy loss for each section.

(a) Pipe properties (Table C.4 250-psi cast iron)

Standard size	*Inside diameter–ft*
12 in.	1.013
16 in.	1.345
20 in.	1.680
24 in.	2.018

From Table 4.9 for average new cast iron pipe, $C_{HW} = 130$.

(b) Flow through parallel section B to C:

$$H_{f12} = \frac{4.728 L_{12} Q_{12}^{1.852}}{D_{12}^{4.870} C_{HW}^{1.852}} = H_{f16} = \frac{4.728 L_{16} Q_{16}^{1.852}}{D_{16}^{4.870} C_{HW}^{1.852}} \tag{4.58}$$

Solving this equation for Q_{16}/Q_{12} yields the following:

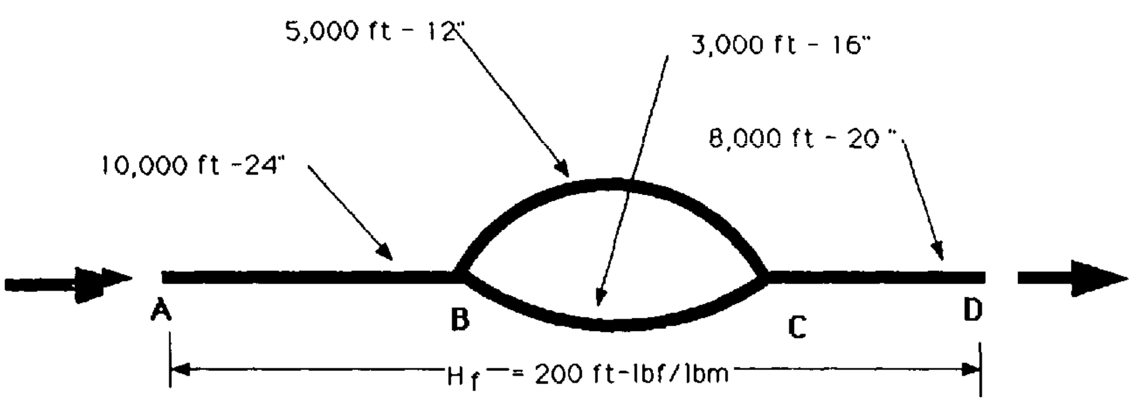


Figure 4.7 Notation for Example 4.9.

$$\frac{Q_{16}}{Q_{12}} = \left[\frac{L_{12}}{L_{16}}\left(\frac{D_{16}}{D_{12}}\right)^{4.870}\right]^{\frac{1}{1.852}}$$

$$\frac{Q_{16}}{Q_{12}} = \left[\frac{5000}{3000}\left(\frac{1.345}{1.013}\right)^{4.870}\right]^{0.54} = 2.7769$$

The total flow Q through both pipes is:

$$Q = Q_{16} + Q_{12} = 2.777Q_{12} + Q_{12} = 3.777Q_{12}$$

$$Q_{12} = 0.2648Q$$

(c) Flow through system

$$H_f = H_{fAB} + H_{fBC} + H_{fCD}$$

Substituting in equation (4.58) yields the following:

$$H_f = \frac{4.728L_{24}Q^{1.852}}{D_{24}^{4.870}C_{HW}^{1.852}} + \frac{4.728L_{12}(0.2648Q)^{1.852}}{D_{12}^{4.870}C_{HW}^{1.852}} + \frac{4.728L_{20}Q^{1.852}}{D_{20}^{4.870}C_{HW}^{1.852}} \qquad (4.58)$$

$$200 = \frac{4.728 \times 10{,}000Q^{1.852}}{2.018^{4.870}130^{1.852}} + \frac{4.728 \times 5{,}000 \times 0.2648^{1.852}Q^{1.852}}{1.013^{4.870}130^{1.852}} + \frac{4.728 \times 8{,}000Q^{1.852}}{1.680^{4.870}130^{1.852}}$$

$$200 = 0.7861Q^{1.852}$$

$$Q = 19.9\,\text{ft}^3/\text{s}$$

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5

Flow in Open Channels

5.1 INTRODUCTION

It it the intent of this chapter briefly to describe and illustrate some of the more important points regarding flow in open channels. Included is sufficient information to design rectangular and triangular weirs using American Society of Mechanical Engineers coefficients and corrections. The examples in this chapter are solved in USCS units only, but all the equations may be used with either set of units.

5.2 BACKGROUND

An *open* channel is a conduit in which a liquid flows with a free surface subjected to a constant pressure. Flow of water in natural streams, artificial canals, irrigation ditches, sewers, and flumes are examples where the water surface is subject to atmospheric pressure. The flow of any liquid in a pipe where there is a free liquid surface is an example of open channel flow where the liquid surface may be subjected to the pressure existing in the pipe.

The flow of fluids in open channels was first described in detail by Sextus Julius Frontinus (A.D. 40–103), who published a treatise on Roman methods of water distribution. Antoine Chezy (1718–1798) formulated similarity parameters for predicting flow characteristics of one channel from measurements carried out on another.

5.3 DEFINITIONS

Energy relations for open channel flow may be determined by modifying the Bernoulli equation (3.24) to include a term for friction loss. The resulting equation becomes:

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + H_L \tag{5.1}$$

H_L in this equation is the head lost due to friction. Note that all the terms in equation (5.1) have dimensions of length. Figure 5.1 is a plot of equation (5.1) and assumes that the channel is of uniform cross section.

The *energy grade line* is the sum of all the available energy $(z + V^2/2g + p/\gamma)$.

The *hydraulic grade line* is the sum of the potential and pressure energies and is also the liquid surface $(z + p/\gamma)$.

The distance between the liquid surface and the bottom of the channel is sometimes called the *stage* and is denoted by the symbol y in Figure 5.1.

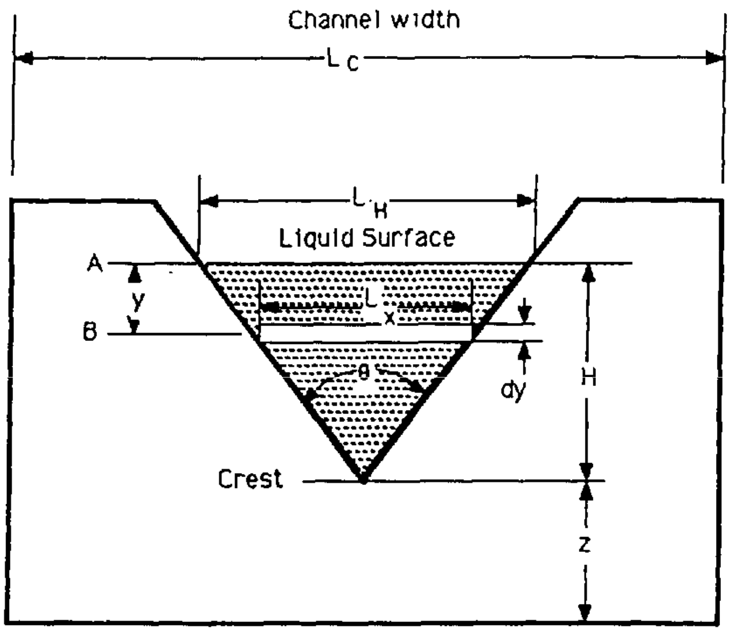


Figure 5.1 Notation for open channel flow study.

When the stages between the sections are not uniform, that is, $y_1 \neq y_2$, or the cross section of the channel changes, or both, then the flow is said to be *varied*.

When a liquid flows in a channel of uniform cross section and the slope of the surface is the same as the slope of the bottom of the channel ($y_1 = y = y_2$), then the flow is said to be *uniform*.

The *slope* of a channel, S_z, is the change in elevation per unit of horizontal distance. For small slopes, this is equivalent to dividing the change in elevation by the distance L measured along the channel bottom between two sections. For steady, uniform flow, the velocity is uniform at all sections of the channel, so the energy grade line has the same angle as the bottom of the channel. S_z can therefore be expressed as follows:

$$S_z = \frac{z_1 - z_2}{L} \tag{5.2}$$

For uniform flow, $v_1 = v_2$, and at any streamtube $p_1 = p_2$, so equation (5.1) reduces to the following:

$$H_L = z_1 - z_2 \tag{5.3}$$

Substituting equation (5.3) in equation (5.2) yields equation (5.4):

$$S_z = \frac{z_1 - z_2}{L} = \frac{H_L}{L} \tag{5.4}$$

5.4 PARAMETER FOR OPEN CHANNEL FLOW

Consider the steady, uniform flow of a liquid in an open channel produced by a gravity of g. The liquid flows with a velocity of v and has a density of ρ, viscosity of μ, and a surface tension of σ. The channel has an absolute surface roughness of ε. The flow has a hydraulic radius of R_h. It can be shown from dimensional analysis (see, for example, Manning, 1896) that the following relationships are valid:

$$v = F\sqrt{R_h g} \tag{5.5}$$

where

$$F = f(\text{Re}, W, \frac{\varepsilon}{4R_h})$$

The Weber number (W) is a dimensionless parameter representing the ratio of inertia forces to surface tension forces. W can be written as $(\rho L v^2/\sigma g_c)$.

It is conventional practice to write equation (5.5) in the following form:

$$v = C_z\sqrt{R_h g} \tag{5.6}$$

C_z in this equation is known as the *Chezy coefficient*. The functional relationship of C_z can be expressed as follows:

$$C_z = f(\text{Re}, W, \frac{\varepsilon}{4R_h}, g)$$

Relationship Between Friction Factor and Chezy Coefficient

The head lost due to friction H_L and the energy lost due to friction H_f are related as follows:

$$H_L = \frac{g_c}{g} H_f \tag{5.7}$$

H_f can be written in terms of the friction factor, f, according to equation (4.5). Substituting this expression into equation (5.7), and noting from equation (4.53) that $D = 4R_h$, results in the following:

$$H_l = \frac{fL}{D}\frac{v^2}{2g} = \frac{fLv^2}{8R_h g} \tag{5.8}$$

Solving equation (5.8) for velocity, and noting from equation (5.4) that $S_Z = H_L/L$, yields equation (5.9):

$$v = \sqrt{\frac{8H_L R_h g}{fL}} = \sqrt{\frac{8S_z R_h g}{f}} \tag{5.9}$$

Setting equation (5.8) equal to equation (5.6) and solving for C_z gives the following relationship:

$$v = \sqrt{\frac{8S_z R_h g}{f}} = C_z\sqrt{R_h S_z}$$

or

$$C_z = \sqrt{\frac{8g}{f}} \tag{5.10}$$

For open channel flow, the Chezy coefficient is calculated by means of the Manning equation, which was developed from the examination of experimental results of water tests. The Manning relation is stated as equation (5.11):

$$C_z = \frac{0.260\sqrt{g}R_h^{1/6}}{n} \tag{5.11}$$

where n is a roughness factor with units of $ft^{1/6}$ ($m^{1/6}$).

Replacing C_z in equation (5.6) with its value from equation (5.11) gives the following equation for velocity:

$$v = C_z\sqrt{R_h S_z} = \left(\frac{0.2620\sqrt{g}R_h^{1/6}}{n}\right)\sqrt{R_h S_z} = \frac{0.2620 g^{1/2} R_h^{2/3} S_z^{1/2}}{n} \tag{5.12}$$

From the dimensional analysis, n should be a function of Reynolds number, Weber number, and relative roughness. Since only water test data obtained at ordinary temperatures support this value, it must be assumed that n is the value for turbulent flow only. Since surface tension is a weak property, the effects of Weber number variation are negligible, leaving n to be some function of surface roughness. Design values of n are given in Table 5.1.

Example 5.1: *Flow in an Open Channel.* Water flows in a rubble-lined trapezoidal channel at a depth of 13 ft. The sides slope at 45°, and the bottom width is 66 ft. If the channel drops 9ft per 10,000 ft, estimate (a) the average velocity and (b) the volumetric flow rate.

Solution

This example is solved by the application of equation (5.12) and the continuity equation.

1. *Roughness coefficient n*: From Table 5.1, for a rubble-lined channel, $n = 0.025\ ft^{1/6}$.
2. *Channel properties*: From Table C.2 for a trapezoidal channel with a 45° slope:

$$A = (b + h)h = (66 + 13) = 1027 ft^2$$

$$R_h = \frac{(b+h)h}{b + 2.828h} = \frac{1027}{66 + 2.828 \times 13} = 9.993 ft$$

3. *Hydraulic slope*: From equation (5.4), $S_z = (z_1 - z_2)/L = 9/10{,}000 = 0.0009$ ft/ft.

a. Velocity [from equation (5.12)]:

Table 5.1 Values of Manning's Roughness Factor, *n*

Type of surface	*n*	
	$ft^{1/6}$	$m^{1/6}$
Brick	0.016	0.013
Cast iron	0.015	0.012
Concrete, finished	0.012	0.010
Concrete, unfinished	0.014	0.012
Corrugated metal	0.022	0.018
Earth, good condition	0.025	0.021
Earth, with stones and weeds	0.035	0.029
Gravel	0.029	0.024
Riveted steel	0.018	0.015
Rubble	0.025	0.021
Wood, planed	0.012	0.010
Wood, unplaned	0.013	0.011

$$v = \frac{0.2620 g^{1/2} R_h^{2/3} S_z^{1/2}}{n}$$

$$v = \frac{0.2620(32.17)^{1/2}(9.993)^{2/3}(0.0009)^{1/2}}{0.025} \tag{5.12}$$

$$v = 8.273 \text{ ft/s}$$

b. *Flow rate*:

$$Q = Av = 1027 \text{ ft}^2 \times 8.273 \text{ ft/s} = 8,496 \text{ ft}^3/\text{s}$$

5.5 MAXIMUM HYDRAULIC RADIUS

If equation (5.1) is substituted for v in the continuity equation, then the following expression for flow rate is obtained:

$$Q = Av = \frac{0.2620 A g^{1/2} R_h^{2/3} S_z^{1/2}}{n} \tag{5.13}$$

Examination of equation (5.13) indicates that, for a given roughness, slope, and area, the volume rate of flow will be a maximum when the hydraulic radius is a maximum.

The maximum hydraulic radius may be determined as follows: From Table C.2, the hydraulic radius of a trapezoid is given by equation (5.14):

$$R_h = \frac{A}{b + h(\text{cosec}\ \alpha + \text{cosec}\ \beta)} \tag{5.14}$$

The area of a trapezoid is given as follows:

$$A = [b + \frac{1}{2}h(\cot\alpha + \cot\beta)]h \tag{5.15}$$

Solving equation (5.15) for b and substituting in equation (5.14) yields the following expression for hydraulic radius:

$$R_h = \frac{Ah}{A + h^2\left(\text{cosec}\ \alpha + \text{cosec}\ \beta - \dfrac{\cot\alpha + \cot\beta}{2}\right)} \tag{5.16}$$

Let

$$\text{cosec}\ \alpha + \text{cosec}\ \beta - \frac{\cot\alpha + \cot\beta}{2} = K \text{ (a constant)} \tag{5.17}$$

Substituting equation (5.17) in equation (5.16) simplifies the expression for hydraulic radius:

$$R_h = \frac{Ah}{A + Kh^2} \tag{5.18}$$

Differentiating equation (5.18) with respect to h gives the following:

$$dR_h = \frac{(A + Kh^2)A\ dh - Ah(2Kh\ dh))}{(A + Kh^2)^2} \tag{5.19}$$

To find the maximum value of R_h, set equation (5.19) equal to zero:

$$\frac{dR_h}{dh} = 0 = \frac{(A + Kh^2)A - 2AKh^2}{(A + Kh^2)^2}$$

or

$$A = Kh^2 \tag{5.20}$$

Substituting equation (5.20) in equation (5.18) gives the final result for maximum hydraulic radius:

$$R_{h\ \max} = \frac{Kh^2h}{Kh^2 + Kh^2} = \frac{h}{2} \tag{5.21}$$

Since rectangular, square, and triangular channels are a special case of the trapezoidal channel, $R_{h\ max} = h/2$ must also apply. Table C.2 gives values of $R_{h\ max}$ for various channel shapes.

Example 5.2: *Maximum Hydraulic Radius.* A brick-lined rectangular channel is to carry 200 ft^3/s of water. If the channel slope is 1 in 10,000, what should be the size of the channel?

Solution

This simple example is solved by application of the principles and equations given in this section.

1. *Roughness coefficient*, n: From Table 5.1 for a brick-lined channel, $n = 0.016\ ft^{1/6}$.
2. *Channel properties*: From Table C.2 for a rectangular channel maximum hydraulic radius:

$$\frac{h}{b} = 0.5 \quad \text{or} \quad b = 2h$$

$$A = bh = 2h^2 \quad \text{(a)}$$

$$R_{h\ max} = \frac{h}{2} \quad \text{(b)}$$

Substituting equation (a) and equation (b) in equation (5.13) yields:

$$Q = \frac{0.2620 A g^{1/2} R_h^{2/3} S_z^{1/2}}{n}$$

$$200 = \frac{0.2620(2h^2)(32.17)^{1/2}(h/2)^{2/3}(1/10{,}000)^{1/2}}{0.016} \quad (5.13)$$

$$h = 6.875\text{ft}$$

$$b = 2h = 6.875 \times 2 = 13.75 \text{ ft}$$

5.6 SPECIFIC ENERGY

Specific energy is defined as the energy of the fluid referred to the bottom of the channel as the datum, as shown in Figure 5.2. Thus, the specific energy, E, at any section is given by the following:

$$E = y + \frac{v^2}{2g} \quad (5.22)$$

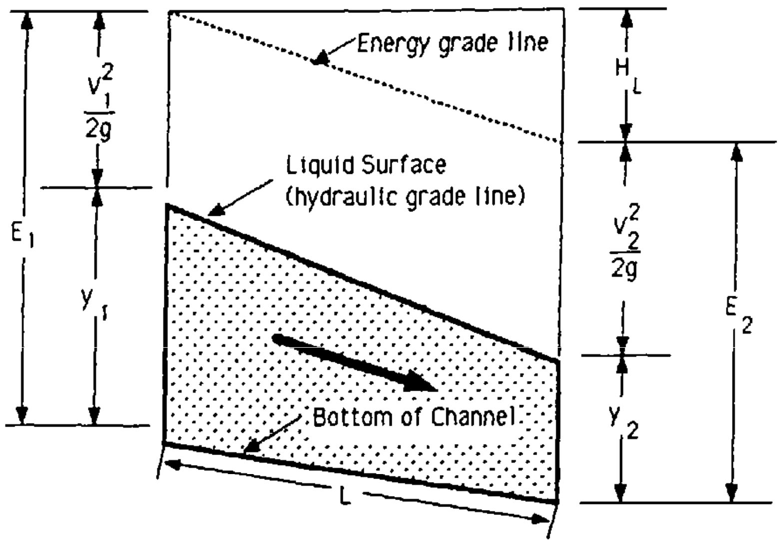


Figure 5.2 Notation for specific energy study.

Equation (5.22) may be written in terms of volumetric flow rate by substituting from the continuity equation, $v = Q/A$:

$$E = y + \frac{(Q/A)^2}{2g} \tag{5.23}$$

For a rectangular channel whose width is b, $A = by$; and if q is defined as the flow rate per unit width ($q = Q/b$), then equation (5.23) may be written as follows:

$$E = y + \frac{(Q/A)^2}{2g} = y + \frac{(qb/by)^2}{2g} = y + \frac{(q/y)^2}{2g} \tag{5.24}$$

Critical Values for Rectangular Channels

Critical values of specific energy, E_c, depth, y_c, and unit flow rate, q_c, may be derived by differentiating equation (5.24) with respect to y and setting the first derivative equal to zero:

$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{(q/y)^2}{2g}\right] = 0 = 1 + \frac{-2q^2}{2gy^3}$$

or

$$q_c^2 = y_c^3 g \tag{5.25}$$

Substituting equation (5.25) in equation (5.24) yields the following expression for critical specific energy:

$$E_c = y_c + \frac{(q_c/y_c)^2}{2g} = y_c + \frac{y_c^3 g/y_c^2}{2g} = y_c + \frac{y_c}{2} = \frac{3y_c}{2} \tag{5.26}$$

Critical velocity, v_c, may be obtained from equation (5.22) and equation (5.26):

$$E_c = \frac{3y_c}{2} = y_c + \frac{v_c^2}{2g}$$

or

$$v_c = \sqrt{gy_c} \tag{5.27}$$

Figure 5.3 shows the relation of depth to specific energy for a constant flow rate, and Figure 5.4 shows the relation between depth and flow rate for a constant specific energy. If the depth is greater than the critical value, then the flow is *subcritical*; at critical depth it is *critical*; and at depths below critical, the flow is *supercritical*. For a given specific energy there is a maximum unit flow rate that can exist. It should be noted from equation (5.27) that

$$\frac{v_c}{\sqrt{gy_c}} = 1 = F_c \qquad \text{(critical Froude number)}$$

Substituting the general relation for Froude number ($F = v/(yg)^{1/2}$) in equation (5.22) yields the following:

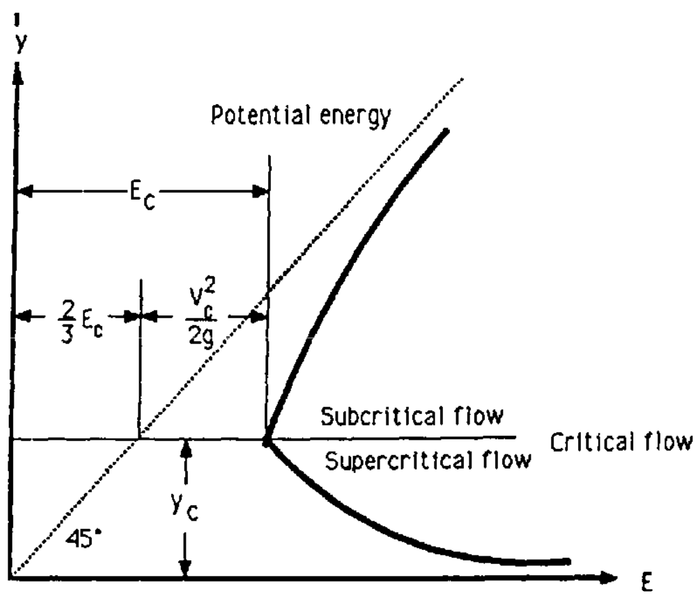


Figure 5.3 Specific energy diagram, q = constant.

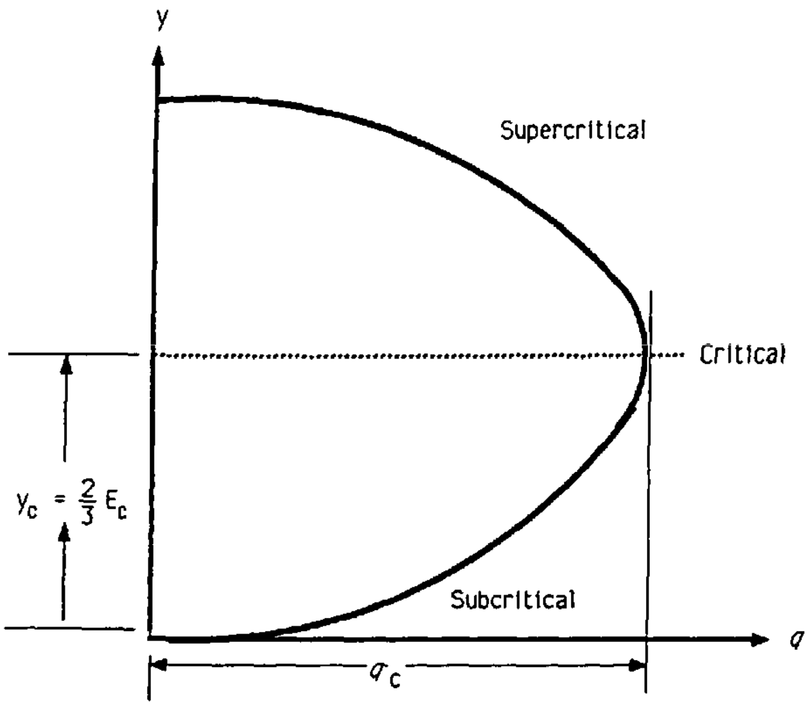


Figure 5.4 Depth vs. unit flow rate for constant specific energy.

$$E = y + \frac{v^2}{2g} = y + \frac{gyF^2}{2g} = y\left(1 + \frac{F^2}{2}\right) \tag{5.28}$$

Equation (5.28) may be rearranged in the form shown in equation (5.29):

$$\frac{y}{E} = \frac{1}{y(1 + F^2/2)} = \frac{2}{2 + F^2} \tag{5.29}$$

Examination of equation (5.29) indicates the following:

$F < 1$,	$y/E < 2/3$	flow is subcritical
$F = 1$,	$y/E = 2/3$	flow is critical
$F > 1$,	$y/E > 2/3$	flow is supercritical

It is seen that for open channel flow, the Froude number determines the types of flow in the same manner that Mach number does for compressible flow.

Example 5.3: *Subcritical Flow in a Rectangular Channel.* A rectangular channel lined with planed lumber has a width of 27 feet. When the depth of water is 3.5 feet, the flow is 370 ft^3/s. Determine (a) the required slope, (b) the specific energy, and (c) the type of flow.

Solution

This example is solved by using the Chezy equation (5.6) for the slope, the energy equation (5.22) for the specific energy, and the Froude relationships for the type of flow.

1. *Roughness coefficient n*: From Table 5.1 for a planed wood, $n = 0.012$ $\text{ft}^{1/6}$.
2. *Channel properties*: From Table C.2 for a rectangular channel:

$$A = bh = 27 \times 3.5 = 94.5 \text{ ft}^2$$

$$R_h = \frac{bh}{2h + b} = \frac{27 \times 3.5}{2 \times 3.5 + 27} = 2.779 \text{ ft}$$

3. *Velocity*:

$$v = \frac{Q}{A} = \frac{370}{94.5} = 3.915 \text{ ft/s}$$

4. *Chezy coefficient*:

$$C_z = \frac{0.2620\sqrt{g}R_h^{1/6}}{n}$$

$$C_z = \frac{0.2620\sqrt{32.17}(2.779)^{1/6}}{0.012} \tag{5.11}$$

$$C_z = 146.83 \text{ ft}^{1/2}/\text{s}$$

a. *Slope*:

$$v = C_z\sqrt{R_h S_z} \qquad \text{or} \qquad S_z = \frac{(v/C_z)^2}{R_h}$$

$$S_z = \frac{(3.915/146.83)^2}{2.779} \tag{5.6}$$

$$S_z = 2.558 \times 10^{-4} = 1/3{,}909$$

b. *Specific energy*:

$$E = y + \frac{v^2}{2g}$$

$$E = 3.5 + \frac{3.915^2}{2 \times 32.17} \tag{5.22}$$

$$E = 3.738 \text{ ft}$$

c. *Type of flow*: Solve equation (5.28) for the Froude number:

$$F = \sqrt{2\left(\frac{E}{y} - 1\right)} = \sqrt{2\left(\frac{3.738}{3.5}\right)} = 1 + 0.3688 < 1 \qquad \text{(flow is subcritical)}$$

5.7 HYDRAULIC JUMP

Under certain conditions in open channel flow, a stream of water flowing at supercritical velocity may change abruptly at subcritical flow. This phenomenon is accompanied by a change in surface elevation and is know as a *hydraulic jump*. Consider the hydraulic jump shown in Figure 5.5 as a free-body diagram. Application of the impulse-momentum equation (1.11) to Figure 5.5 as a free-body diagram. Application of the impulse-momentum equation (1.11) to Figure 5.5 results in the following:

$$F_1 - F_x = \frac{\dot{m}(v_2 - v_1)}{g_c} \tag{5.30}$$

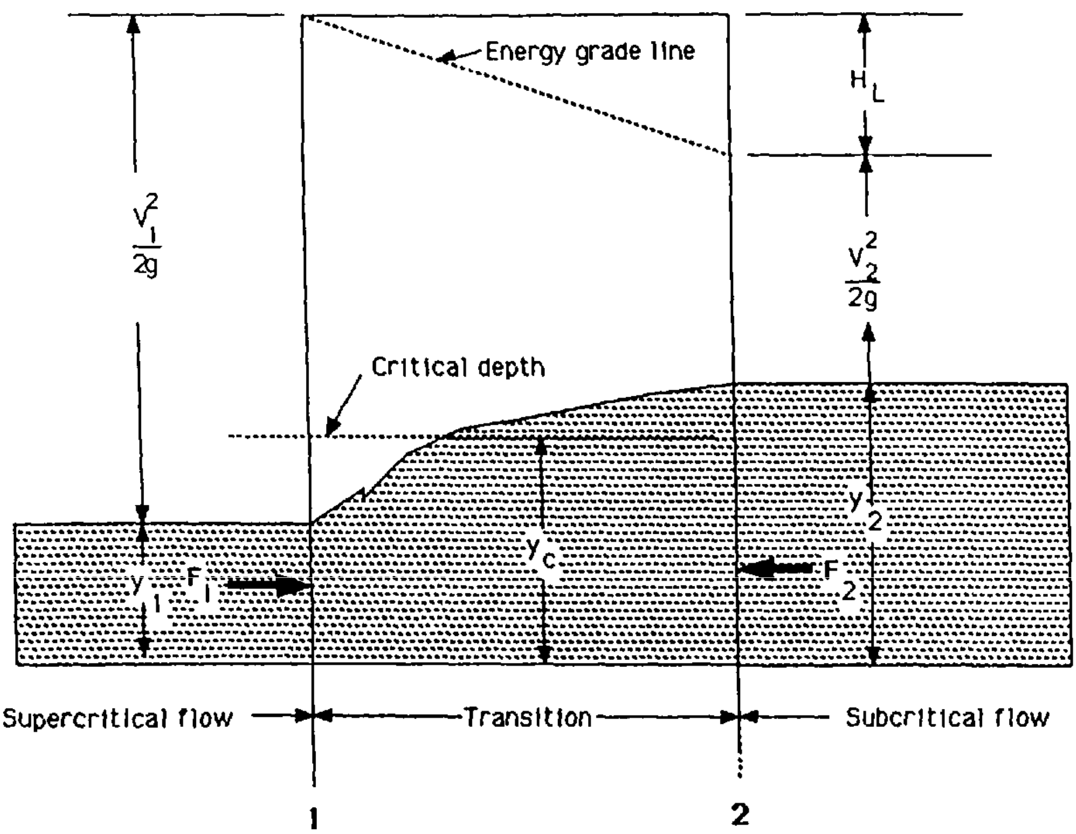


Figure 5.5 Notation for hydraulic jump study.

From buoyancy principles, $F = \gamma h_c A$. Also note from equation (1.14) that $\gamma = \rho g/g_c$ and that by definition $h_c = y/2$. For a channel of width b, $A = by$, and force may be written as follows:

$$F = \gamma h_c A = \left(\frac{\rho g}{g_c}\right)\left(\frac{y}{2}\right)(by) = \left(\frac{\rho g b y^2}{2g_c}\right) \tag{5.31}$$

Continuity is expressed as $\dot{m} = \rho A v$; substituting $A = by$, $Q = Av$, and $Q = bq$ (from Section 5.6) yields the following form for continuity:

$$\dot{m} = \rho A v = \rho Q = \rho b q \tag{5.32}$$

From these substitutions, the following can be written for velocity:

$$v = \frac{Q}{A} = \frac{bq}{by} = \frac{q}{y} \tag{5.33}$$

Substituting equations (5.31), (5.32), and (5.33) in equation (5.30) yields the following:

$$F_1 - F_2 = \frac{\dot{m}(v_2 - v_1)}{g_c} = \frac{\rho g b y_1^2}{2g_c} - \frac{\rho g b y_2^2}{2g_c} = \frac{(\rho b q)\left(\dfrac{q}{y_2} - \dfrac{q}{y_1}\right)}{g_c}$$

which reduces to:

$$\frac{q^2}{g y_1} + \frac{y_1^2}{2} = \frac{q^2}{g y_2} + \frac{y_2^2}{2} \tag{5.34}$$

Equation (5.34) may be solved explicitly for either y_1 or y_2:

$$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + \frac{8q^2}{g y_2^3}}\right) \tag{5.35}$$

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + \frac{8q^2}{g y_1^3}}\right) \tag{5.36}$$

The lost energy due to a hydraulic jump may be computed from equation (5.22):

$$H_L = E_1 - E_2 = \left(y_1 + \frac{v_1^2}{2g}\right) - \left(y_2 + \frac{v_2^2}{2g}\right) = y_1 - y_2 + \frac{v_1^2 - v_2^2}{2g} \tag{5.37}$$

Example 5.4: *Energy Loss for a Hydraulic Jump.* Water flows at a rate of 350 ft^3/s in a rectangular channel 15 feet wide at a depth of 1 foot. What is the energy loss in a hydraulic jump that has occurred from this flow?

Solution

This example is solved by the principles set forth in this section. Note that it is necessary to determine if a hydraulic jump can take place.

1. *Initial area and velocity*:

$$A_1 = by_1 = 18 \times 1 = 18 \text{ ft}^2$$
$$v_1 = \frac{Q}{A_1} = \frac{350}{18} = 19.44 \text{ ft/s}$$

2. *Unit flow rate*:

$$q = \frac{Q}{b} = \frac{350}{18} = 19.44 \text{ ft}^2\text{/s}$$

3. *Determine if jump can take place*:

$$y_c = \left(\frac{q_c^2}{g}\right)^{1/3}$$
$$y_c = \left(\frac{19.44^2}{32.17}\right)^{1/3} \tag{5.25}$$
$$y_c = 2.273 \text{ ft} > y_1 = 1 \text{ ft}$$

Therefore, jump can take place

4. *Conditions at section 2*:

$$y_2 = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8 \times 19.44^2}{32.17 \times 1^3}}\right) = 4.373 \text{ ft} \tag{5.36}$$

$$y_2 = 4.373 \text{ ft} > y_c \qquad \text{(subcritical)}$$

5. *Compute head loss*:

$$A_2 = by_2 = 18 \times 4.373 = 78.71 \text{ ft}^2$$
$$v_2 = \frac{Q}{A_2} = \frac{350}{78.71} = 4.45 \text{ ft/s}$$

$$H_L = y_1 - y_2 + \frac{v_1^2 - v_2^2}{2g}$$

$$H_L = 1 - 4.373 + \frac{19.44^2 - 4.45^2}{2 \times 32.17} \qquad (5.37)$$

$$H_L = 2.192 \text{ ft}$$

5.8 WEIRS

Weirs are used to measure the flow of liquids in open channels or in conduits that do not flow full, i.e., in which there is a free liquid surface. A weir is, in effect, a dam over which the liquid is forced to flow. Weirs are almost exclusively used for measuring water flow, although small ones have been used for metering other liquids.

Weirs are classified according to the form of their notch or opening as follows: rectangular notch, the original form; the V or triangular notch; and special notches, such as the trapezoidal, hyperbolic, and parabolic notches, which are designed to have a constant discharge coefficient or to have the head vary directly with the flow.

Velocity–Height Relations

Conventional practice is to base weir computations on the ideal flow of an incompressible fluid and to correct to actual flow conditions by use of a coefficient of discharge, adjustment lengths, and head corrections.

Consider the flow of a liquid over a weir as shown in Figure 5.6. A is a fluid particle located on the surface of the liquid a distance upstream from the weir. B is a fluid particle in the free jet issuing from the weir and is a distance y below the surface point A. Both A and B are at atmospheric pressure. The equation of motion for constant pressure ($dp = 0$) and for frictionless flow ($\tau = 0$) is written as follows:

$$\frac{g}{g_c}dz + \frac{U\,dU}{g_c} + v\,dp + v\tau dL\left(\frac{dp}{dA}\right) = 0 = \frac{g}{g_c}d\ + \frac{U\,dU}{g_c} + 0 + 0$$

or

$$g\,dz + U\,du = 0 \qquad (5.38)$$

Integrating equation (5.38) between the limits of A and B leads to equation (5.39):

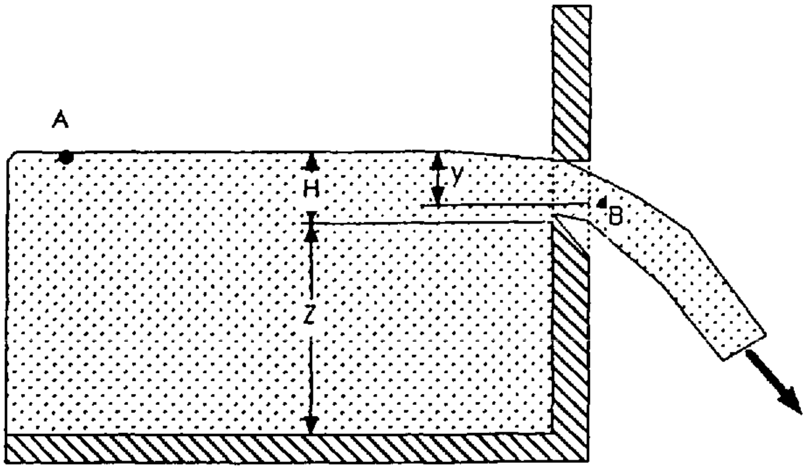


Figure 5.6 Flow over a weir.

$$g\int_{z_A}^{z_B} dz + \int_{U_A}^{U_B} U\,dU = 0 = g(z_A - z_A) + \frac{U_B^2 - U_A^2}{2}$$

or

$$U_B = \sqrt{2g(z_A - z_A) + U_A^2} \tag{5.39}$$

Conventional practice is to assume that point A is located in a channel of infinite length, so $U_A \to 0$. From Figure 5.6, $z_A - z_b = y$, and let $U_B = U_i$ (ideal jet velocity); then equation (5.39) may be written as follows:

$$U_i = \sqrt{2gy} \tag{5.40}$$

5.9 RECTANGULAR WEIRS

Consider the jet issuing from the rectangular weir shown in Figure 5.7. The flow area of the fluid element shown at a distance y below the surface is $dA = Lw\ dy$. The ideal jet velocity is given by equation (5.40).

Substituting these values in the continuity equation and integrating between the liquid surface (0) and the weir crest (H) yields the following:

$$Q_i = \int_0^A U_i\,dA = \int_0^H \sqrt{2gy}(L_w\,dy) = L_w\sqrt{2g}\left[\frac{2y^{3/2}}{3}\right]_0^H \tag{5.41}$$

$$Q_i = \frac{2}{3}L_w\sqrt{2g}[H^{3/2}] \qquad \text{(ideal flow rate)}$$

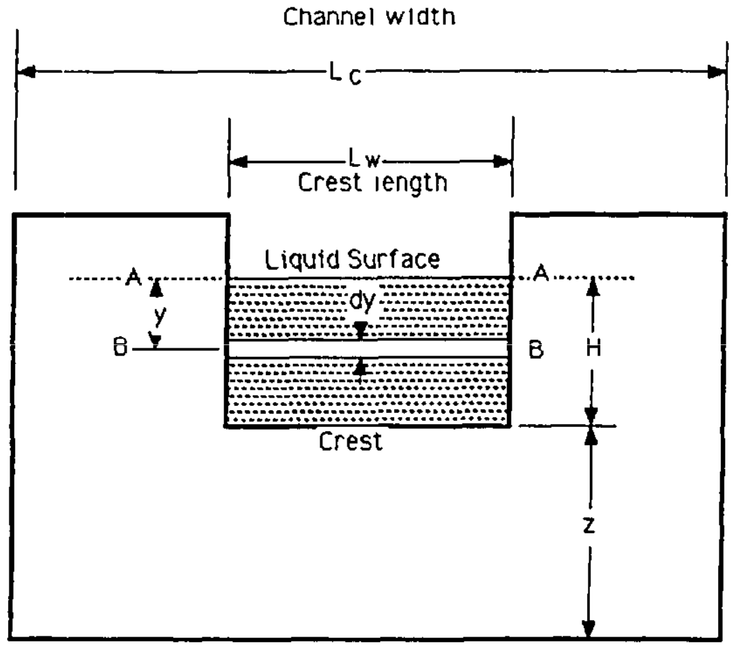


Figure 5.7 Notation for rectangular weir study.

For water measurement, the ASME Research Committee on Fluid Meters (Bean, 1971) recommends the following:

$$Q = \frac{2}{3} C L_a \sqrt{2g}[H_a^{3/2}] \tag{5.42}$$

where

Q = actual discharge from weir
C = coefficient of discharge $= f(L_w/L_c, H/z)$ (Table 5.2)
L_a = adjusted crest length $= Lw + \Delta L$ (Table 5.2)
H_a = adjusted weir head $= H + 0.003$ ft or $H + 0.9$ mm

Example 5.5: *Rectangular Weir Flow Measurement.* Water flows in a channel whose width is 10 ft. At the end of this channel is a weir whose crest height is 4 ft and whose width is the same as that of the channel. The water flows at a height of 3 ft over the crest of the weir. Estimate the flow in cubic feet per second.

Solution

This example is solved using equation (5.42) and Table 5.2.

1. *Weir properties:*

Table 5.2 Values of C and ΔL for Use in Equation (5.42)

	Crest length/channel width $= L_w/L_c$							
	0	0.2	0.4	0.6	0.7	0.8	0.9	1.0
H/z	Coefficient of discharge: C							
0	0.587	0.588	0.590	0.593	0.595	0.597	0.599	0.602
0.15	0.587	0.587	0.593	0.604	0.611	0.619	0.629	0.640
1.0	0.586	0.586	0.595	0.614	0.627	0.642	0.659	0.679
1.5	0.585	0.585	0.598	0.624	0.643	0.664	0.689	0.718
2.0	0.584	0.583	0.600	0.635	0.659	0.687	0.719	0.756
2.5	0.584	0.582	0.603	0.645	0.674	0.709	0.749	0.795
3.0	0.583	0.581	0.605	0.655	0.690	0.732	0.779	0.834
	Adjustment for crest length: ΔL							
ft	0.007	0.008	0.009	0.012	0.013	0.014	0.013	0.005
m	0.0021	0.0024	0.0027	0.0037	0.0040	0.0043	0.0040	0.0015

$$\frac{L_w}{L_c} = \frac{10}{10} = 1$$

$$\frac{H}{z} = \frac{3}{4} = 0.75$$

From Table 5.2 at $L_w/L_c = 1$ and $H/z = 0.75$:

$$C = 0.660 \qquad \text{(by linear interpolation)}$$

$$\Delta L = -0.005 \text{ ft}$$

$$L_a = L_w + \Delta L = 10 + (-0.005) = 9.995 \text{ ft}$$

$$H_a = H + 0.003 = 3.003 \text{ ft}$$

2. *Calculate flow using equation (5.42):*

$$Q = \frac{2}{3} C L_a \sqrt{2g}[H_a^{3/2}]$$

$$Q = \frac{2}{3} \times 0.660 \times 0.995 \times \sqrt{2 \times 32.17} \times 3.003^{3/2} \tag{5.42}$$

$$Q = 183.6 \text{ ft}^3/\text{s}$$

5.10 TRIANGULAR WEIRS

Consider the jet issuing from the triangular weir shown in Figure 5.8. The flow area of the fluid element at a distance below the surface of y is $dA = L_x\ dy$. From geometry and trigonometry, the following expressions can be derived:

$$\frac{L_x}{L_H} = \frac{H - y}{H} \quad \text{or} \quad L_x = \frac{L_H(H - y)}{H}$$

$$L_H = 2H \tan(\theta/2)$$

Therefore:

$$L_x = \frac{L_H(H - y)}{H} = \frac{2H \tan(\theta/2)}{H} = 2 \tan(\theta/2)(H - y)$$

and

$$dA = 2 \tan(\theta/2)(H - y)\ dy$$

The ideal jet velocity is given by equation (5.40). Substituting these values for dA and U_t in the equation of continuity and integrating between the liquid surface (0) and the crest (H) yields the following:

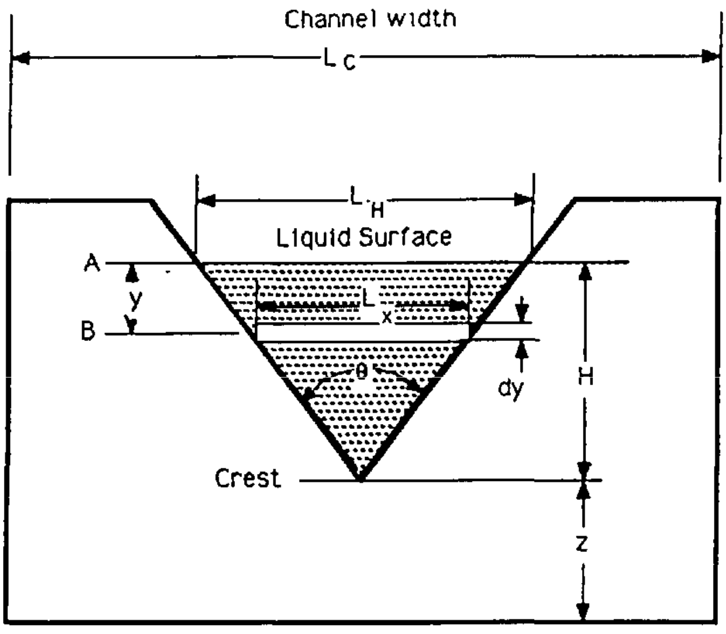


Figure 5.8 Notation for triangular weir study.

$$Q_i = \int_0^A U_t\, dA = \int_0^H \sqrt{2gy}[2\tan(\theta/2)(H - y)]\, dy = 2\sqrt{2g}\ \tan(\theta/2)$$

$$\times \left[\frac{2Hy^{3/2}}{3} - \frac{2y^{5/2}}{5}\right]_0^H$$

$$Q_i = \frac{8}{15}\tan(\theta/2)\sqrt{2g}[H^{5/2}] \qquad \text{(ideal volume flow rate)} \qquad (5.43)$$

The value of the discharge coefficient for triangular weirs is dependent primarily on the notch angle, θ, and only slightly on the head–crest ratio, H/z. The fluid viscosity and surface tension may affect the value of the discharge coefficient slightly, but experimental data are inadequate to define such effects. The ASME Research Committee on Fluid Meters (Bean, 1971) recommends the following for the measurement of liquid flow through triangular weirs:

$$Q = \frac{8}{15}C\ \tan(\theta/2)\sqrt{2g}(H + \Delta H)^{5/2} \qquad (5.44)$$

where:

Q = actual discharge from weir

C = coefficient of discharge $= f(\theta)$ (Table 5.3)

ΔH = correction for head/crest ratio $= f(\theta)$ (Table 5.3)

Example 5.6: *Regulating Flow with a Triangular Weir.* It is desired to maintain a flow of 180 ft^3/s of water in an open channel whose width is 10 ft at a height of 10 ft by locating a triangular weir at the end of the channel. The weir has a crest height of 4 ft. What angle of notch is required to maintain these conditions?

Solution

This example is solved by application of equation (5.44). Because the value of the coefficient of discharge C and the head/crest correction ΔH are both a function of the weir angle θ, a trial-and-error solution is necessary.

1. *Weir data*: If the total depth of the channel $H + z = 10$ ft, and the crest height z is 4 ft, then $H = 10 - z = 10 - 4 = 6$ ft.

Table 5.3 Values of C and ΔH for use in Equation (5.44)

Weir notch angle θ (degrees)	Coefficient of discharge C	Correction for head/crest ratio ΔH (feet)	ΔH (meters)
20	0.593	0.0094	0.0029
30	0.587	0.0072	0.0022
40	0.582	0.0055	0.0017
45	0.580	0.0049	0.0015
50	0.579	0.0044	0.0014
60	0.576	0.0038	0.0011
70	0.576	0.0034	0.0010
75	0.576	0.0033	0.0010
80	0.576	0.0032	0.0010
90	0.578	0.0030	0.0009
100	0.581	0.0027	0.0008

2. *First trial*: From Table 5.3, $C \approx 0.58$ for all values of θ, and ΔH is small compared with H. Use $C = 0.58$ for the first trial:

$$Q = \frac{8}{15} C \tan(\theta/2) \sqrt{2g}(H + \Delta H)^{5/2}$$

$$180 = \frac{8}{15} \times 0.58 \times \tan(\theta/2) \sqrt{2 \times 32.17}(6)^{5/2} \qquad (5.44)$$

$$\tan(\theta/2) = 0.8227$$

$$\theta = 78.88^\circ$$

3. *Second trial*: Use $\theta = 80^\circ$ (nearest tabular value):

$$C = 0.576, \qquad \Delta H = 0.0032 \text{ ft}$$

$$Q = \frac{8}{15} C \tan(\theta/2) \sqrt{2g}(H + \Delta H)^{5/2}$$

$$180 = \frac{8}{15} \times 0.576 \times \tan(\theta/2) \sqrt{2 \times 32.17}(6.0032)^{5/2} \qquad (5.44)$$

$$\tan(\theta/2) = 0.8272$$

$$\theta = 79.22$$

Further trials are unnecessary; therefore use 79.22°.

REFERENCES

Bean, H. S., ed., *Fluid Meters— Their Theory and Application*, American Society of Mechanical Engineers, New York, N.Y., 6th Ed., 1971.

Manning, R. T., Flow of water in open channels and pipes, *Transactions of the Institute of Civil Engineers (Ireland)*, Vol. 20, 1896, p. 161.

6
Flow Measurement in Closed Conduits

6.1 INTRODUCTION

This chapter presents the concepts involved in the measurement of fluid flow. Six types of primary elements were selected to illustrate the fundamentals. They are all nonproprietary and may be fabricated from drawings given in the standards referenced in this chapter. *Flow measurement is an extremely complex business. Before using this chapter for design, the references cited should be reviewed for installation information.*

Although the examples are in USCS units only, all equations are structured so that SI units may also be used.

6.2 BACKGROUND

The first real flow measurement was described by Hero of Alexandria (1 AD). Hero's flow measurement system included a tank for volume measurement and a sundial for time measurement. The flow rate was calculated using the continuity equation. More recent are: 1502, Leonardo da Vinci established the principle of continuity; 1748, Daniel Bernoulli developed his famous equation; 1895, Clement Herschel invented the Venturi tube.

6.3 FLOW MEASUREMENT ACCURACY

It is not possible, except by accident, to manufacture two objects *exactly* the same. Because of this limitation, no two meters, even of the same type, are likely to give the same indication when the same quantity of fluid is passing through them. The degree to which this applies is not the same for all types of meters. For this reason, "uncertainties" are assigned to different types of primary elements to describe their inherent accuracy. *Uncertainties* have to do with practically unavoidable differences between apparently duplicate primary elements. *Uncertainty* in flow measurement is defined as a range of values within which the true value is estimated to lie with 95% probability. Calibration of a primary element at conditions of use reduces this uncertainty to the level of the accuracy of the calibration laboratory.

Uncertainties given in this chapter are for uncalibrated primary elements only. Installation, pressure sensing, and other factors may cause flow measurement errors many times the uncertainty of a primary element.

6.4 DIMENSIONAL ANALYSIS OF PIPELINE FLOW METERS

Consider a compressible fluid of density ρ, dynamic viscosity μ, and bulk modulus of elasticity E, flowing with a velocity v through a primary element (Venturi, nozzle, or orifice) whose diameter is d, as shown in Figure 6.1. The primary element is located in a horizontal meter tube of diameter D and an absolute roughness of ε. The flow through the primary element produces a pressure differential of Δp sensed by pressure taps of diameter δ located a distance L_1 before and L_2 after the primary element. From dimensional analysis, Equation (6.1) can derived for fluid velocity:

$$v = K\sqrt{\frac{2\Delta p\, g_c}{\rho}} \tag{6.1}$$

where $K = f(\mathrm{Re}_d, M, L_1/D, L_2/D, \delta/D, \beta, \varepsilon/D)$.

6.5 PHYSICAL ANALYSIS OF PIPELINE FLOW METERS

Substituting v from equation (6.1) in the continuity equation ($\dot{m} = \rho A v$) results in the following:

$$\dot{m} = \rho A v = \rho A K\sqrt{\frac{2\ \Delta p\ g_c}{\rho}} = KA\sqrt{2g_c\ \Delta p\ \rho} \tag{6.2}$$

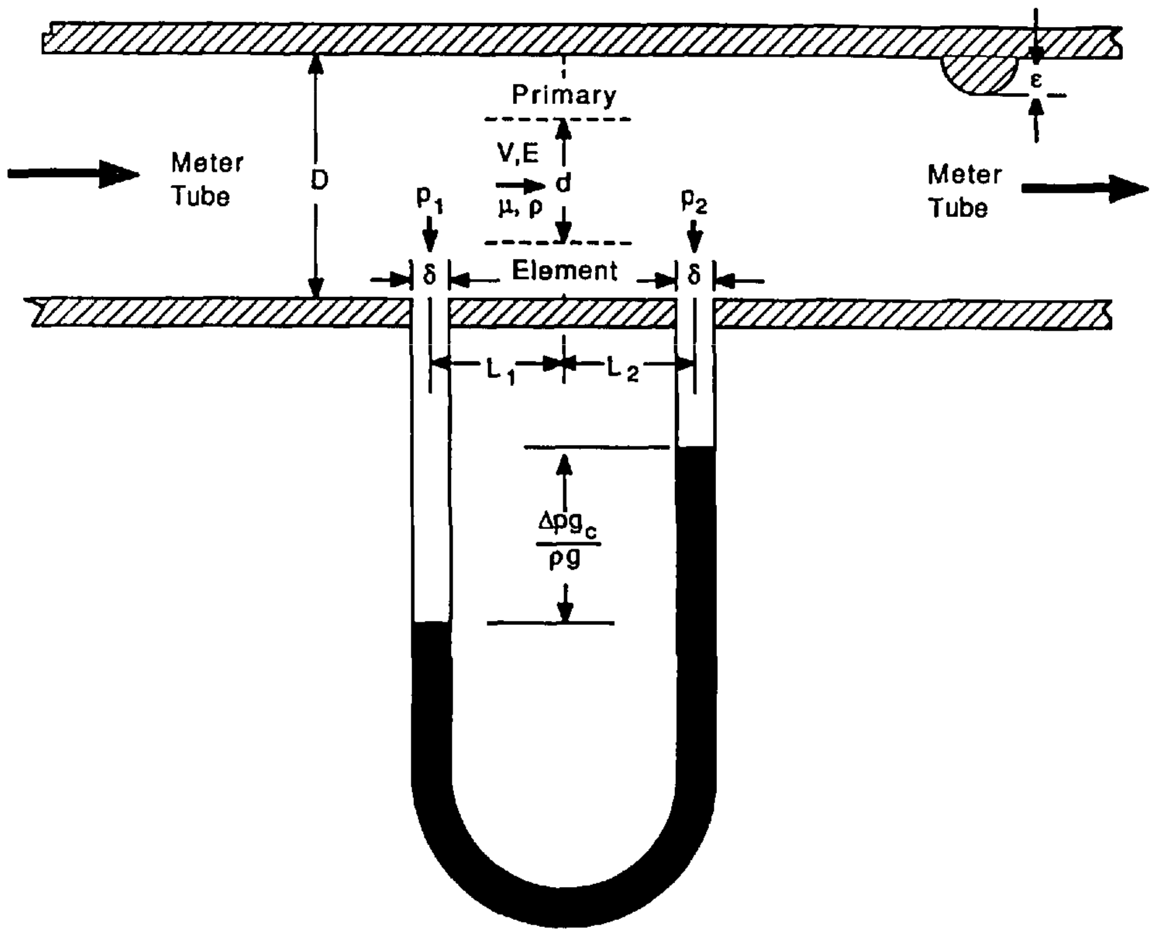


Figure 6.1 Notation for dimensional analysis of flow meters.

In Chapter 3, equation (3.122) was developed for ideal compressible flow through nozzles. Noting that by definition $\rho = 1/v$ and that for this application $A_2 = A_d$:

$$\dot{m}_{\text{ideal}} = YA_2\sqrt{\frac{2g_c(p_1 - p_2)}{v_1(1 - \beta^4)}} = YA_d\sqrt{\frac{2g_c\ \Delta p\ \rho_1}{1 - \beta^4}} \tag{6.3}$$

where Y, the expansion factor, is defined by the following:

$$Y = \frac{\text{mass flow rate of a compressible fluid}}{\text{mass flow rate of an incompressible fluid}}$$

For actual flow, equation (6.3) may be written as follows:

$$\dot{m} = CYA_d\sqrt{\frac{2g_c\ \Delta p\ \rho_1}{1 - \beta^4}} \tag{6.4}$$

where C is the coefficient of discharge.

Comparing equation (6.1) with equation (6.4), we can write C as follows:

$$C = \frac{K\sqrt{1-\beta^4}}{Y} \tag{6.5}$$

Conventional practice is to express C in terms of Re_d as follows:

$$C = C_\infty + \Delta C = C_\infty + \frac{a}{\mathrm{Re}_d^b} \tag{6.6}$$

In equation (6.6), C_∞ is the coefficient of discharge when the Reynolds number is infinity and ΔC is the change in the coefficient from a Reynolds number of infinity to Re_d. The numerical value of the slope, a, depends on the type of primary element; that of the exponent, b, depends upon the type of flow.

Pipe Reynolds Numbers

In Section 4.5, equations for calculating Reynolds number were developed for flow in pipes. In flow measurement, the Reynolds number may be calculated in terms of the primary element diameter d or in terms of the pipe diameter D. The relation between these is as follows:

$$\frac{\mathrm{Re}_d}{\mathrm{Re}_D} = \frac{4\dot{m}/\pi d\mu g_c}{4\dot{m}/\pi D\mu g_c} = \frac{D}{d}$$

or

$$\mathrm{Re}_D = \frac{D}{d}\mathrm{Re}_d = \beta\mathrm{Re}_d \tag{6.7}$$

6.6 ASME VENTURI TUBES

The advantages of the Venturi tube are its ability to transport materials in suspension without clogging and its low overall energy loss. The disadvantage is the long "laying length" required to maintain low loss. Traditional applications of this device have been in low-pressure gas lines and water and sewage mains.

The Venturi tubes described in this section are those conforming with ASME Fluid Meters Research Committee on Fluid Meters Report (Bean, 1971) and ASME Flow Measurement Standard (*Measurement of Gas Flow*, 1987). They are also called classical Venturi tubes as well as Herschel-type tubes. The form of the Venturi tube is shown in Figure 6.2. The ASME Venturi tube consists essentially of a cylindrical inlet, convergent entrance, throat, and divergent outlet. The divergent outlet is present to reduce the

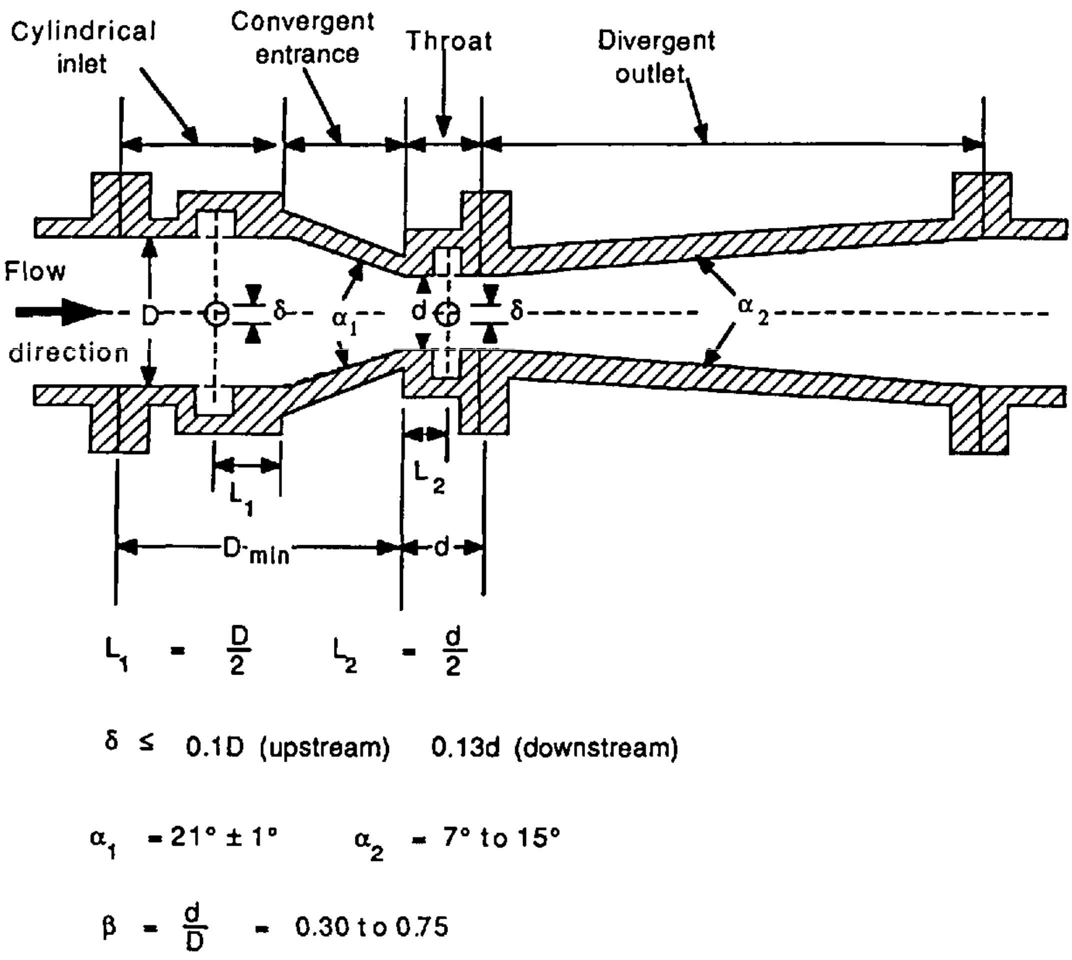


Figure 6.2 ASME Venturi tube.

overall loss of the meter; its removal will have no effect on the coefficient of discharge. Pressure is sensed through a series of holes in the inlet section and throat. Each set of holes leads to an annular chamber; the two chambers are connected to a pressure differential measurement device.

Classification of Venturi Tubes

Venturi tubes are classified into three types according to the method of manufacture.

1. *Rough-cast convergent.* This type is manufactured from castings. The throat is machined, and the junctions between the various sections are rounded.
2. *Machined convergent.* This type is manufactured in the same manner as type 1, except that entrance section, convergent, and throat are

machined as one assembly from a bar forging or other suitable material.

3. *Fabricated convergent.* This type is manufactured from formed metal sheet and is joined together by welding. The junctions between the various sections are not rounded.

In the United States, type 1 is almost always used; in Europe, all three types are used.

Discharge Coefficients and Uncertainty

1. *Rough-cast convergent.* For this type of Venturi, the ASME Code (*Measurement of Fluid Flow*, 1989) specifies $C = 0.984$ with an uncertainty of 1.0% subject to the following limits:

Pipe Reynolds number, Re_D, from 200,000 to 6,000,000
Beta ratio, β, from 0.30 to 0.75
Nominal pipe size from 4 in. to 48 in. (may be used up to 84 in. with reliability)

The coefficient of discharge for type 1 Venturis may be estimated using the following equation:

$$C = 1.0087 - \frac{10.386}{\mathrm{Re}_D^{1/2}} \tag{6.8}$$

The uncertainty is estimated from equation (6.9):

$$\text{uncertainty } \% = 14.3 - 2.57 \log_{10} (\mathrm{Re}_D) \qquad [\text{minimum } 1\%] \tag{6.9}$$

Equations (6.8) and (6.9) are valid for a pipe Reynolds number ranging from 10,000 to 200,000. Table 6.1 gives tabulated values calculated from equations (6.8) and (6.9).

2. *Machined convergent.* For this type of Venturi, the ASME Code (*Measurement of Fluid Flow*, 1989) specifies $C = 0.995$ with an uncertainty of 1.0% subject to the following limits:

Pipe Reynolds number Re_D from 200,000 to 2,000,000
Beta ratio, β, from 0.30 to 0.75
Nominal pipe size from 2 in. to 10 in.

The coefficient of discharge and uncertainty for type 2 Venturis may be obtained from Table 6.2. Note that the coefficient of discharge for type 2 Venturi tubes is shown as a function of the throat Reynolds number, Re_d.

Table 6.1 Coefficients of Discharge and Uncertainties for ASME Rough-Cast Convergent Venturi Tubes (Type 1)

Pipe Reynolds number, Re_D	Coefficient of discharge, C	Uncertainty, %	Pipe Reynolds number, Re_D	Coefficient of discharge, C	Uncertainty, %
10,000	0.905	4.0	42,000	0.958	2.4
11,000	0.910	3.9	44,000	0.956	2.4
12,000	0.914	3.8	46,000	0.960	2.3
13,000	0.918	3.7	48,000	0.961	2.3
14,000	0.921	3.6	50,000	0.962	2.2
15,000	0.924	3.6	55,000	0.964	2.1
16,000	0.927	3.5	60,000	0.966	2.0
17,000	0.929	3.4	65,000	0.968	1.9
18,000	0.931	3.4	70,000	0.969	1.8
19,000	0.933	3.3	75,000	0.971	1.8
20,000	0.935	3.2	80,000	0.972	1.7
21,000	0.937	3.2	85,000	0.973	1.6
22,000	0.939	3.1	90,000	0.974	1.6
23,000	0.940	3.1	95,000	0.975	1.5
24,000	0.942	3.0	100,000	0.976	1.5
25,000	0.943	3.0	110,000	0.977	1.3
26,000	0.944	3.0	120,000	0.979	1.2
27,000	0.945	2.9	130,000	0.980	1.1
28,000	0.947	2.9	140,000	0.981	1.0
29,000	0.948	2.8	150,000	0.982	1.0
30,000	0.949	2.8	160,000	0.983	1.0
32,000	0.951	2.7	170,000	0.984	1.0
34,000	0.952	2.7	180,000	0.984	1.0
36,000	0.954	2.6	190,000	0.984	1.0
38,000	0.955	2.5	2×10^5–6×10^6	0.984	1.0
40,000	0.957	2.5			

3. *Fabricated convergent*. For this type of Venturi, the ASME Code (*Measurement of Fluid Flow*, 1989) specifies $C = 0.984$ with an uncertainty of 2.0% subject to the following limits:

Pipe Reynolds number R_D from 200,000 to 6,000,000
Beta ratio, β, from 0.30 to 0.75

Table 6.2 Coefficients of Discharge and Uncertainties for ASME Machined Convergent Venturi Tubes (Type 2)

Throat Reynolds number, Re_d	Coefficient of discharge, C	Uncertainty, %	Throat Reynolds number, Re_D	Coefficient of discharge, C	Uncertainty, %
10,000	0.964	3.1	220,000	0.994	2.1
20,000	0.966	3.0	240,000	0.995	2.1
30,000	0.967	3.0	260,000	0.996	2.0
40,000	0.968	3.0	280,000	0.997	1.9
50,000	0.970	3.0	300,000	0.998	1.5
60,000	0.971	2.9	320,000	0.998	1.5
70,000	0.973	2.8	340,000	0.999	1.5
80,000	0.974	2.8	360,000	0.999	1.5
90,000	0.976	2.7	380,000	0.998	1.4
100,000	0.977	2.5	400,000	0.998	1.3
120,000	0.981	2.5	420,000	0.998	1.3
140,000	0.984	2.5	440,000	0.997	1.2
160,000	0.987	2.5	460,000	0.996	1.1
180,000	0.990	2.5	480,000	0.996	1.0
200,000	0.992	2.5	500,000	0.995	1.0

Nominal pipe size from 4 in. to 48 in.

The coefficient of discharge and uncertainties for type 3 Venturis may be estimated from Table 6.3

Expansion Factor *Y*

The expansion factor Y for all types of Venturi tubes is the adiabatic expansion factor Y, developed in Chapter 3. Values of Y may be calculated using equation (3.120) or taken from Table 3.2.

Pressure Loss Caused by Venturi Tubes

The pressure loss in terms of pressure developed by a Venturi is dependent primarily on the exit cone angle and the beta ratio. The following equations may be used to estimate this loss for all type of Venturi tubes where Δp_L is the pressure loss in the same units as Δp:

Table 6.3 Coefficients of Discharge and Uncertainties for ASME Fabricated Convergent Venturi Tubes (Type 3)

Pipe Reynolds number, Re_D	Coefficient of discharge, C	Uncertainty, %
10,000	0.930	3.5
20,000	0.940	3.0
30,000	0.940	3.0
40,000	0.960	2.5
50,000	0.960	2.5
60,000	0.970	2.5
70,000	0.970	2.5
80,000	0.980	2.5
90,000	0.980	2.5
100,000	0.980	2.5
150,000	0.980	2.3
200,000	0.984	2.0

For maximum loss (maximum exit angle):

$$\Delta p_L = (0.436 - 0.86\beta + 0.59\beta^2)\ \Delta p \qquad (\alpha_2 = 15^\circ) \tag{6.10}$$

For minimum loss (minimum exit angle):

$$\Delta p_L = (0.218 - 0.42\beta + 0.38\beta^2)\ \Delta p \qquad (\alpha_2 = 7^\circ) \tag{6.11}$$

Example 6.1: *Venturi Tube Flow Measurement* Benzene at 68°F flows through a horizontal rough-cast convergent Venturi tube (type 1). The inlet section has a diameter of 8 inches and the throat of 3.5 inches. The exit cone has an angle of 7°. The differential pressure is measured by a mercury manometer having benzene on top of the mercury. The mercury level in the throat leg is 4 inches above the mercury in the inlet leg. Estimate (1) the volume flow rate and (2) the overall pressure loss.

Solution

This example is solved by computing the pressure differential from the manometer levels using equation (2.8). The mass flow rate is then computed using equation (6.4). Because coefficient of discharge is a function of

Reynolds number, a trial-and-error solution is necessary. The overall pressure loss is computed using equation (6.11).

(a) Fluid properties
From Table D.1 for 68 F:

Benzene:

$$\rho_f = 54,79 \text{ lbm/ft}^3, \qquad \mu = 13.60 \times 10^{-6} \text{ lbf s/ft}^2$$

$$\gamma_f = \frac{\rho_f g}{g_c} = \frac{54.79 \times 32.17}{32.17} = 54.79 \text{ lbf/ft}^2$$

Mercury: $\rho_m = 845.67 \text{ lbm/ft}^3$

$$\gamma_m = \rho_m g/g_c = \frac{845.67 \times 32.17}{32.17} = 845.67 \text{ lbf/ft}^2$$

(b) Tube properties

$$\beta = \frac{d}{D} = \frac{3.5}{8} = 0.4375$$

$$A_d = \frac{\pi d^2}{4} = \frac{\pi(3.5/12)^2}{4} = 0.06681$$

(c) Pressure differential

$$\Delta p - (\gamma_m - \gamma_f)h_m = (845.67 - 54.79)\frac{4}{12} = 263.62 \text{ lbf/ft}^2 \qquad (2.8)$$

(d) First trial
Assume $C = 0.984$ (maximum for type 1 tubes). Note that $Y = 1$ for incompressible fluids (liquids):

$$\dot{m} = CYA_d\sqrt{\frac{2g_c\ \Delta p\ \rho_1}{1-\beta^4}}$$

$$\dot{m} = 0.984 \times 1 \times 0.06681 \times \sqrt{\frac{2 \times 32.17 \times 263.62 \times 54.79}{1 - 0.4375^4}} \qquad (6.4)$$

$$\dot{m} = 64.57 \text{ lbm/s}$$

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu g_c} = \frac{4 \times 64.57}{\pi \times (8/12) \times 13.60 \times 10^{-6} \times 32.17} = 281,866$$

From Table 6.2, $C = 0.984$ for $Re_D = 200,000$ to 6,000,000. Therefore, further trials are unnecessary.

1. Volume flow rate:

$$Q = \frac{\dot{m}}{\rho} = \frac{64.57}{54.79} = 1.178 \text{ ft}^3/\text{s}$$

2. Overall pressure loss—7 exit cone:

$$\Delta p_L = (0.218 - 0.42\beta + 0.38\beta^2)\Delta p$$

$$\Delta p_L = (0.218 - 0.42 \times 0.4375 + 0.38 \times 0.4375^2)263.52$$

$$\Delta p_L = \frac{28.91 \text{ lbf/ft}^2}{144} = 0.195 \text{ psi}$$

6.7 ASME FLOW NOZZLES

The ASME flow nozzle (Figures 6.3 and 6.4) consists of an approach portion shaped in the form of an ellipse whose major diameter is parallel to the flow, followed by a cylindrical section. Pressure differential is sensed by taps located one pipe diameter upstream of the nozzle inlet and one-half diameter downstream. The downstream taps may be located to sense the fluid

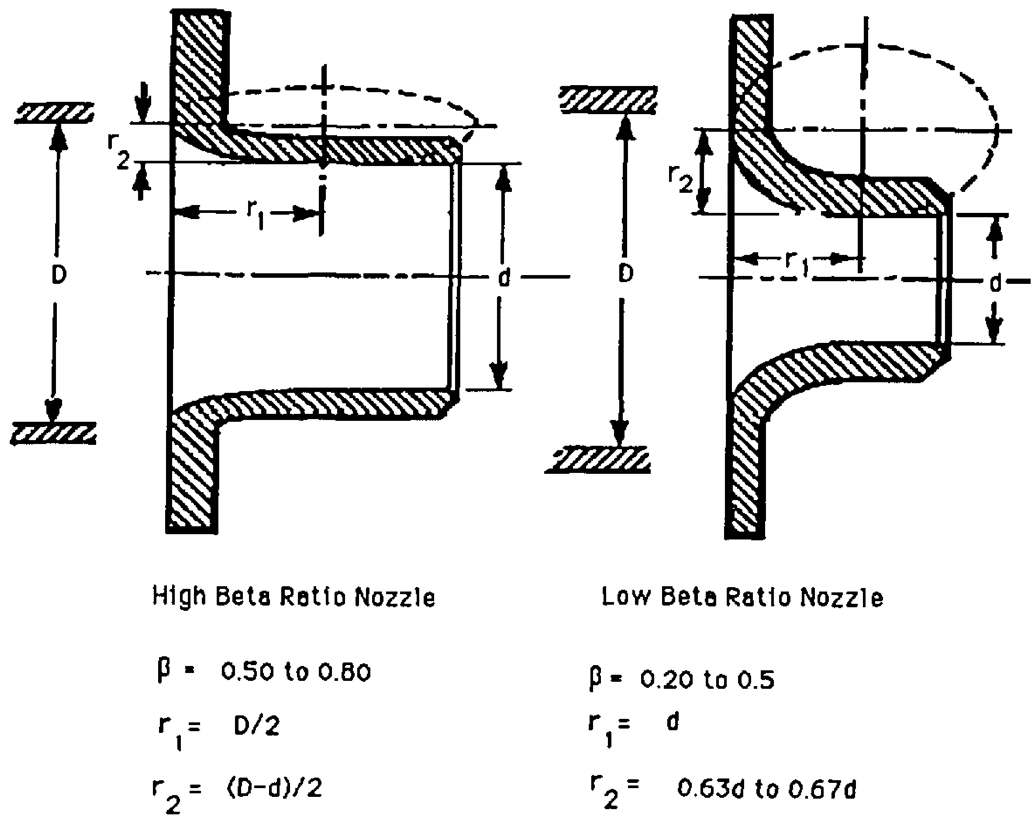
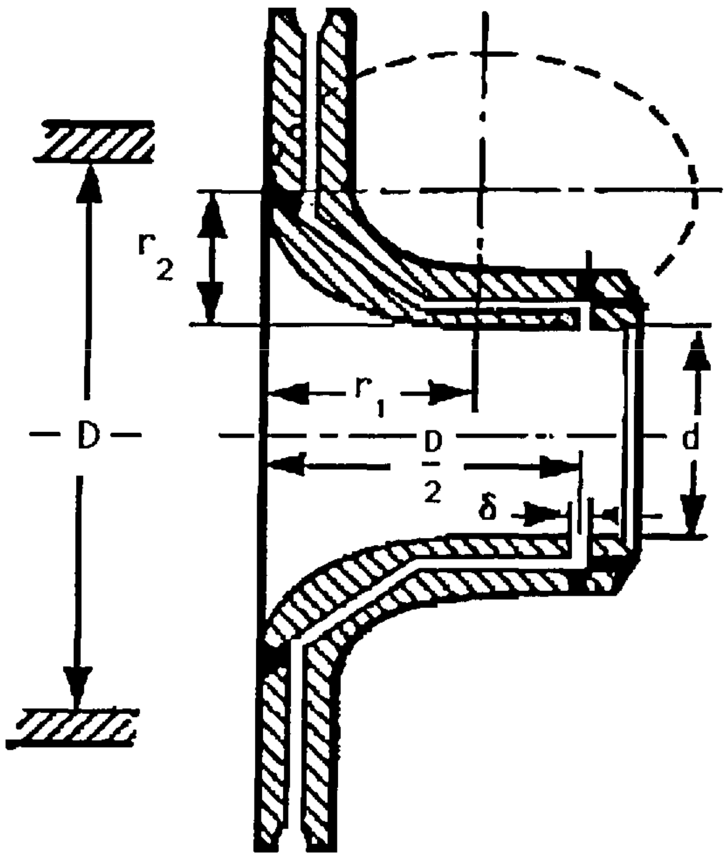


Figure 6.3 ASME wall tap flow nozzles.



$\beta = \frac{d}{D} = 0.25$ to 0.50

$r_1 = d$

$r_2 = 0.63d$ to $0.67d$

$\delta = 0.13$ in. to 0.25 in

Figure 6.4 ASME throat tap nozzles.

pressure in the cylindrical portion (throat) of the nozzle or the fluid pressure between the nozzle and the pipe wall. The former are known as "throat" taps and the latter as "pipe wall" taps.

Wall tap flow nozzles are used to measure the flow of air, water, steam, and most fluids that are not highly viscous. The throat tap flow nozzle, because of the high cost of fabrication, is used almost exclusively for the measurement of condensate flow in the testing of steam turbines.

The flow nozzles described in this section are those conforming with ASME Fluid Meters Research Committee on Fluid Meters Report (Bean, 1971), ASME Flow Measurement Standard (*Measurement of Fluid Flow*, 1989), and PTC6 "Steam Turbines" (*Performance Test Code*, 1976).

Classification of Flow Nozzles

Flow nozzles are classified into three types according to their beta ratio and the location of their pressure sensing taps.

1. *Wall tap flow nozzle (high beta ratio).* In this type of nozzle, the beta ratio ranges from 0.50 to 0.80.
2. *Wall tap flow nozzle (low beta ratio).* The beta ratio for this type of nozzle ranges from 0.2 to 0.5.
3. *Throat tap flow nozzle.* In this type of nozzle, the beta ratio ranges from 0.25 to 0.5.

Discharge Coefficients and Uncertainty

Types 1 and 2

The following equations may be used for both type 1 and type 2 nozzles:

$$C = 0.9975 - \frac{6.53}{\mathrm{Re}_d^{1/2}} \tag{6.12}$$

Equation (6.12) is valid for throat Reynolds numbers from 10,000 to 1,000,000. For higher throat Reynolds numbers, use equation (6.13) to determine the discharge coefficient:

$$C = 0.9975 - \frac{0.1035}{\mathrm{Re}_d^{1/5}} \tag{6.13}$$

The uncertainty associated with types 1 and 2 is 2.0%.

Type 3

For throat tap nozzles the following equations may be used:

$$C = 1.007 - \frac{7.376}{\mathrm{Re}_d^{1/2}} \tag{6.14}$$

Equation (6.14) is valid for throat Reynolds numbers from 10,000 to 450,000. For throat Reynolds numbers from 450,000 to 1,000,000, use equation (6.15):

$$C = 0.9953 - \frac{1073}{\mathrm{Re}_d} \tag{6.15}$$

For throat Reynolds numbers greater than 1,000,000, equation (6.16) should be used:

$$C = 1.0054 - \frac{0.186}{\mathrm{Re}_d^{1/5}}\left(1 - \frac{361,239}{\mathrm{Re}_d}\right)^{4/5} \tag{6.16}$$

The uncertainty of throat tap nozzles is about 1% for Reynolds numbers below 1,000,000; the uncertainty is 0.5% above this value for uncalibrated nozzles. When used for turbine testing according to ANSI/ASME PTC6 (*Performance Test Code*, 1976), nozzles must be calibrated and match equation (6.16) within 0.25% before they can be used.

Coefficients of discharge for all three types of nozzles are shown in Table 6.4.

Expansion Factor *Y*

The expansion factor Y for all types of flow nozzles is the adiabatic expansion factor Y developed in Section 3.4.3. Values of Y may be calculated using equation (3.120) or taken from Table 3.2.

Table 6.4 Coefficients of Discharge for ASME Flow Nozzles

Throat Reynolds number, Re_d	Wall taps C	Throat taps, C	Throat Reynolds number, Re_d	Wall taps C	Throat taps C
10,000	0.9322	0.9332	1,000,000	0.9910	0.9972
15,000	0.9442	0.9468	1,500,000	0.9915	0.9968
20,000	0.9513	0.9548	2,000,000	0.9918	0.9967
30,000	0.9598	0.9644	3,000,000	0.9923	0.9969
40,000	0.9649	0.9701	4,000,000	0.9926	0.9972
50,000	0.9683	0.9740	5,000,000	0.9928	0.9974
60,000	0.9708	0.9769	6,000,000	0.9929	0.9976
70,000	0.9728	0.9791	7,000,000	0.9931	0.9978
80,000	0.9744	0.9809	8,000,000	0.9932	0.9980
90,000	0.9757	0.9824	9,000,000	0.9933	0.9981
100,000	0.9769	0.9837	10,000,000	0.9934	0.9980
150,000	0.9806	0.9880	15,000,000	0.9937	0.9986
200,000	0.9829	0.9905	20,000,000	0.9939	0.9990
300,000	0.9856	0.9935	30,000,000	0.9942	0.9995
400,000	0.9872	0.9953	40,000,000	0.9944	0.9998
500,000	0.9883	0.9962	50,000,000	0.9945	1.0001
600,000	0.9891	0.9965	60,000,000	0.9946	1.0003
700,000	0.9897	0.9968	70,000,000	0.9947	1.0004
800,000	0.9902	0.9970	80,000,000	0.9948	1.0005
900,000	0.9906	0.9971	90,000,000	0.9948	1.0007

Pressure Loss Caused by Flow Nozzles

The pressure loss in terms of pressure developed by a flow nozzle is dependent primarily on the beta ratio. The following equation may be used to estimate this loss for all types of flow nozzles.

$$\Delta p_L = (0.218 - 0.42\beta + 0.38\beta^2)\Delta p \tag{6.17}$$

Δp_L is the pressure loss in the same units as Δp.

Example 6.2: *Design of a Flow Nozzle* An ASME wall tap flow nozzle is to be designed to meter 7,000 lbm/hr of carbon dioxide with a pressure differential equivalent to 100 inches of 68°F water. The carbon dioxide enters the nozzle at a temperature of 122°F and 20 psia through an 8-inch standard-weight steel pipe. Determine the throat diameter and the beta ratio of the nozzle.

Solution

This example is solved by trial and error. The coefficient of discharge is function of the throat Reynolds number, which cannot be computed until the throat diameter is calculated. The expansion factor also requires that the throat diameter be known. Since the coefficient of discharge is near unity, the best procedure is to assume for the first trial that $C = 1$ and that the fluid is incompressible $Y = 1$.

(a) Fluid properties
Carbon diozide: From Table D.1, $M = 44.010$ lbm/lbmol. From Table D.2 for 122°F, $\mu = 0.337 \times 10^{-6}$ lbf·s/ft^2, $k = 1.279$.

$$\rho_1 = \frac{p}{RT} = \frac{144 \times 20}{(1545/44.01) \times (122 + 460)} = 0.1410 \text{ lbm/ft}^3$$

(b) Pipe properties
Table C.3, standard steel pipe, $D = 0.6651$ ft.

(c) Pressure differential

$$\Delta p = 100 \text{ in. } H_2O \times 3.6065 \times 10^{-2} \text{ psi/in. } H_2O \times 144 = 519.34 \text{ lbf/ft}^2$$

$$p_2 = p_1 - \Delta p = 0.20 - \frac{519.34}{144} = 16.39 \text{ psia}$$

$$\frac{p_2}{p_1} = \frac{16.39}{20} = 0.8196$$

(d) First trial
Assume $C = 1$, $Y = 1$, $\beta = 0$:

$$\dot{m} = CYA_d\sqrt{\frac{2g_c\ \Delta p\ \rho_1}{1-\beta^4}}$$

$$(7000/3600) = 1 \times 1 \times A_d\sqrt{\frac{2 \times 32.17 \times 519.34 \times 0.1410}{1-0^4}}$$

$$A_d = 0.02832\ \text{ft}^2 \tag{6.4}$$

$$d = \sqrt{\frac{4A_d}{\pi}} = \sqrt{\frac{4 \times 0.02832}{\pi}} = 0.1899\ \text{ft}$$

$$\beta_1 = \frac{0.1899}{0.6651} = 0.2855$$

Expansion factor: From Table 3.2 at $k = 1.279$, $\beta = 0.288$, $p_2/p_1 = 0.8196$:

$Y_1 = 0.8879$ (interpolated)

Throat Reynolds number:

$$\text{Re}_{d1} = \frac{4\dot{m}}{\pi d \mu g_c} = \frac{4 \times (7000/3600)}{\pi \times 0.1899 \times 0.337 \times 10^{-6} \times 32.17} = 1{,}202{,}500$$

From Table 6.4 at $\text{Re}_{d1} = 1,202,500$:

$C_1 = 0.9912$ (interpolated)

(e) Second trial
Assume $C = 0.9912$, $Y = 0.8879$ $\beta = 0.2855$:

$d_2 = 0.2023$ ft, $\beta_2 = 0.3042$, $\text{Re}_{d2} = 1,281,000$

$Y_2 = 0.8878$, $C_2 = 0.9913$

Note that the differences between the assumed and calculated values of C and Y are in the fourth decimal place, and further trials are unnecessary.

Therefore, $d = 0.2032 \times 12 = 2.44$ in. and $\beta = 0.3055$.

Example 6.3. *Calibration of a Throat Tap Nozzle* Calibration data for an ASME throat tap nozzle are as follows:

Fluid	water
Pipe inside diameter, inches	14.9900
Throat diameter, inches	7.4990
Scale weight, pounds	190.179
Time for weight, seconds	531.995
Differential reading, inches of mercury	4.956
Manometer temperature, F	79
Water temperature, °F	130

Does this calibration meet the PTC6 requirement that the coefficient be within ±0.25% of the value calculated using equation (6.16)?

Solution

This example is solved by calculating the coefficient of discharge from the calibration data and matching it with the coefficient obtained from equation (6.16).

(a) Fluid properties
Table D.1 (interpolated values):

Water at 79°F: $\rho_f = 62.2 \text{ lbm/ft}^3$

$$\gamma_f = \frac{\rho_f g}{g_c} = \frac{62.21 \times 32.17}{32.17} = 62.21 \text{ lbf/ft}^3$$

Water at 130°F: $\rho_1 = 61.55 \text{ lbm/ft}^3$, $\mu = 10.63 \times 10^{-6} \text{ lbf}\cdot\text{s/ft}^2$
Mercury at 79°F: $\rho_m = 844.67 \text{ lbm/ft}^3$

$$\gamma_m = \frac{\rho_m g}{g_c} = \frac{844.67 \times 32.17}{32.17} = 844.67 \text{ lbf/ft}^3$$

(b) Nozzle properties

$$\beta = \frac{d}{D} = \frac{7.4990}{14.990} = 0.5003$$

$$A_d = \frac{\pi d^2}{4} = \pi \frac{(7.4990/12)^2}{4} = 0.3067 \text{ ft}^2$$

(c) Pressure differential

$$\Delta p = (\gamma_m - \gamma_f)h_m = (844.67 - 62.21)\left(\frac{4.956}{12}\right) = 323.16 \text{ lbf/ft}^2 \qquad (2.8)$$

(d) Mass flow rate

$$\dot{m} = \frac{\text{mass}}{\text{time}} = \frac{190,179}{531.995} = 357.48 \text{ lbm/s}$$

(e) Throat Reynolds number

$$\text{Re}_d = \frac{4\dot{m}}{\pi d \mu g_c} = \frac{4 \times 357.48}{\pi \times (7.4990/12) \times 10.63 \times 10^{-6} \times 32.17} = 2,130,000$$

(f) Calibrated coefficient of discharge

$$C = \frac{\dot{m}}{YA_d}\sqrt{\frac{1-\beta^4}{2g_c\ \Delta p\ \rho_1}} = \frac{357.48}{1 \times 0.3067}\sqrt{\frac{1-0.5003^4}{2 \times 32.17 \times 323.16 \times 61.55}} = 0.9975$$

(g) PTC6 coefficient

$$C = 1.0054 - \frac{0.185}{\text{Re}_d^{1/5}}\left(1 - \frac{361,239}{\text{Re}_d}\right)^{4/5}$$

$$C = 1.0054 - \frac{0.185}{2,130,000^{1/5}}\left(1 - \frac{361,239}{2,130,000}\right)^{4/5}$$

$$C = 0.9975$$

(h) Meets requirement of 0.25%:

$$\text{difference} = \left[\frac{0.9968 - 0.9975}{0.9968}\right] \times 100 = 0.07\%$$

6.8 ASME ORIFICE METERS

The ASME thin-plate sharp-edged concentric orifice is simply a circular plate with a hole bored in its center. When fluid forces require that the plate be thick, the hole is beveled on the downstream side at an angle of 45° so that the cylindrical portion is between one-tenth and one-eighth of the bore diameter ratio.

Figure 6.5 shows the relative pressure difference due to the presence of the orifice plate. Because the location of the pressure taps is critical, it is necessary to specify the exact position of the upstream and downstream pressure taps. The jet contraction amounts to about 60% of the orifice area, so orifice coefficients are on the order of 0.6, compared with nearly unity obtained with Venturi tubes and flow nozzles.

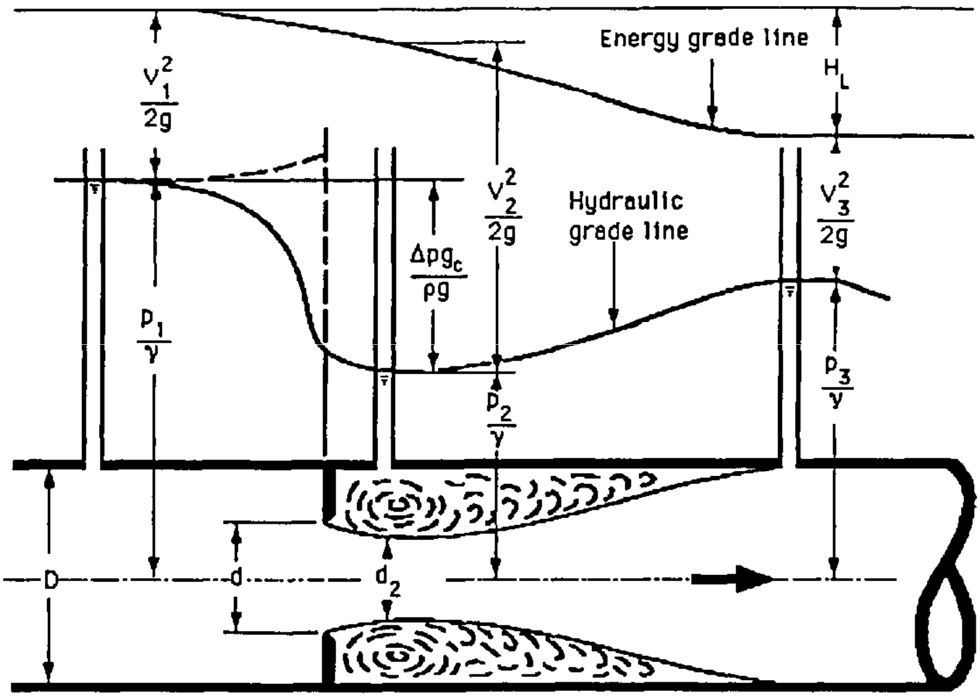


Figure 6.5 Pressure gradients for flow through an orifice plate.

The orifices described in this section are those conforming with ASME Fluid Meters Research Committee on Fluid Meters Report (Bean, 1971) and the ASME Flow Measurement Standard (*Measurement of Fluid Flow*, 1989).

Standard Tap Locations

Three tap locations are specified by the ASME for measuring pressure differential: flange taps, *D* and *D*/2 taps, and corner taps.

1. *Flange taps*. These taps are always located 1 inch from either face of the orifice plate, regardless of the size of the pipe. Flange taps are used because they can be prefabricated and because flanges with the holes drilled at the correct locations may be purchased as off-the-shelf items, thus saving the cost of field fabrication. The disadvantages of this type is that they are not symmetrical with respect to pipe size and discharge coefficients vary with pipe size.

2. D *and* D/2 *taps*. The upstream tap is located one pipe diameter from the inlet face of the orifice plate and the downstream tap one-half pipe diameter from the inlet face of the orifice plate. The downstream tap is located so that it essentially senses the minimum pressure and thus produces the maximum differential pressure. *D* and *D*/2 taps are symmetrical, and

their discharge coefficients are independent of pipe size. The disadvantage of this type of tap is the cost of installation.

3. *Corner taps.* As the name suggests, the upstream tap is located at the inlet corner of the plate and the downstream at the outlet corner. These taps are symmetrical, and the coefficient of discharge is independent of pipe size.

Discharge Coefficients

The general equation for the coefficient of discharge for an ASME orifice is as follows:

$$C = 0.5959 + 0.031\beta^{2.1} - 0.1840\beta^{8} + \frac{91.71\beta^{5/2}}{\mathrm{Re}_D^{3/4}} + \frac{0.0900L_1\beta^4}{1-\beta^4} - 0.0337L_2\beta^3 \tag{6.18}$$

where

$$\begin{aligned} L_1 &= \text{dimensionless correction for upstream tap location} \\ &= \frac{L_1}{D}\text{ measured from the upstream face} \\ L_2 &= \text{dimensionless correction for downstream tap location} \\ &= \frac{L_2}{D}\text{ when measured from the upstream face} \\ &= \frac{L_2 - \text{plate thickness}}{D}\text{ when measured from the downstream face} \end{aligned}$$

The general equation can be written in the following form:

$$C = C_\infty + \Delta C_{\mathrm{tap}} + \Delta C_{\mathrm{Reynolds}} \tag{6.19}$$

where

$$\begin{aligned} C_\infty &= \text{coefficient of discharge at Reynolds number of infinity} \\ &= 0.5959 + 0.031\beta^{2.1} - 0.1840\beta^{8} \end{aligned} \tag{6.20}$$

$$\Delta C_{\mathrm{tap}} = \text{correction for tap location} = \frac{0.0900L_1\beta^4}{1-\beta^4} - 0.0337L_2\beta^3 \tag{6.21}$$

$$\Delta C_{\mathrm{Reynolds}} = \text{correction for Reynolds number} = \frac{91.71\beta^{5/2}}{\mathrm{Re}_D^{3/4}} \tag{6.22}$$

Corner taps

$L_1 = L_2 = 0$, $\Delta C_{tap} = 0$, so equation (6.19) reduces to the following:

$$C_{corner} = C_{\infty} + \Delta C_{Reynolds} \tag{6.23}$$

D and D/2 Taps

$L_1 = 0.4333$ and $L_2 = 0.47$, so equation (6.21) becomes the following:

$$\begin{aligned} \Delta C_{tap} &= \frac{0.0900 L_1 \beta^4}{1-\beta^4} - 0.0337 L_2 \beta^3 \\ \Delta C_{tap(D+D/2)} &= \frac{0.0390\beta^4}{1-\beta^4} - 0.01584\beta^3 \end{aligned} \tag{6.24}$$

Equation (6.19) becomes the following:

$$C_{D+D/2} = C_{\infty} + \Delta C_{tap} + \Delta C_{Reynolds} \tag{6.25}$$

Flange Taps

Because flange taps are not symmetrical, the coefficient equations are divided into two groups, one for smaller pipes (internal diameters between 2 and 2.3 in.) and the other for pipes of internal diameter equal to or greater than 2.3 in.

For pipes with internal diameters between 2 and 2.3 in., $L_1 = 0.4333$ and $L_2 = 1/D$ (D is in inches). Equation (6.21) becomes the following:

$$\begin{aligned} \Delta C_{tap} &= \frac{0.0900 L_1 \beta^4}{1-\beta^4} - 0.0337 L_2 \beta^3 \\ \Delta C_{tap(flange)} &= \frac{0.0900\beta^4}{1-\beta^4} - \frac{0.0337\beta^3}{D} \qquad (D = 2\text{--}2.3 \text{ in.}) \end{aligned} \tag{6.26}$$

For pipes with internal diameters 2.3 in. and larger, $L_1 = L_2 = 1/D$ (D is in inches) and equation (6.21) becomes the following:

$$\begin{aligned} \Delta C_{tap} &= \frac{0.0900 L_1 \beta^4}{1-\beta^4} - 0.0337 L_2 \beta^3 \\ \Delta C_{tap(flange)} &= \frac{0.0900\beta^4}{1-\beta^4} - \frac{0.0337\beta^3}{D} \qquad (D \geq 2.3 \text{ in.}) \end{aligned} \tag{6.27}$$

Equation (6.19) becomes the following for flange taps (all pipe sizes):

$$C_{flange} = C_{\infty} + \Delta C_{tap(flange)} + \Delta C_{Reynolds} \tag{6.28}$$

Table 6.5 gives values for the computation of equation (6.19).

Table 6.5 Coefficients of Discharge for ASME Orifices

Pipe internal diameter, in.	Beta ratio 0.20	0.30	0.40	0.50	0.60	0.70	0.75
	Coefficient of discharge at Reynolds number of infinity– C_∞						
All	0.5970	0.5984	0.6003	0.6024	0.6034	0.6000	0.5944
	Tap correction for $D + D/2$ taps $\Delta C_{\text{tap}}(D + D/2)$						
All	−0.0001	−0.0001	0.0000	0.0006	0.0024	0.0069	0.0114
	Tap correction for flange taps– $\Delta C_{\text{tap(flange)}}$						
2	−0.0001	−0.0001	−0.0001	0.0005	0.0022	0.0065	0.0109
2.3	0.0000	−0.0001	0.0001	0.0008	0.0026	0.0073	0.0119
3	0.0000	−0.0001	0.0001	0.0006	0.0020	0.0056	0.0091
4	0.0000	0.0000	0.0001	0.0004	0.0015	0.0042	0.0069
6	0.0000	0.0000	0.0000	0.0003	0.0010	0.0028	0.0046
8	0.0000	0.0000	0.0000	0.0002	0.0008	0.0021	0.0034
12	0.0000	0.0000	0.0000	0.0001	0.0005	0.0014	0.0023
24	0.0000	0.0000	0.0000	0.0001	0.0003	0.0007	0.0011
48	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0006
96	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003

Pipe Reynolds number, RE_D	Beta ratio 0.20	0.30	0.40	0.50	0.60	0.70	0.75
	Reynolds number correction – $\Delta C_{\text{Reynolds}}$						
2,000	0.0055	0.0151	0.0310	0.0542	0.0855	0.1257	0.1494
3,000	0.0040	0.0112	0.0229	0.0400	0.0631	0.0928	0.1102
4,000	0.0033	0.0090	0.0185	0.0322	0.0508	0.0748	0.0888
5,000	0.0028	0.0076	0.0156	0.0273	0.0430	0.0632	0.0751
6,000	0.0024	0.0066	0.0136	0.0238	0.0375	0.0552	0.0655
8,000	0.0019	0.0053	0.0110	0.0192	0.0302	0.0444	0.0528
10,000	0.0016	0.0045	0.0093	0.0162	0.0256	0.0376	0.0447
20,000	0.0010	0.0027	0.0055	0.0096	0.0152	0.0224	0.0266
40,000	0.0006	0.0016	0.0033	0.0057	0.0090	0.0133	0.0158

Table 6.5 (continued) Coefficients of Discharge for ASME Orifices

Pipe Reynolds number, RE_D	Beta ratio 0.20	0.30	0.40	0.50	0.60	0.70	0.75
	Reynolds number correction $\Delta C_{\text{Reynolds}}$ (*continued*)						
60,000	0.0004	0.0012	0.0024	0.0042	0.0067	0.0098	0.0117
80,000	0,0003	0.0010	0.0020	0.0034	0.0054	0.0079	0.0094
100,000	0.0003	0.0008	0.0017	0.0029	0.0045	0.0067	0.0079
200,000	0.0002	0.0005	0.0010	0.0017	0.0027	0.0040	0.0047
400,000	0.0001	0.0003	0.0006	0.0010	0.0016	0.0024	0.0028
600,000	0.0001	0.0002	0.0004	0.0008	0.0012	0.0017	0.0021
800,000	0.0001	0.0002	0.0003	0.0006	0.0010	0.0014	0.0017
1,000,000	0.0001	0.0001	0.0003	0.0005	0.0008	0.0012	0.0014
2,000,000	0.0000	0.0001	0.0002	0.0003	0.0005	0.0007	0.0008
4,000,000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0004	0.0005
6,000,000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0004
8,000,000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003
10×10^6	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0003
20×10^6	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
40×10^6	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
80×10^6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
10×10^7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Uncertainty for Orifice Meters

For pipe Reynolds numbers from 2,000 to 10,000 the uncertainty is equal to $(0.6 + \beta)\%$ for beta ratios between 0.20 and 0.75%. For pipe Reynolds numbers from 10,000 to 100,000,000, the uncertainty is 0.6% for beta ratios from 0.20 to 0.6 and equal to $\beta\%$ for higher beta ratios.

Expansion Factor *Y*

As shown in Figure 6.5, the minimum flow area for an orifice is located downstream of the orifice. The stream of a compressible fluid is not restrained as it leaves the orifice throat and is free to expand transversely and longitudinally to the point of minimum flow area. Thus, the maximum contraction of the jet will be less for a compressible fluid than for a liquid. Because of this, the adiabatic expansion factor determined from equation

(3.120) or Table 3.2 may not be used with orifices. Neither may the critical pressure ratio equation, equation (3.115), be used, because the phenomenon of critical flow has not been observed during testing of orifice meters.

For orifice meters, equation (6.29), which is based on experimental data, is used to determine the expansion factor:

$$Y = 1 - (0.41 + 0.35\beta^4)\frac{\Delta p}{kp_1} \tag{6.29}$$

Pressure Loss Caused by Orifice Meter

The pressure loss in terms of pressure developed by an orifice is dependent primarily on the beta ratio and the coefficient of discharge. The following equation may be used to estimate this loss for all orifices:

$$\Delta p_L = \left(\frac{\sqrt{1-\beta^4} - C\beta^2}{\sqrt{1-\beta^4} + C\beta^2}\right)\Delta p \tag{6.30}$$

Δp_L is the pressure loss in the same units as Δp.

Example 6.4: *ASME Orifice Meter Coefficients* Air at 122 F and 150 psia flows in a 10-in. schedule 40 steel pipe at a volumetric flow rate of 3,575 ft^3/min. An ASME orifice whose throat diameter is 7.5 in. is to be installed to meter this flow with a readout device that has a maximum indication of 150 in. of water at 68°F. What standard tap location should be used to match this requirement most nearly?

Solution

This example may be solved directly by the application of equation (6.4) and related coefficient and expansion factor equations.

(a) Fluid properties—air at 122°F
From Table D.1, $M = 28.97$ lbm/lbmol. From Table D.2, $\mu = 0.410 \times 10^{-6}$ lbf·s/ft^2, $k = 1.401$:

$$\rho = \frac{p}{RT} = \frac{144 \times 150}{(1545/28.97) \times (122 + 460)} = 0.6959\ \text{lbm/ft}^3$$

(b) Pipe properties
From Table C.3, for 10-in. schedule 40 pipe:

$$D = 0.8350\ \text{ft} = 0.8350 \times 12 = 10.02\ \text{inches}$$

(c) Orifice properties

$$d = \frac{7.5}{12} = 0.625 \text{ ft}$$

$$\beta = \frac{d}{D} = 0.625/0.8350 = 0.7485$$

$$A_d = \frac{\pi d^2}{4} = \frac{\pi(0.625)^2}{4} = 0.3068 \text{ ft}^2$$

(d) Pressure differential

1 inch of water at 68°F = 3.6065×10^{-2}psi

$$\Delta p = 150 \times 3.6065 \times 10^{-2} \times 144 = 778.8 \text{ lbf/ft}^2$$

$$\frac{\Delta p}{p_1} = \frac{778.8}{150 \times 144} = 0.03606$$

(e) Mass flow rate

$$\dot{m} = \rho Q = 0.6959 \times \left(\frac{3575}{60}\right) = 41.46 \text{ lbm/s}$$

(f) Pipe Reynolds number

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu g_c} = \frac{4 \times 41.46}{\pi \times 0.8350 \times 0.410 \times 10^{-6} \times 32.17} = 4,793,000$$

(g) Expansion factor Y

$$Y = 1 - (0.41 + 0.35\beta^4)\frac{\Delta p}{kp_1}$$

$$Y = 1 - (0.41 + 0.35 \times 0.7485^4) \times \frac{778.8}{1.401 \times (150 \times 144)} \tag{6.29}$$

$$Y = 0.9866$$

(h) Required coefficient of discharge

$$\dot{m} = CYA_d\sqrt{\frac{2g_c\,\Delta p\,\rho_1}{1-\beta^4}}$$

$$41.46 = C \times 0.9866 \times 0.3068 \times \sqrt{\frac{2 \times 32.17 \times 778.8 \times 0.6959}{1 - 0.7485^4}} \tag{6.4}$$

$$C = 0.6072$$

(i) From Table 6.5 at $\beta \approx 0.75$, $D \approx 10$ in, $\text{Re}_D \approx 5{,}000{,}000$:

	Corner	$D+D/2$	Flange
C_∞	0.5944	0.5944	0.5944
C_{tap}	-	0.0116	0.0029
C_{Reynolds}	0.0005	0.0005	0.0005
Σ	0.5949	0.6063	0.5978

The $D + D/2$ tap coefficient of 0.6063 is closest to the calculated value of C, with an error of 0.15%. The flange tap, with a coefficient of 0.5978, is next, with an error of 1.54%. The corner tap is last, with a coefficient of 0.5949 and an error of 2.03%. The coefficient tolerance is 0.75%, making the choice of the $D + D/2$ taps significant.

6.9 ELBOW FLOW METERS

Although elbows have been used since the turn of the century for flow measurement, they have never been standardized by any technical society. Most of the information contained in this section is obtained from the ASME Fluid Meters Report (Bean, 1971) and from Murdock et al. (1964). Elbows are commercially manufactured to change flow direction, and standards relate to external dimensions. There is considerable difference in internal dimensions among various manufactuers for elbows that will meet the same specifications. *For this reason, elbows used for flow measurement must be carefully measured.*

Derivation of Flow Equation

It can be shown (see, for example, Murdock et al., 1964) that the velocity in a bend (see Figure 6.6) is related to the pressure drop as follows:

$$v = \sqrt{\frac{r_o r_i}{2(r_o - r_i)}}\sqrt{\frac{(p_o - p_i)g_c}{\rho}} = \sqrt{\frac{R}{2D}}\sqrt{\frac{2\ \Delta p\ g_c}{\rho}} \tag{6.31}$$

Application of the continuity equation, $\dot{m} = \rho A v$, results in the following:

$$\dot{m}_{\text{ideal}} = A_D\sqrt{\frac{R}{2D}}\sqrt{2\ \Delta p\ g_c \rho} \tag{6.32}$$

For actual flow conditions, equation (6.32) becomes the following:

$$\dot{m} = CYA_D\sqrt{\frac{R}{2D}}\sqrt{2\ \Delta p\ g_c \rho} \tag{6.33}$$

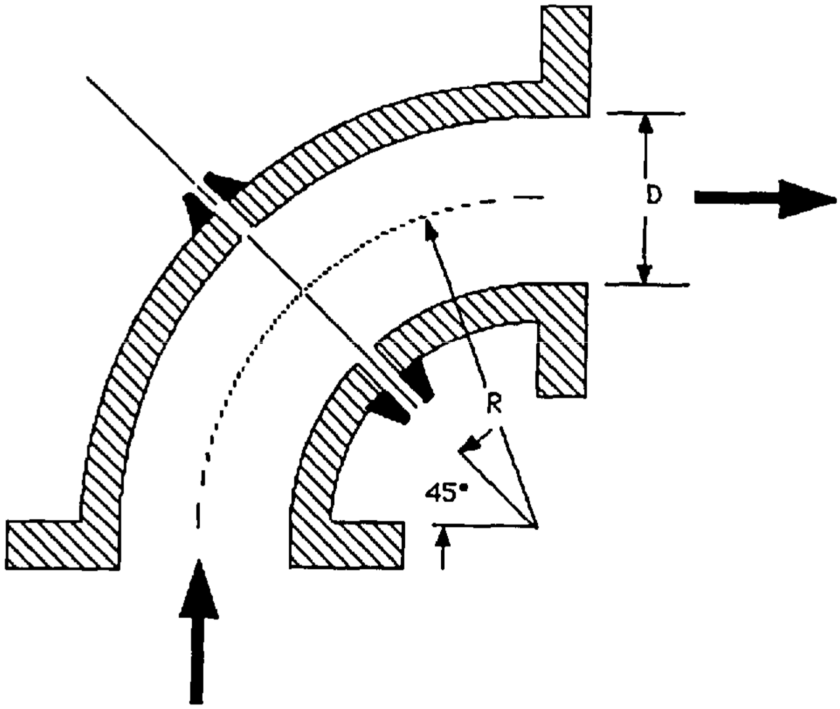


Figure 6.6 Elbow flow meter.

Discharge Coefficient and Uncertainty

The variation of discharge coefficient C with pipe Reynolds number is as follows:

$$C = 1 - \frac{6.5}{Re_D} \tag{6.34}$$

Coefficients of discharge calculated using equation (6.34) are shown in Table 6.6.

The uncertainty is 4% subject to the following limits:

Pipe Reynolds number Re_D from 10,000 to 1,000,000
R/D greater than or equal to 1.25

With calibration, the uncertainty is the same as for other devices, and the repeatability of these devices is excellent.

Expansion Factor Y

The expansion factor Y has not been established, thus limiting the use of elbow flow meters to incompressible fluids.

Table 6.6 Coefficients of Discharge for Elbow Flow Meters

Pipe Reynolds number, Re_D	Discharge coefficient, C	Pipe Reynolds number, Re_D	Discharge coefficient, C	Pipe Reynolds number, Re_D	Discharge coefficient C
10,000	0.9350	50,000	0.9709	240,000	0.9867
11,000	0.9380	55,000	0.9723	260,000	0.9873
12,000	0.9407	60,000	0.9735	280,000	0.9877
13,000	0.9430	65,000	0.9745	300,000	0.9881
14,000	0.9451	70,000	0.9754	320,000	0.9885
15,000	0.9469	75,000	0.9763	340,000	0.9889
16,000	0.9486	80,000	0.9770	360,000	0.9892
17,000	0.9501	85,000	0.9777	380,000	0.9895
18,000	0.9516	90,000	0.9783	400,000	0.9897
19,000	0.9528	95,000	0.9989	450,000	0.9903
20,000	0.9540	100,000	0.9794	500,000	0.9908
22,000	0.9562	110,000	0.9804	550,000	0.9912
24,000	0.9580	120,000	0.9812	600,000	0.9916
26,000	0.9597	130,000	0.9820	650,000	0.9919
28,000	0.9612	140,000	0.9826	700,000	0.9922
30,000	0.9625	150,000	0.9832	750,000	0.9925
32,000	0.9637	160,000	0.9838	800,000	0.9927
34,000	0.9647	170,000	0.9842	850,000	0.9929
36,000	0.9657	180,000	0.9847	900,000	0.9931
38,000	0.9667	190,000	0.9851	1,000,000	0.9933
40,000	0.9675	200,000	0.9855		
45,000	0.9694	220,000	0.9861		

Pressure Loss Caused by Elbow Flow Meters

There is no additional pressure loss if the elbow is required in the piping system.

Example 6.5: *Elbow Flow Meter calculation* Water at 83°F and atmospheric pressure flows through an elbow flow meter. The measured radius of curvature is 2.81 in., and the internal diameter is 2.24 in. Determine the mass flow rate in lbm/hr if the differential pressure produced is 50.6 in. of water at 68°F.

Solution

This example requires a trial-and-error solution, because the coefficient of discharge is a function of pipe Reynolds number, which requires that the flow rate be known. The number of iterations is reduced by assuming the initial pipe Reynolds number to be infinity.

(a) Fluid properties
From Table D.1 for water at 83°F:

$$\rho_1 = 62.18 \text{ lbm/ft}^3, \qquad \mu = 17.31 \times 10^{-6} \text{ lbf·s/ft}^2$$

(b) Meter properties

$$\left(\frac{R}{2D}\right)^{1/2} = \left[\frac{2.81}{(2 \times 2.24)}\right]^{1/2} = 0.7920$$

$$A_D = \frac{\pi D^2}{4} = \frac{\pi(2.24/12)^2}{4} = 0.02737 \text{ ft}^2$$

(c) Pressure differential

$$1 \text{ in. of water at } 68^\circ\text{F} = 3.6065 \times 10^{-2} \text{ psi}$$

$$\Delta p = 50.6 \times 3.6065 \times 10^{-2} \times 144 = 262.8 \text{ lbf/ft}^2$$

(d) First trial calculation
Assume $\text{Re}_c = \infty$. Then from equation (6.34), $C = 1$ and the mass flow rate is as follows:

$$\dot{m} = CYA_D\sqrt{\frac{R}{2D}}\sqrt{2\ \Delta p\ g_c\rho} \qquad (6.33)$$

$$\dot{m} = 1 \times 1 \times 0.2737 \times 0.7920\sqrt{2 \times 262.8 \times 32.17 \times 62.18}$$

$$\dot{m} = 22.23 \text{ lbm/s}$$

Reynolds number:

$$\text{Re}_{D1} = \frac{4\dot{m}}{\pi D \mu g_c} = \frac{4 \times 22.23}{\pi \times (2.24/12) \times 17.31 \times 10^{-6} \times 32.17} = 272{,}300$$

From Table 6.6 at $\text{Re}_D = 272{,}300$, $C = 0.9875$:

$$\dot{m} = 0.9875 \times 22.23 \times 3600 = 79.028 \text{ lbm/hr}$$

Further trials did not change the result.

Note, this example was taken from an actual calibration. The flow measured by weigh tanks was 77.217 lbm/hr. The calculated flow is 2.3% higher in this example.

6.10 PITOT TUBES

A pitot tube is a device shaped to sense stagnation pressure. The first description of a tube used for measurement of stagnation pressure for the determination of velocity was given by Henri Pitot in 1732. The name "Pitot tube" has been applied to two general classifications of instruments. The first is a tube that measures the impact or stagnation pressures only. The second is a combined tube that measures both impact and static pressures with a single primary instrument. The combined sensor is also called a Pitot-static tube.

Figure 6.7 shows an impact tube located at radius r from the centerline of a circular pipe to sense the stagnation pressure p_o produced by the streamline velocity U. The static pressure p_s is sensed by the wall tap. The differential, $p_o - p_s = \Delta p$ is then calculated from the depression of the manometer.

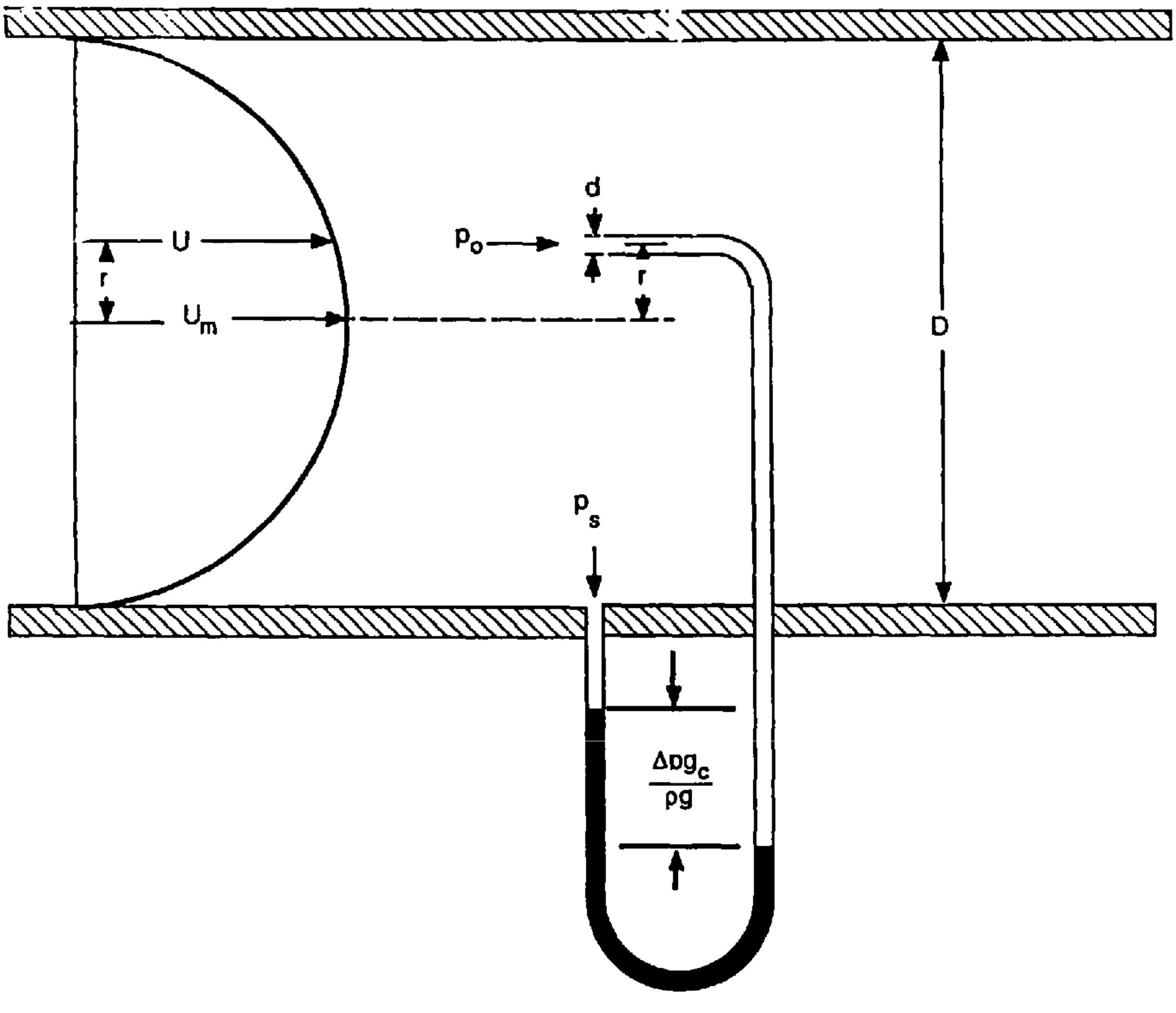


Figure 6.7 Notation for Pitot tube study.

If the equation of motion [equation (3.18)] is written for horizontal frictionless flow, the result is as follows:

$$\frac{U\,du}{g_c} + \frac{dp}{\rho} = 0 \tag{6.35}$$

Incompressible Flow

Integrating equation (6.35) for constant density and noting that by definition the stagnation velocity is 0 yields the following:

$$\int_{U_{\text{ideal}}}^{0} \frac{U\,du}{g_c} + \frac{1}{\rho}\int_{p_s}^{p_o} dp = \frac{0^2 - U_{\text{ideal}}^2}{2g_c} + \frac{p_o - p_s}{\rho} = 0$$

or

$$U_{\text{ideal}} = \sqrt{\frac{2g_c\,\Delta p}{\rho}} \tag{6.36}$$

Conventional practice is to define the Pitot tube coefficient C_{Pitot} as follows:

$$C_{\text{Pitot}} = \frac{\text{actual streamtube velocity}}{\text{ideal streamtube velocity}} = \frac{U}{U_{\text{ideal}}}$$

or

$$U = C_{\text{Pitot}}\sqrt{\frac{2g_c\,\Delta p}{\rho}} \tag{6.37}$$

Compressible Flow

The stagnation process as U is reduced to 0 if the impact tube is assumed to be frictionless and adiabatic: The path of such a process, from equation (1.33), is $pv^k = p_s v_s^k = p_o v_o^k$. From equation (1.15), $v = 1/\rho$; therefore, the following can be written:

$$\frac{p}{\rho^k} = \frac{p_o}{\rho_o^k} = \frac{p_s}{\rho_s^k} \qquad \text{or} \qquad \rho = \rho_o\left(\frac{p}{p_o}\right)^{1/k} = \rho_s\left(\frac{p}{p_s}\right)^{1/k} \tag{6.38}$$

Substituting equation (6.38) in the second term of equation (6.35) and integrating yields the following:

$$\int_{p_s}^{p_o} \frac{dp}{\rho} = \frac{p_s^{1/k}}{\rho_s}\int_{p_s}^{p_o} \frac{dp}{p} = \left(\frac{k}{k-1}\right)\left(\frac{p_s}{\rho_s}\right)\left[\left(\frac{p_o}{p_s}\right)^{k-1/k} - 1\right] \tag{6.39}$$

The adiabatic expansion factor Y_{Pitot} is defined as follows:

$$Y_{\text{Pitot}} = \frac{\text{ideal compressible velocity}}{\text{ideal incompressible velocity}} \tag{6.40}$$

Based on this definition, Y_{Pitot} is the square root of the ratio of the integration of the second term of equation (6.35), or the following:

$$Y_{\text{Pitot}} = \sqrt{\frac{\left(\dfrac{k}{k-1}\right)\left(\dfrac{p_s}{\rho_s}\right)\left[\left(\dfrac{p_o}{p_s}\right)^{k-1/k} - 1\right]}{\dfrac{p_o - p_s}{\rho_s}}} \tag{6.41}$$

$$Y_{\text{Pitot}} = \sqrt{\frac{\left(\dfrac{k}{k-1}\right)[(p_o/p_s)^{k-1/k} - 1]}{(p_o/p_s) - 1}}$$

Table 6.7 contains values of the adiabatic compression factor Y_{Pitot} as a function of pressure and specific heat ratios.

Since the Pitot tube can sense only a streamtube velocity profile, a coefficient, C_{profile}, is needed and is defined as follows:

$$C_{\text{profile}} = \frac{\text{average velocity}}{\text{streamtube velocity}} = \frac{v}{U} \tag{6.42}$$

Combining equations (6.37), (6.40), (6.41), and (6.42) with the continuity equation results in the following expression for mass flow rate:

$$\dot{m} = \rho A v = C_{\text{Pitot}} C_{\text{profile}} Y_{\text{Pitot}} A_D \sqrt{2g_c \, \Delta p \, \rho_s} \tag{6.43}$$

Velocity Profiles

The best way to determine the velocity profile in a pipe is to make a Pitot tube traverse. If it is impractical to make a traverse, then the Pitot tube should be located on the pipe centerline to sense the maximum velocity. The relationship for the centerline velocity can then be calculated from the following equations:

Laminar Flow

In Section 4.6, the velocity profile for laminar flow was shown to be parabolic, and from equation (4.18) $U_m = 2v$:

$$C_{\text{profile}} - \frac{v}{U} = \frac{v}{U_m} - \frac{1}{2} \qquad \text{(laminar flow)} \tag{6.44}$$

Table 6.7 Adiabatic Compression Factor, Y_{Pitot}

Pressure ratio, p_o/p_s	Ratio of specific heats, $k(= c_p/c_v)$					
	1.1	1.2	1.3	1.4	1.5	5/3
1.01	0.9977	0.9979	0.9981	0.9982	0.9983	0.9985
1.02	0.9955	0.9959	0.9962	0.9965	0.9967	0.9970
1.03	0.9933	0.9938	0.9943	0.9947	0.9951	0.9956
1.04	0.9911	0.9918	0.9925	0.9930	0.9935	0.9941
1.05	0.9989	0.9898	0.9906	0.9913	0.9919	0.9927
1.06	0.9868	0.9879	0.9888	0.9896	0.9903	0.9912
1.07	0.9846	0.9859	0.9870	0.9879	0.9887	0.9898
1.08	0.9825	0.9840	0.9852	0.9862	0.9871	0.9884
1.09	0.9805	0.9821	0.9834	0.9846	0.9856	0.9870
1.10	0.9784	0.9802	0.9817	0.9830	0.9841	0.9856
1.20	0.9588	0.9621	0.9649	0.9674	0.9695	0.9725
1.30	0.9408	0.9455	0.9495	0.9530	0.9560	0.9603
1.40	0.9242	0.9302	0.9353	0.9396	0.9435	0.9489
1.50	0.9089	0.9159	0.9220	0.9272	0.9318	0.9383
1.60	0.8946	0.9027	0.9096	0.9156	0.9209	0.9283
1.70	0.8813	0.8903	0.8980	0.9047	0.9106	0.9190
1.80			0.8871	0.8944	0.9009	0.9101
1.90				0.8847	0.8917	0.9017
2.00					0.8830	0.8937

Turbulent Flow (Smooth Pipes)

An empirical relation for turbulent flow velocity distribution in smooth circular pipes known as the "law of the wall" is as follows:

$$\frac{u}{U_m} = \left(1 - \frac{r}{r_o}\right)^a \tag{6.45}$$

C_{profile} can be expressed for smooth pipe turbulent flow as follows:

$$C_{\text{profile}} = \frac{v}{U} = \frac{v}{U_m} = \frac{2}{(a+1)(2a+1)} \quad \text{(turbulent flow, smooth pipes)} \tag{6.46}$$

Values of a may be estimated using equation (6.46) as follows:

$$a = 0.2463 - 2.287 \log_{10} \text{Re}_D \tag{6.47}$$

Turbulent Flow (Rough Pipes)

For rough pipes, the following relationship is widely used to express turbulent flow velocity profiles:

$$\frac{v}{U} = \left[1 + 1.326\sqrt{f} - 2.04\sqrt{f}\ \log_{10}\left(\frac{r_o}{r_o - r}\right)\right]^{-1} \tag{6.48}$$

Values of the friction factor, f, may be calculated from equation (4.25).

At the pipe centerline, where $r = 0$ and $U = U_m$, equation (6.48) reduces to the following:

$$C_{\text{profile}} = \frac{v}{U} = \frac{v}{U_m} = \frac{1}{1 + 1.326\sqrt{f}} \qquad \text{(turbulent flow, rough pipes)} \tag{6.49}$$

Example 6.6: *Pitot Tube Differential* Carbon dioxide flows at 122°F and 20 psia at an average velocity of 500 ft/s through an 8-in. schedule 40 wrought steel pipe. The tube has been calibrated, and C_{Pitot} was found to be 0.9802. What pressure differential should be indicated when the Pitot tube is located on the pipe centerline?

Solution

A trial-and-error solution is necessary, because the adiabatic compression factor Y_{Pitot} cannot be determined until the stagnation pressure is calculated. The number of iterations is reduced by assuming incompressible flow for the first trial.

(a) Fluid properties for CO_2 at 68°F

From Table D.1, $M = 44.010$ lbm/lbmol. From Table D.2, $\mu = 0.337 \times 10^{-6}$ lbf s/ft^2, $k = 1.279$:

$$\rho_s = \frac{p_s}{RT} = \frac{20 \times 144}{(1545/44.010)(112 + 460)} = 0.1410 \text{ lbm/ft}^3$$

(b) Pipe properties

From Table C.3 for 8-in. schedule 40 pipe:

$$D = 0.6651 \text{ ft} \qquad \frac{\varepsilon}{D} = 2.255 \times 10^{-4}$$

(c) Reynolds number

$$\text{Re}_D = \frac{\rho D v}{\mu g_c} = \frac{0.1410 \times 0.6651 \times 500}{0.337 \times 10^{-6} \times 32.17} = 4{,}325{,}000$$

(d) Friction factor

$$f = \left[1.14 - 2\ \log_{10}\left(\frac{\varepsilon}{D} + \frac{21.25}{\mathrm{Re}^{0.9}}\right)\right]^{-2}$$

$$f = \left[1.14 - 2\ \log_{10}\left(2.255 \times 10^{-4} + \frac{21.25}{4{,}325{,}000^{0.9}}\right)\right]^{-2} \qquad (4.25)$$

$$f = 0.01434$$

(e) Centerline velocity

$$U_m = (1 + 1.326\sqrt{f})v$$

$$U_m = (1 + 1.326\sqrt{0.01434}) \times 500 \qquad (6.49)$$

$$U_m = 579.4\ \mathrm{ft/s}$$

(f) Incompressible stagnation pressure

$$\Delta p = \left(\frac{U_m}{C_{\mathrm{Pitot}}}\right)^2 \frac{\rho_s}{2g_c} = \left(\frac{579.4}{0.9802}\right)^2 \frac{0.1410}{2 \times 32.17} = 765.7\ \mathrm{lbf/ft^2}$$

$$p_o = p_s + \Delta p = 20 \times 144 + 765.7 = 3645.7\ \mathrm{lbf/ft^2} \qquad (6.36)$$

$$p_o = \frac{3645.7}{144} = 25.32\ \mathrm{lbf/in.^2}$$

(g) Compressible pressure differential
First trial: Use incompressible pressure differential:

$$\frac{p_o}{p_s} = \frac{25.32}{20} = 1.266$$

From Table 6.7 at $p_o/p_s = 1.266$ and $k = 1.279$, $Y_{\mathrm{Pitot}} = 0.9540$ (interpolated):

$$\Delta p = \frac{\Delta p_{\mathrm{incompressible}}}{(Y_{\mathrm{Pitot}})^2} = \frac{765.7}{0.9540^2} = 841.3\ \mathrm{lbf/ft^2}$$

$$p_o = p_s + \Delta p = 20 \times 144 + 841.3 = 3721.2\ \mathrm{lbf/ft^2} = \frac{3721.3}{144} = 25.84\ \mathrm{lbf/in.^2}$$

Second trial: Use first trial values:

$$\frac{p_o}{p_s} = \frac{25.84}{20} = 1.292$$

From Table 6.7 at $p_o/p_s = 1.266$ and $k = 1.279$, $Y_{\mathrm{Pitot}} = 0.9499$ (interpolated):

$$\Delta p = \frac{\Delta p_{\text{incompressible}}}{(Y_{\text{Pitot}})^2} = \frac{765.7}{0.9499^2} = 848.6\ \text{lbf/ft}^2$$

Subsequent trials resulted in $\Delta p = 849.3/144 = 5.90$ lbf/in.^{2}.

6.11 ASME CRITICAL FLOW VENTURI NOZZLES

ASME critical flow Venturi nozzles are used to measure the flow of gases. The Venturi nozzles described in this section are limited to those conforming with ASME Flow Measurement Standards (*Measurement of Gas Flow*, 1987).

Classification of Venturi Nozzles

Venturi nozzles are classified into two types:

1. *Toroidal throat.* For this type, the inlet is in the shape of a partial torus followed by a diverging section, as shown in Figure 6.8.
2. *Cylindrical throat.* For this type, the inlet is the arc of a circle followed by a cylindrical section of one throat diameter followed by a diverging section, as shown in Figure 6.9.

Flow Equation

Unlike the other devices covered in this chapter, the flow is independent of the differential produced (provided that the downstream pressure is low enough). The flow rate is a direct function of the inlet stagnation pressure

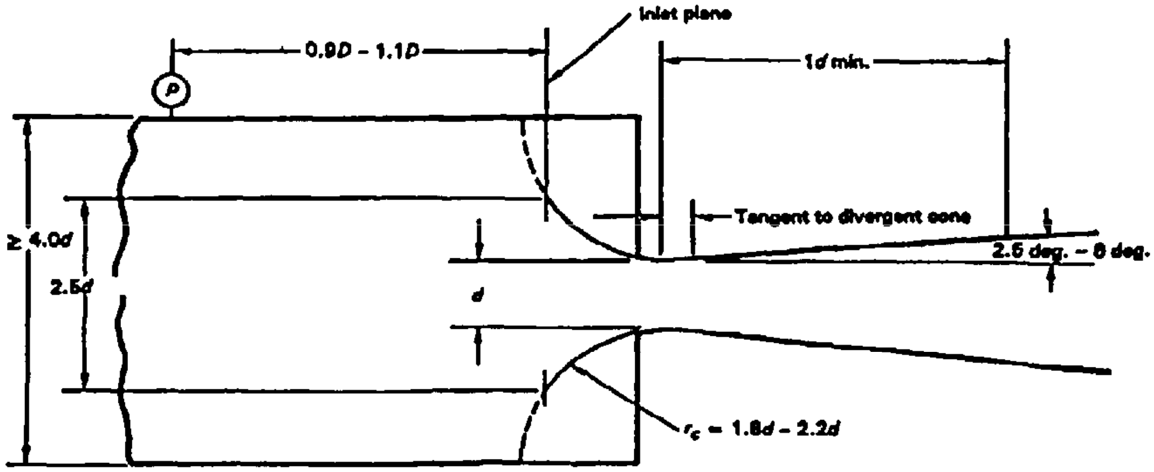


Figure 6.8 Toroidal throat Venturi nozzle.

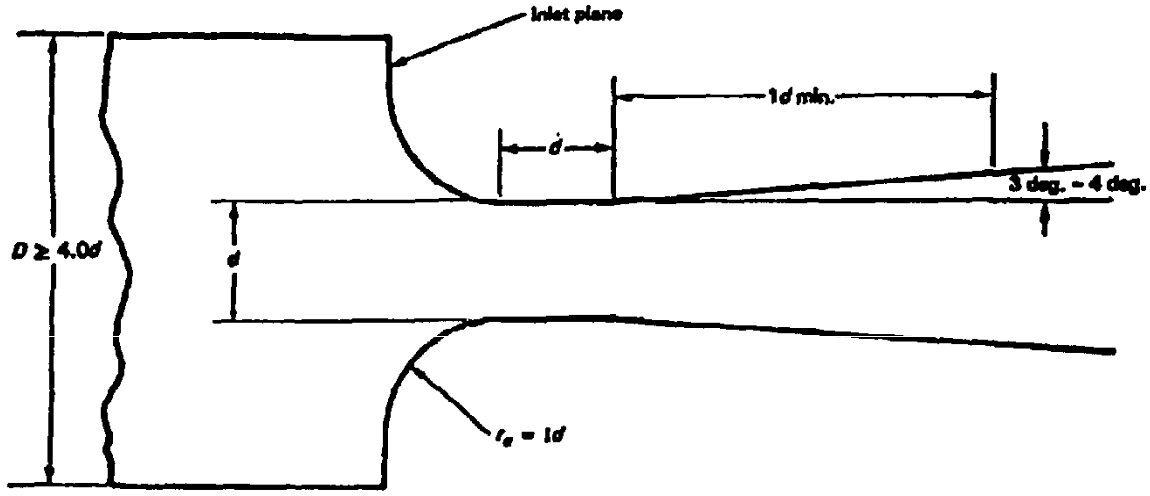


Figure 6.9 Cylindrical throat Venturi nozzle.

and an inverse function of the square root of the inlet absolute temperature. The theoretical aspects of critical flow are covered in Chapter 3.

An equation for ideal flow of ideal gases was derived in Chapter 3 as follows:

$$\dot{m}^*_{\text{ideal}} = \frac{A^* p_o}{\sqrt{T_o}} \sqrt{\frac{k g_c}{R} \left[\frac{2}{k+1}\right]^{(k+1)/(k-1)}} \qquad \text{(ideal gas)} \qquad (3.109)$$

Conventional practice is to write equation (3.109) as follows for the actual flow of a real gas:

$$\dot{m}^* = \frac{C Y_{CR} A_d p_o}{\sqrt{R T_o / g_c}} \qquad (6.50)$$

In equation (6.50), C is the coefficient of discharge, Y_{CR} is the critical flow function, and A_d is the throat area.

Discharge Coefficients and Uncertainty

1. *Toroidal throat*

Throat Reynolds numbers from 10,000 to 100,000; uncertainty = 1%:

$$C = 0.99844 - \frac{3.032}{\text{Re}_d^{1/2}} \qquad (6.51)$$

Throat Reynolds numbers from 100,000 to 10,000,000; uncertainty = 0.5%:

$$C = 0.9935 - \frac{1.525}{\text{Re}_d^{1/2}} \qquad (6.52)$$

2. *Cylindrical throat.* For this Venturi nozzle, uncertainty = 0.5% for all ranges.

Throat Reynolds numbers from 10,000 to 350,000:

$$C = 1.0000 - \frac{7.21}{\mathrm{Re}_d^{1/2}} \tag{6.53}$$

Throat Reynolds number from 350,000 to 2,500,000:

$$C = 0.9887 \tag{6.54}$$

Throat Reynolds number from 2,500,000 to 10,000,000:

$$C = 1.0000 - \frac{0.2165}{\mathrm{Re}_d^{1/2}} \tag{6.55}$$

Values of the coefficient of discharge for both types are given in Table 6.8.

Table 6.8 Coefficients of Discharge for ASME Critical Flow Venturi Nozzles

Throat Reynolds number, Re_d	Toroidal throat C	Cylindrical throat C	Throat Reynolds number, Re_d	Toroidal throat C	Cylindrical throat C
10,000	0.9681	0.9279	500,000	0.9913	0.9887
20,000	0.9770	0.9490	600,000	0.9915	0.9887
30,000	0.9809	0.9584	700,000	0.9917	0.9887
40,000	0.9833	0.9640	800,000	0.9918	0.9887
50,000	0.9849	0.9678	900,000	0.9919	0.9887
60,000	0.9861	0.9706	1,000,000	0.9920	0.9887
70,000	0.9870	0.9727	1,500,000	0.9923	0.9887
80,000	0.9877	0.9745	2,000,000	0.9924	0.9887
90,000	0.9883	0.9860	2,500,000	0.9925	0.9887
100,000	0.9887	0.9772	3,000,000	0.9926	0.9890
200,000	0.9901	0.9839	4,000,000	0.9927	0.9896
250,000	0.9905	0.9856	5,000,000	0.9928	0.9901
300,000	0.9907	0.9868	6,000,000	0.9929	0.9905
350,000	0.9909	0.9887	8,000,000	0.9930	0.9910
400,000	0.9911	0.9887	10,000,000	0.9930	0.9914

Critical Flow Function Y_{CR}

It is evident by comparing equation (3.109) with equation (6.50) that the critical flow function Y_{CR} for an ideal gas may be expressed by the following:

$$Y_{CR} = \sqrt{k\left[\frac{2}{k+1}\right]^{(k+1)/(k-1)}} \tag{6.56}$$

In Section 1.5, it was shown for the process path $pv^n = c$ that n can be expressed as follows:

$$n = -\frac{v}{p}\frac{dp}{dv} \tag{1.32}$$

If the isentropic exponent κ is defined as the path $pv^\kappa = c$ taken by a real gas undergoing an isentropic process, then:

$$\kappa = -\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_s \tag{6.57}$$

The equation of state of a real gas may be expressed as $pv = ZRT$ [equation (1.43)], where Z is the compressibility factor. With the preceding definitions of a real gas, the critical flow function becomes:

$$Y_{CR} = \sqrt{\frac{\kappa}{Z}\left[\frac{2}{\kappa+1}\right]^{(\kappa+1)/\kappa-1)}} \qquad \text{(real gas)} \tag{6.58}$$

Values of Y_{CR} for air, nitrogen, oxygen, and steam are given in Table 6.9. Note that for an ideal gas, $Z = 1$ and $\kappa = k$ (ratio of specific heats), so equation (6.58) reduces to equation (6.56).

Example 6.7: *Mass Flow Rate of an ASME Toroidal Venturi Nozzle* Air at 115 psia and 100°F flows in a 24-in. schedule 40 horizontal steel pipe. Installed in this pipe is an ASME toroidal Venturi nozzle whose throat diameter is 1 in. The Venturi nozzle discharges into the atmosphere. Estimate the mass flow rate of air through the pipe.

Solution

With the very large pipe discharging into a small nozzle, the static pressures and temperatures are essentially the same as stagnation. Although a trial-and-error solution is indicated, the coefficient of discharge is very close to unity, and at higher Reynolds numbers its variation is so small that the ideal mass flow may be used to calculate the Reynolds number. The discharge

Table 6.9 Critical Flow Functions (Y_{CR}) of Air, Nitrogen, Oxygen, and Steam

T_{o1}, R	Inlet stagnation pressure, p_{o1} psia						
	0	100	200	400	600	800	1000
				Air			
400	0.6849	0.6897	0.6947	0.7050	0.7161	0.7276	0.7401
450	0.6849	0.6882	0.6916	0.6986	0.7057	0.7130	0.7203
500	0.6848	0.6873	0.6897	0.6945	0.6992	0.7041	0.7089
550	0.6847	0.6864	0.6882	0.6916	0.6950	0.6983	0.7015
600	0.6847	0.6860	0.6871	0.6896	0.6919	0.6942	0.6965
650	0.6846	0.6855	0.6864	0.6881	0.6897	0.6913	0.6929
700	0.6844	0.6850	0.6856	0.6868	0.6881	0.6892	0.6901
				Nitrogen			
400	0.6847	0.6893	0.6839	0.6978	0.7138	0.7243	0.7351
450	0.6847	0.6879	0.6910	0.6975	0.7040	0.7108	0.7174
500	0.6847	0.6869	0.6891	0.6936	0.6980	0.7025	0.7068
550	0.6847	0.6863	0.6879	0.6909	0.6940	0.6970	0.6999
600	0.6846	0.6858	0.6869	0.6891	0.6912	0.6932	0.6953
650	0.6846	0.6854	0.6862	0.6877	0.6891	0.6905	0.6919
700	0.6845	0.6882	0.6856	0.6866	0.6876	0.6885	0.6893
				Oxygen			
400	0.6846	0.6903	0.6963	0.7090	0.7231	0.7386	0.7553
450	0.6845	0.6886	0.6927	0.7014	0.7106	0.7018	0.7305
500	0.6844	0.6874	0.6904	0.6965	0.7029	0.7095	0.7183
550	0.6841	0.6862	0.6885	0.6930	0.6976	0.7023	0.7069
600	0.6838	0.6854	0.6871	0.6904	0.6937	0.6971	0.7005
650	0.6833	0.6827	0.6858	0.6882	0.6908	0.6932	0.6958
700	0.6828	0.6745	0.6847	0.6865	0.6884	0.6901	0.6919
				Steam			
200	0.6724	0.6727					
300	0.6714	0.6717					
400	0.6706	0.6708					
500	0.6693	0.6695	0.6757				
600	0.6683	0.6685	0.6732	0.6957			
700	0.6670	0.6675	0.6708	0.6855	0.7075	0.7772	
800	0.6657	0.6664	0.6685	0.6789	0.6938	0.7316	0.8803
1000	0.6633	0.6640	0.6649	0.6708	0.6788	0.6962	0.7168
1200	0.6608	0.6618	0.6628	0.6665	0.6714	0.6815	0.6925
1400	0.6587	0.6592	0.6600	0.6626	0.6655	0.6719	0.6791
1600	0.6567	0.6564	0.6561	0.6582	0.6600	0.6637	0.6678

into the atmosphere provides a pressure ratio of 14.7/115 = 0.13, which ensures sonic flow in the throat.

(a) Pipe properties
From Table C.3 for a schedule 40 steel pipe:

$$D = 1.885 \text{ ft}, \qquad A_D = 2.792 \text{ ft}^2$$

(b) Venturi nozzle properties

$$A_d = \frac{\pi d^2}{4} = \frac{\pi(1/12)^2}{4} = 0.005454 \text{ ft}^2$$

$$\beta = \frac{d}{D} = \frac{1/12}{1.885} = 0.0442 \qquad (\beta < 0.25)$$

(c) Fluid properties for air
From Table D.1, $M = 28.97$ lbm/lbmol:

$$R = \frac{R_u}{M} = \frac{1545}{28.97} = 53.33 \text{ ft·lbf/(lbm·°R)}$$

From Table D.2 for 100°F, $\mu = 0.398 \times 10^{-6}$ lbf · s/ft². From Table 6.9 at $p_{o1} = 115$ psia and $T_{o1} = 100 + 460 = 560$ R, $Y_{CR} = 0.6866$.

(d) Reynolds number
Ideal mass flow rate:

$$\dot{m}^*_{\text{ideal}} = \frac{C Y_{CR} A_d p_o}{\sqrt{R T_o / g_c}}$$

$$\dot{m}^*_{\text{ideal}} = \frac{1 \times 0.6866 \times 0.005454 \times (1144 \times 115)}{\sqrt{53.33 \times 560/32.17}} \qquad (6.50)$$

$$\dot{m}^*_{\text{ideal}} = 2.035 \text{ lbm/s}$$

$$\text{Re}_d = \frac{4\dot{m}}{\pi d \mu g_c} = \frac{4 \times 2.035}{\pi(1/12) \times 0.398 \times 10^{-6} \times 32.17} = 2{,}428{,}000$$

(e) Mass flow rate
From Table 6.8 at $\text{Re}_d = 2{,}428{,}000$, $C = 0.9925$:

$$\dot{m} = C\dot{m}^*_{\text{ideal}} = 0.9925 \times 2.035 = 2.020 \text{ lbm/s}$$

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7
Forces on Immersed Objects

7.1 INTRODUCTION

The number of types of structures that are subject to fluid forces is almost infinite. For this reason, our consideration has to be limited to some simple shapes, such as the flat plate, the sphere, and the cylinder. For the purpose of illustration, consideration is given to typical engineering situations in connection with vortex-induced vibration, resistance of ships, properties of lifting vanes, and characteristics of propellers.

Although examples are solved in USCS units, all equations are structured to enable their use with SI units as well.

7.2 BACKGROUND

Men whose significant contributions underlie the material covered in this chapter are Leonardo da Vinci (1452–1519), who first sketched and described vortex formation, and Ludwig Prandtl (1875 1953), Paul Heinrich Blasius (1883 ?), Theodore von Karman (1881-1963), and Hermann Schlichting (1907-?), all of whom made contributions to boundary layer theory.

7.3 DRAG AND LIFT

Figure 7.1 shows the effect of an object placed in a fluid stream. The impingement of the fluid on the object produces a force F. The horizontal component of this force is the *drag force*; the vertical component is the *lift force*. The presence of this object may produce a *wake*, in which eddies are shed regularly from alternate sides at a definite frequency. This shedding pattern is generally known as *Karman vortex trails*, in honor of Theodore von Karman, whose studies formalized this phenomenon.

Consider a fluid of density ρ, viscosity μ, and bulk modulus of elasticity E approaching an object whose characteristic length is L at velocity v. The impingement of the fluid on the object produces a drag force of F_D and a lift force of F_L. The wake sheds eddies with a frequency of f.

Dimensional analysis gives the following functional relationship:

$$f(C_D, C_L, \text{Re}, M, S) = 0 \tag{7.1}$$

In equation (7.1), the Strouhal number is a dimensionless parameter representing the ratio of inertia to vibration forces ($S = Lf/v$). The formation of

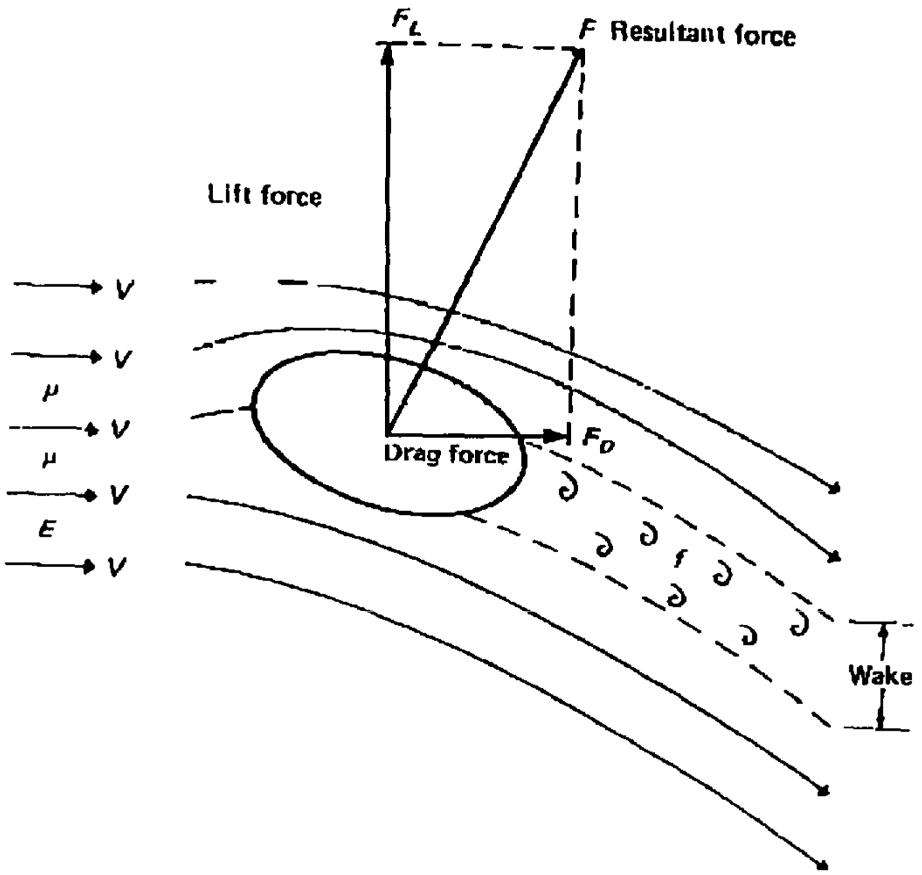


Figure 7.1 Notation for drag and lift.

a wake with shedding eddies depends upon the ratio of inertia to viscous forces or Reynolds number, so it follows that the Strouhal number must be some function of Reynolds number:

$$S = f(\text{Re}) \tag{7.2}$$

This function will be developed in a later section on cylinders. Equation (7.1) may now be simplified as follows:

$$f(C_D, C_L, \text{Re}, M) = 0 \tag{7.3}$$

Since the lift and drag forces are independent of each other, equation (7.3) can be written as follows:

$$F_D = C_D \frac{\rho v^2 A}{2g_c} \tag{7.4}$$

where the drag coefficient $C_D = f(\text{Re}, M)$ and A is the characteristic area. In a similar manner, the lift force can be written as follows:

$$F_l = C_L \frac{\rho v^2 A}{2g_c} \tag{7.5}$$

The drag force shown in Figures 7.1 and 7.2 arises from two sources, the interference of the object to the flow, called the *pressure* or *shape drag*, and the friction drag due to the wall shear stress τ_o, called *skin friction drag*. The drag coefficient is made up of two parts, as follows:

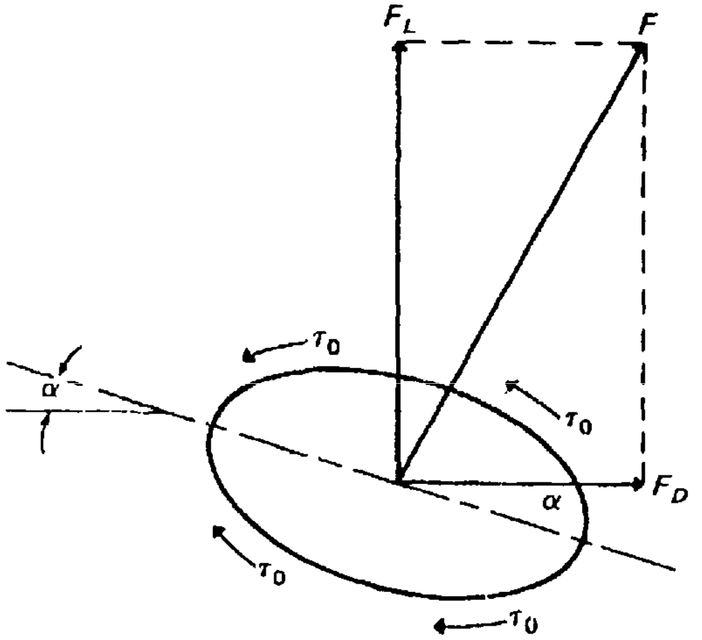


Figure 7.2 Notation for physical analysis.

$$F_D = F_p + F_f - C_D \frac{\rho v^2 A}{2g_c} = C_f \frac{\rho v^2 A_s}{2g_c} + C_s \frac{\rho v^2 A}{2g_c} \tag{7.6}$$

or

$$C_D = C_s \frac{C_f A_s}{A} \tag{7.7}$$

where C_s is the shape coefficient, C_f is the skin friction coefficient, and A_s is the characteristic area for shear.

The lift force shown in Figures 7.1 and 7.2 is a function of shape and the angle of attack, α, as well as Reynolds and Mach numbers.

Example 7.1: *Lift and Drag* A kite (Figure 7.3) weighs 3 lbf and has an area of 10 ft^2. The tension in the kite string is 8 lbf when the string makes an angle of 45° with the horizontal. For air at 80 F and 14.7 psi with a wind velocity of 20 mph, what are the coefficients of lift and drag?

Solution

This example is solved by direct application of the definitions of lift and drag.

(a) Unit conversion

$$v = \frac{20\text{mph} \times 5280\ \text{ft/mile}}{3600\ \text{s/hr}} = 29.33\text{ft/s}$$

(b) Fluid properties – air
From Table D.1, $M = 28.97$ lbm/lbmole:

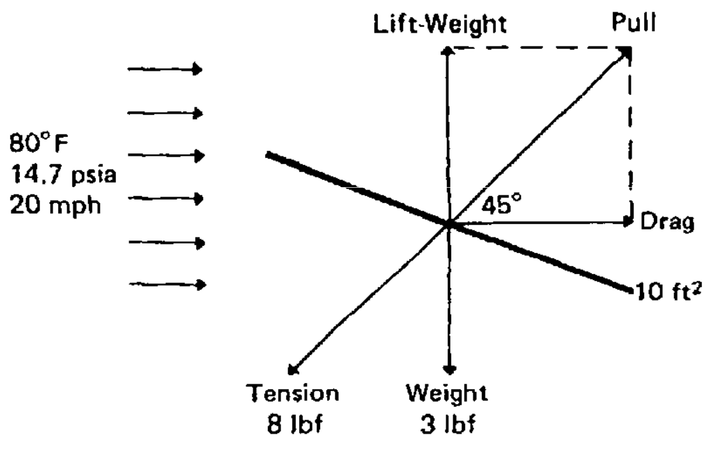


Figure 7.3 Notation for Example 7.1.

$$\rho = \frac{p}{RT}$$

$$\rho = \frac{14.7\ \text{lbf/in.}^2 \times 144\ \text{in.}^2/\text{ft}^2}{(1545/28.97)\ \text{ft·lbf/(lbm·°R)} \times (80 + 460)\text{°R}} \tag{1.39}$$

$$\rho = 0.0735\ \text{lbm/ft}^3$$

(c) Compute drag coefficient

$$F_D = \text{tension} \times \cos 45° = 8 \times \cos 45° = 5.657\ \text{lbf}$$

$$C_D = \frac{2g_c F_D}{\rho v^2 A}$$

$$C_D = \frac{2 \times 32.17 \times 5.657}{0.0735 \times 29.33^2 \times 10} \tag{7.4}$$

$$C_D = 0.5756$$

(d) Compute lift coefficient

$$F_l = \text{tension} \times \sin 45° + \text{weight} = 8 \times \sin 45° + 3\ \text{lbf} = 8.657\ \text{lbf}$$

$$C_L = \frac{2g_c F_L}{\rho v^2 A}$$

$$C_L = \frac{2 \times 32.17 \times 8.657}{0.0735 \times 29.33^2 \times 10} \tag{7.5}$$

$$C_L = 0.8809$$

7.4 SKIN FRICTION DRAG

Boundary Layer

Figure 7.4 shows a fluid approaching a flat plate with a uniform velocity profile of v. As the fluid passes over the plate, the velocity at the plate surface is zero and increases to v at some distance from the surface. The region in which the velocity varies from 0 to v is called the *boundary layer*. The thickness of this layer is δ. For some distance along the plate, the flow within the boundary layer is laminar, with the viscous forces predominating. But in the transition zone as the inertial forces begin to exceed the laminar, a turbulent layer begins to form that increases as the laminar layer decreases.

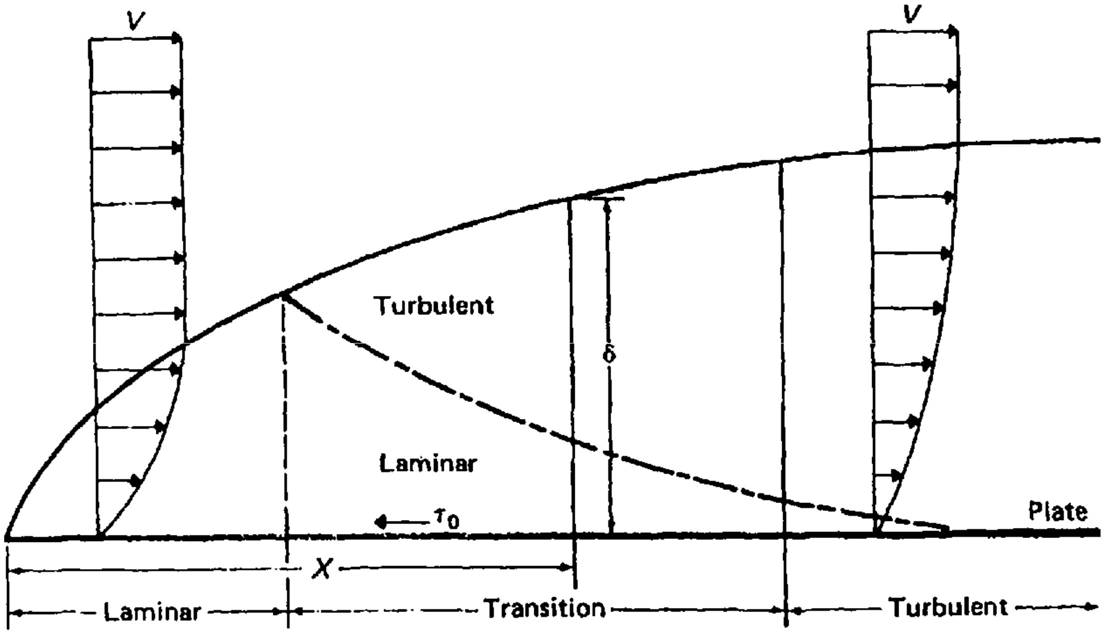


Figure 7.4 Boundary layer along a smooth, flat plate.

Reynolds Number

The Reynolds number used for skin friction drag is based on the distance x from the leading edge of the flat plate. Substituting x for the characteristic length in the Reynolds number equation yields equation (7.8):

$$\mathrm{Re}_x = \frac{\rho x v}{\mu g_c} = \frac{xv}{\nu} \tag{7.8}$$

Equations

Skin friction drag coefficients for incompressible flow over smooth, flat plates may be calculated from the following equations (see Figure 7.5).

Laminar

Laminar flow starts at the leading edge of a flat plate and continues until a Reynolds number of about 500,000, depending upon the surface roughness and the degree of turbulence:

$$C_f = \frac{1.328}{\mathrm{Re}_x^{1/2}} \qquad \text{for } 0 < \mathrm{Re}_x < 500{,}000 \tag{7.9}$$

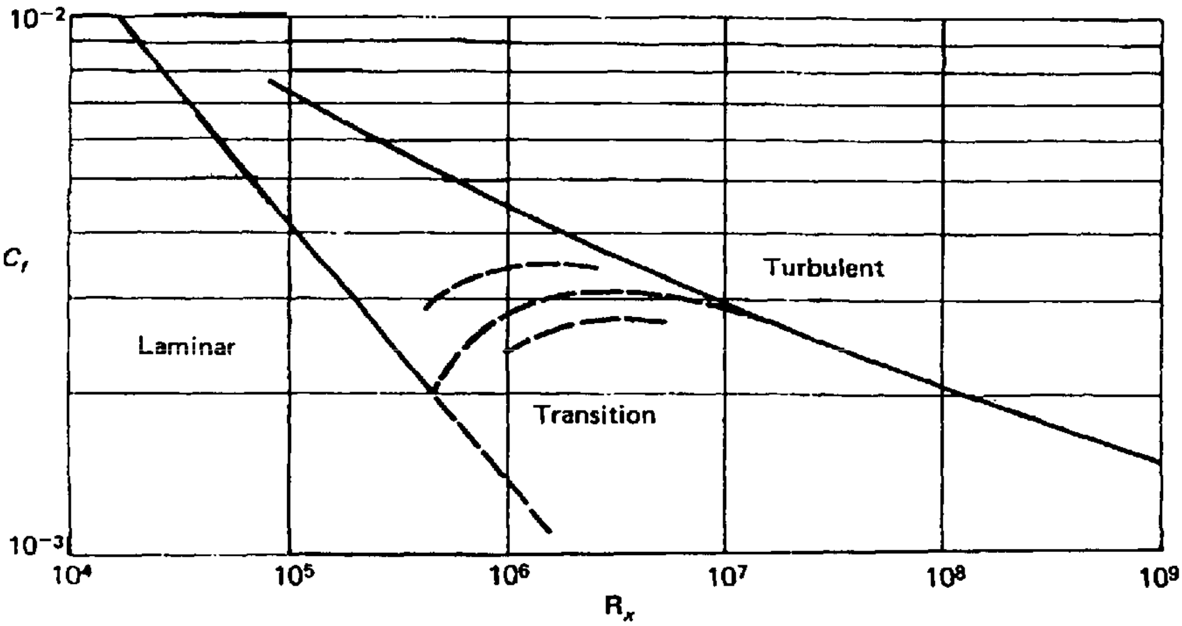


Figure 7.5 Skin friction coefficients for smooth, flat plates.

Transition

The Reynolds number where the boundary layer changes depends upon the roughness of the plate and the degree of turbulence. The generally accepted number is 500,000, but the transition can take place at Reynolds numbers higher or lower. For transition at Reynolds number Re_x of 500,000, C_f can be calculated as follows:

$$C_f = \frac{0.0735}{Re_x^{1/5}} - \frac{1700}{Re_x} \quad \text{for } 500{,}000 < Re_x < 10{,}000{,}000 \tag{7.10}$$

Turbulent

Depending upon the transition Reynolds number, the surface roughness, and the degree of turbulence, turbulent flow can take place in the boundary layer at Reynolds numbers as low as 200,000:

$$C_f = \frac{0.0735}{Re_x^{1/5}} \quad \text{for } 200{,}000 < Re_x < 10{,}000{,}000 \tag{7.11}$$

$$C_f = \frac{0.455}{[\log_{10} Re_x]^{2.58}} \quad \text{for } 10{,}000{,}000 < Re_x < 1{,}000{,}000{,}000 \tag{7.12}$$

Example 7.2: *Drag of a Smooth, Flat Plate* A smooth, flat plate 10 ft wide and 100 ft long is towed through 100°F water at a velocity of 20 ft/s. Compute the tension on the towing cable (neglecting its effect) in pounds force.

Solution

This example is solved by direct application of the appropriate equations given in this section.

(a) Fluid properties water at 100°F
From Table D.1, $\rho = 62.00$ lbm/ft^3, $\mu = 14.24 \times 10^{-6}$ lbf·s/ft^2.

(b) Plate properties

$$x = 100 \text{ ft}$$

$$A_s = 100 \times 10 = 1{,}000 \text{ ft}^2 \qquad \text{(one side)}$$

(c) Reynolds number

$$\text{Re}_x = \frac{\rho x v}{\mu g_c} = \frac{62.000 \times 100 \times 20}{14.24 \times 10^{-6} \times 32.17} = 270{,}680{,}000 \tag{7.8}$$

(d) Skin friction coefficient

$$C_f = \frac{0.455}{[\log_{10} \text{Re}_x]^{2.58}} = \frac{0.455}{[\log_{10}(270{,}680{,}000)]^{2.58}} = 0.001858 \tag{7.12}$$

(e) Tension in towing cable

$$F_f = C_f \frac{\rho v^2 A_s}{2g_c} = 0.001858 \frac{62.00 \times 20^2 \times 1000}{2 \times 32.17} = 716 \text{ lbf} \tag{7.6}$$

The towing cable tension is twice that of one side, or $2 \times 716 = 1432$ lbf

7.5 SHAPE DRAG

Experiments with certain objects normal to the flow of a fluid indicate that their drag coefficients are essentially constant at Reynolds numbers over 1000. This means that the drag for Re > 1,000 is "shape" drag. Table 7.1 shows typical values used by engineers for design and estimating purposes.

Table 7.1 Shape Coefficients, Re > 1000

Flat plate	Normal to flow	Length/width	1	5	10	20	30	∞
		C_s	1.18	1.2	1.3	1.5	1.6	1.95
Disk	Normal to flow	$C_s = 1.18$						
Trains	Locomotive + tender, streamlined	$C_s = 0.40$						
	Locomotive + tender, conventional	$C_s = 0.93$						
	Railroad cars, streamlined	$C_s = 0.15$						
	Railroad cars, conventional	$C_s = 0.40$						
Automotive	Automobiles, racing	$C_s = 0.17$						
	Automobiles, streamlined	$C_s = 0.23$						
	Automobiles, fast-back	$C_s = 0.34$						
	Automobiles, conventional	$C_s = 0.52$						
	Truck, conventional	$C_s = 0.60$						
Ship	Passenger, conventional	$C_s = 0.90$						

Example 7.3: *Wind Forces on a Billboard* Estimate the total wind force on a billboard 10 ft high and 50 ft wide when it is subjected to a 20-mph, 32°F, and 14.7-psia wind blowing normal to it.

Solution

In this situation the skin friction drag is negligible and the drag is essentially caused by the shape of the billboard. This example is solved by using equation (7.6) and Table 7.1.

(a) Unit conversion

$$v = \frac{20 \text{ mph} \times 5280 \text{ ft/mile}}{3600 \text{ s/hr}} = 29.33 \text{ ft/s}$$

(b) Fluid properties—air
From Table D.2, $M = 28.97$ lbm/lbmole. From Table D.2 for 32 F, $\mu = 0.360 \times 10^{-6}$ lbf · sec/ft^2:

$$\rho = \frac{p}{RT} = \frac{144 \times 14.7}{(1545/28.97) \times (32 + 460)} = 0.08067 \text{ lbm/ft}^3 \tag{1.39}$$

(c) Properties of the billboard

$$A = \text{projected area} = \text{height} \times \text{width} = 10 \times 50 = 500 \text{ ft}^2$$

$$R_h = \frac{\text{area}}{\text{perimeter}} = \frac{500}{2(10+50)} = 4.1667 \text{ ft}$$

$$D_e = 4 \times R_h = 4.1667 = 16.67 \text{ ft}$$

$$\frac{\text{longest dimension}}{\text{shortest dimension}} = \frac{50}{10} = 5$$

(d) Reynolds number

$$\text{Re} - \frac{\rho D_e v}{\mu g_c} = \frac{0.08067 \times 16.67 \times 29.33}{0.360 \times 10^{-6} \times 32.17} = 3{,}406{,}000 \quad (7.8)$$

Since Re is greater than 1,000, Table 7.1 may be used.

(e) Shape coefficient

From Table 7.1 for a flat plate normal to flow, length/width (longest dimension/shortest dimension) = 5:

$$C_s = 1.2$$

(f) Total wind force

$$F_f = C_s \frac{\rho v^2 A}{2g_c} = 1.2 \frac{0.08067 \times 29.33^2 \times 500}{2 \times 32.17} = 777 \text{ lbf}$$

7.6 DRAG OF A SPHERE

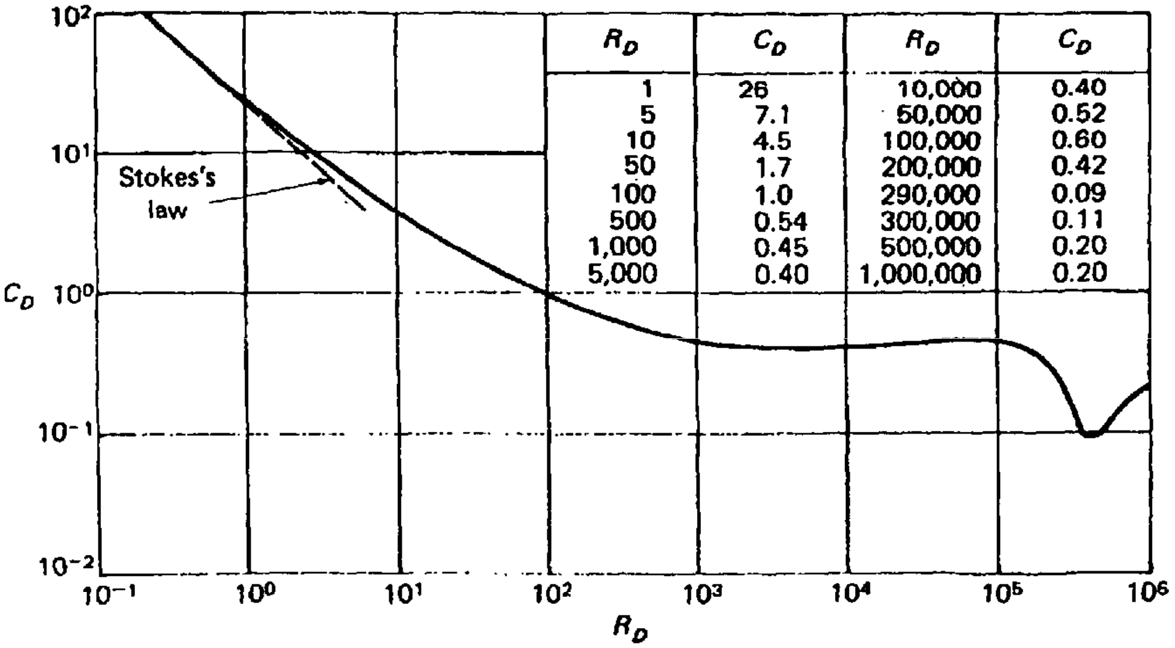
In calculating the drag of spheres, the sphere diameter, D, is used as the characteristic length and the projected area, $\pi D^2/4$, is used as the drag area. The drag of a sphere is obtained by substituting the projected area in equation (7.4):

$$F_D = C_D \frac{\rho v^2 A}{2g_c} = C_D \frac{\rho v^2 (\pi D^2/4)}{2g_c} = C_D \frac{\pi D^2 \rho v^2}{8g_c} \quad (7.13)$$

Values of drag coefficients versus Reynolds numbers for spheres are shown in Figure 7.6.

Laminar Flow

For laminar flow, Stokes' law is used for Reynolds numbers up to 0.5. Stokes' law may be stated as follows:



R_D	C_D	R_D	C_D
1	26	10,000	0.40
5	7.1	50,000	0.52
10	4.5	100,000	0.60
50	1.7	200,000	0.42
100	1.0	290,000	0.09
500	0.54	300,000	0.11
1,000	0.45	500,000	0.20
5,000	0.40	1,000,000	0.20

Figure 7.6 Drag coefficients of spheres.

$$C_D = \frac{24}{\mathrm{Re}_D} = \frac{24\mu g_c}{\rho D v} \quad \text{for } 0 < \mathrm{Re}_D < 0.5 \tag{7.14}$$

From equations (7.13) and (7.14), the drag force of a sphere in laminar flow becomes the following:

$$F_D = C_D \frac{\pi D^2 \rho v^2}{8 g_c} = \left(\frac{24\mu g_c}{\rho D v}\right) \frac{\pi D^2 \rho v^2}{8 g_c} = 3\pi\mu D v \tag{7.15}$$

Example 7.4: *Laminar Drag, Falling Sphere Viscometer* A steel ball whose specific weight is 490 lbf/ft^3 falls at a steady velocity of 0.16 ft/s through a mass of oil whose density is 56.78 lbm/ft^3. What is the dynamic viscosity of the oil? The sphere diameter is 0.06 in. (see Figure 7.7).

Solution

This example is solved by using the "free-body" concept, assuming that Stokes' law applies, and then verifying the assumption.

(a) Fluid properties

$$\gamma_o = \frac{\rho_o g}{g_c} = \frac{56.78 \times 32.17}{32.17} = 56.78 \text{ lbf/ft}^3$$

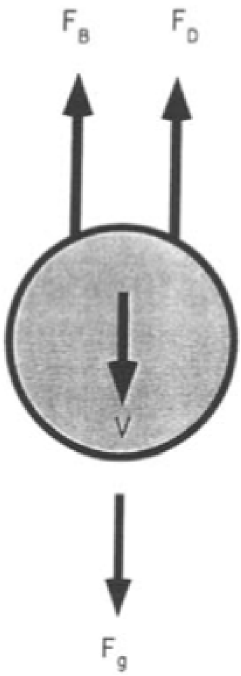


Figure 7.7 Notation for Example 7.4.

(b) Properties of the sphere

$$D = \frac{0.06}{12} = 0.005 \text{ ft}$$

$$V = \frac{\pi D^3}{6}$$

(c) Drag force

$$F_D = 3\pi\mu D v$$

(d) Buoyant force

$$F_B = \gamma_a V$$

(e) Gravity force

$$F_g = \gamma_s V$$

(f) Free body

$$\sum Fz = 0 = F_B - F_g - F_L = F_B - F_g + F_D$$

Substituting the force equations into the free-body equation and solving for μ results in the following:

$$\mu = \frac{(\gamma_s - \gamma_o)V}{3\pi D v} = \frac{(\gamma_s - \gamma_o)(\pi D^3/6)}{3\pi D v} = \frac{(\gamma_s - \gamma_o)(D^2)}{18v}$$

(g) Viscosity of oil

$$\mu = \frac{(\gamma_s - \gamma_o)(D^2)}{18v} = \frac{(450 - 56.78)(0.005^2)}{18 \times 0.16} = 3.761 \times 10^{-3} \text{ lbf} \cdot \text{s/ft}^2$$

(h) Verify flow

$$\text{Re}_D = \frac{\rho D v}{\mu g_c} = \frac{56.78 \times 0.005 \times 0.16}{3.761 \times 10^{-3} \times 32.17} = 0.375$$

Since 0.375 < 0.5, Stokes' law applies.

Turbulent Flow

No equations have been developed for the calculation of drag coefficients for turbulent flow around spheres. For estimating purposes, the tabulated drag coefficients given in Figure 7.6 may be interpolated.

Example 7.5: *Turbulent Drag* A 3-inch-diameter metal ball is towed under fresh water at 68°F by means of a cable fastened to a boat moving steadily in a horizontal direction of 6 ft/s. Neglect the effect of water on the cable. Estimate the power required to pull the ball.

Solution

This example is solved by calculating the drag force and using it to compute the required power.

(a) Fluid properties
From Table D.1 for 68°F water:

$$\rho = 62.31 \text{ lbm/ft}^3, \qquad \mu = 20.95 \times 10^{-6} \text{ lbf} \cdot \text{s/ft}^2$$

(b) Reynolds number

$$\text{Re}_D = \frac{\rho D v}{\mu g_c} = \frac{62.31 \times (3/12) \times 6}{20.95 \times 10^{-6} \times 32.17} = 138{,}700$$

(c) Drag coefficient
From upcoming Figure 7.8 at $\text{Re}_D = 138{,}000$:

$C_D = 0.54$ (by linear interpolation)

(d) Drag force

$$F_D = C_D \frac{\pi D^2 \rho v^2}{8g_c} = 0.54 \frac{\pi(3/12)^2 \times 62.31 \times 6^2}{8 \times 32.17} = 0.9242 \text{ lbf}$$

(e) Required power

$$P = F_D v = 0.9242 \times 6 = 5.545 \text{ ft} \cdot \text{lbf/s} \; (= 5.545/550 \approx 1/100 \text{ hp})$$

Compressible Flow

Table 7.2 may be used for estimating the drag coefficients of spheres at supersonic velocities.

Example 7.6: *Terminal Velocity of a Sphere* A research project requires that a metal ball when dropped through atmospheric air (32°F, 14.7 psia) attain a terminal velocity of Mach 2. The specific weight of metal is 495 lbf/ft^3. Estimate the required sphere diameter.

Solution

This example is solved by assuming that the drag of the sphere is equal to its weight when vertical equilibrium is achieved. The density of the metal compared to that of the air is so large that buoyant force may be ignored.

Table 7.2 Drag Coefficients for Spheres at Supersonic Velocities

Mach number	Reynolds number				
	10^3	10^4	10^5	10^6	10^{7a}
1.0	0.89	0.81	0.89	0.85	0.80
1.2	0.97	0.85	0.93	0.98	0.85
1.4	1.00	0.89	1.01	0.98	0.95
1.6	1.05	0.93	0.99	1.00	1.00
1.8	1.10	0.94	0.98	0.98	0.98
2.0	1.12	0.95	0.96	0.96	0.97
2.2	1.13	0.96	0.95	0.96	0.96
2.4	1.13	0.96	0.94	0.95	0.95
2.6	1.14	0.97	0.93	0.95	0.95
2.8	1.14	0.97	0.93	0.96	0.95
3.0	1.14	0.97	0.92	0.97	0.95

[a]For Reynolds numbers greater than 10^7, use values for 10^7.

(a) Fluid properties—air
From Table D.2, $M = 28.97$ lbm/lbmole. From Table D.2 for 32°F, $\mu = 0.360 \times 10^{-6}$ lbf · s/ft^2, $k = 1.402$:

$$\rho = \frac{p}{RT} = \frac{144 \times 14.7}{(1545/28.97) \times (32 + 460)} = 0.08067 \text{ lbm/ft}^3 \qquad (1.39)$$

(b) Velocity of sphere

$$c = \sqrt{kg_c RT} = \sqrt{1.402 \times 32.17 \times (1545/28.97)(32 + 460)} = 1088 \text{ ft/s}$$
$$v = cM = 1088 \times 2 = 2176 \text{ ft/s}$$

(c) Gravity force

$$F_g = \gamma_s V = \frac{\gamma_s \pi D^3}{6}$$

(d) Equating gravity force with drag force

$$F_D = \frac{\gamma_s \pi D^6}{6} = C_D \frac{\pi D^2 \rho v^2}{8g_c}$$

or

$$D = C_D \frac{6\rho v^2}{8g_c \gamma_s} = C_D \frac{6 \times 0.08067 \times 2176^2}{8 \times 32.17 \times 495} = 17.99 C_D$$

(e) First trial
From Table 7.2 for Mach 2 and $\text{Re}_D = 10^7$, $C_D = 0.97$:

$$D = 17.99 \times 0.97 = 17.45$$

(f) Verify Reynolds number

$$\text{Re}_D = \frac{\rho D v}{\mu g_c} = \frac{0.08067 \times 17.49 \times 2176}{0.360 \times 10^{-6} \times 32.17} = 2.65 \times 10^8$$

The Reynolds number is greater than 10^7. Further trials are not necessary and $D = 17.45$ ft.

7.7 DRAG OF A CYLINDER

In calculating the drag of circular cylinders, the cylinder diameter, D, is used as the characteristic length and the projected area, LD, is used for the drag area. The drag of a cylinder is obtained by substituting the projected area in equation (7.4):

$$F_D = C_D \frac{\rho v^2 A}{2g_c} = C_D \frac{\rho v^2 LD}{2g_c} \tag{7.16}$$

Figure 7.8 gives drag coefficients for cylinders of infinite length; Table 7.3 gives them for cylinders of finite length.

Example 7.7: *Calculation of Drag Coefficient* As shown in Figure 7.9, a cylindrical chimney 3 ft in diameter and 75 ft high is exposed to a 35 mph wind ($\rho = 0.0772$ lbm/ft^3). Measurements at the base of the chimney indicate a bending moment of 9,200 lbf · ft. Compute the drag coefficient.

Solution

This example is solved by computing the drag force from the bending moment.

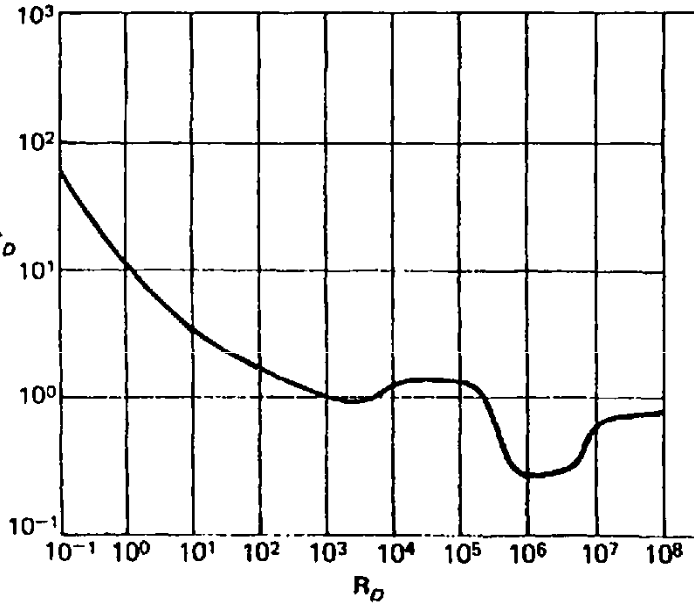
(a) Convert velocity

$$v = \frac{35 \times 5280\text{ft/mile}}{3600 \text{ s/hr}} = 51.33 \text{ ft/s}$$

(b) Drag force from Figure 7.9

$$M = F_D\left(\frac{L}{2}\right)$$

or



R_D	C_D
0.1	67.0
0.5	16.5
1	10.5
5	2.9
10	2.6
50	1.7
100	1.4
500	1.0
1,000	0.9
5,000	0.9
10,000	1.1
50,000	1.2
100,000	1.2
350,000	0.3
3,500,000	0.3
10,000,000	0.7

Figure 7.8 Drag coefficients of a cylinder of infinite length.

Table 7.3 Drag Coefficients for Finite Cylinders, Re > 1000

L/D	1	5	10	20	30	∞
C_D	0.63	0.80	0.83	0.93	1.00	1.20

$$F_D = \frac{2M}{L} = 2 \times 9{,}200/75 = 245.3 \text{ lbf}$$

(c) Drag coefficient

$$C_D = F_D \frac{2g_c}{\rho v^2 LD} = 245.3 \frac{2 \times 32.17}{0.0772 \times 51.33^2 \times 75 \times 3} = 0.3449 \tag{7.16}$$

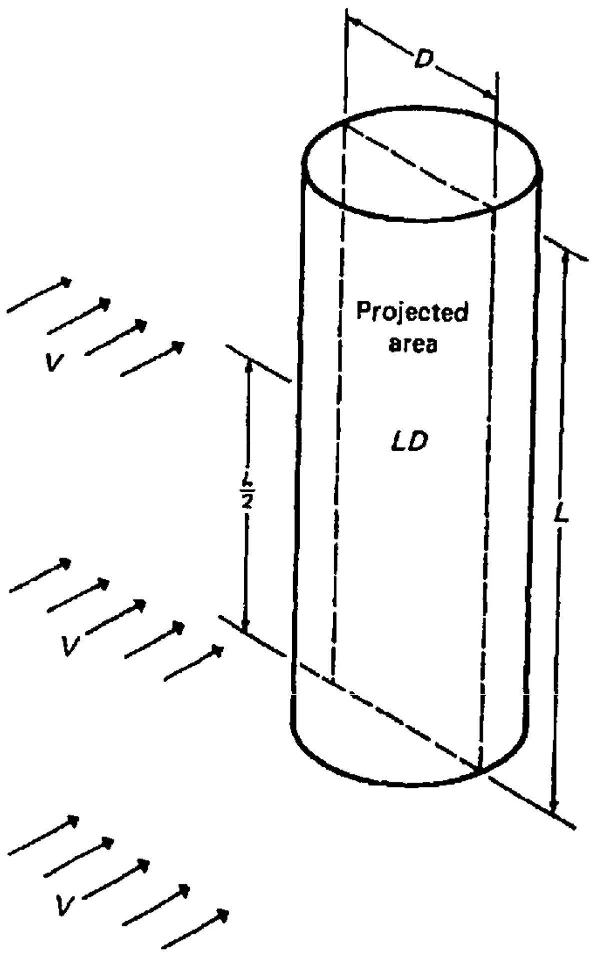


Figure 7.9 Notation for Example 7.7.

7.8 WAKE FREQUENCY

An object in a fluid stream may be subject to the downstream periodic shedding of vortices, first from one side and then from the other. The frequency of the resulting transverse (lift) force is a function of the stream Strouhal number. As the wake frequency approaches the natural frequency of the structure, the periodic lift force increases asymptotically in magnitude; when resonance occurs, the structure fails. Neglecting to take this phenomenon into account in design has been responsible for failures of electric transmission lines, submarine periscopes, smokestacks, bridges, and thermometer wells. The wake frequency characteristics of *cylinders* are shown in Figure 7.10. At a Reynolds number of about 20, vortices begin to shed alternately. Behind the cylinder is a staggered, stable arrangement of vortices known as the *Karman vortex trail*. At a Reynolds number of about 105, the flow changes from laminar to turbulent. At the end of the transition zone (Re = 3.5×10^5), the flow becomes turbulent, the alternate shedding stops, and the wake is aperiodic. At the end of the supercritical zone (Re = 3.5×10^6), the wake continues to be turbulent, but the shedding again becomes alternate and periodic.

The alternating *lift force* is given by the following:

$$F_L(t) = C_L \frac{\rho v^2 A \sin(2\pi f t)}{2g_c} \tag{7.17}$$

where t is the time.

The Strouhal number may be computed from equation (7.18):

$$S = \frac{fD}{v} \tag{7.18}$$

The Strouhal number is nearly constant to Re = 10^5, and a nominal design value of 0.2 is generally used. Above Re = 10^5, data from different experimenters vary widely, as indicated by the cross-hatched zone of Figure 7.10. This wide zone is due to experimental and/or measurement difficulties and the dependence on surface roughness to "trigger" the boundary layer. Examination of Figure 7.10 reveals an inverse relation of Strouhal number to drag coefficient.

Observation of actual structures shows that they vibrate at their natural frequency and with a mode shape associated with their fundamental (first) mode during vortex excitation.

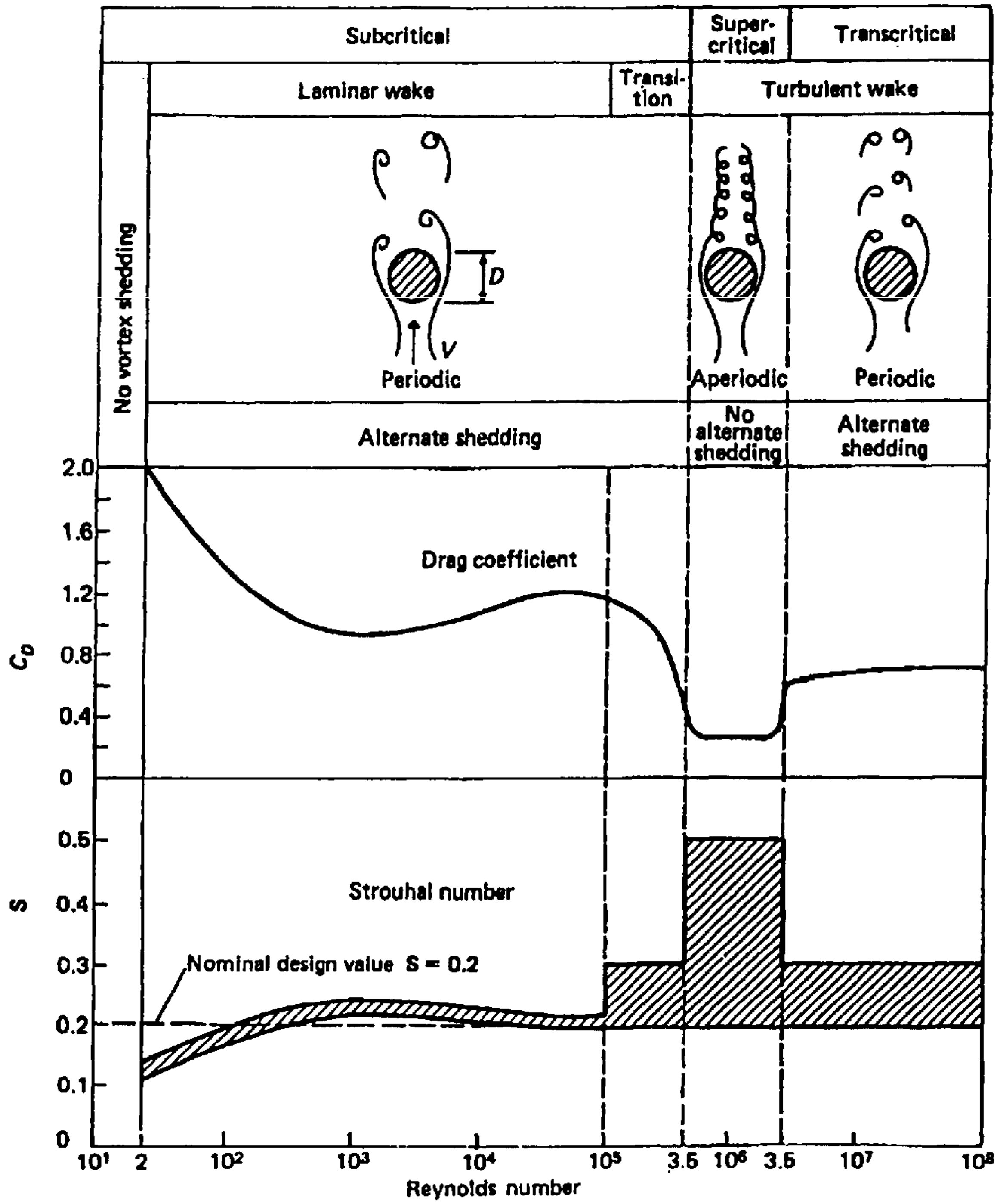


Figure 7.10 Flow around a cylinder.

Example 7.8: *Smokestack Design* A smokestack is to be erected at a location where winter winds up to 50 mph have been observed. The stack design under consideration has a diameter of 16 ft. The natural frequency of this type of stack is given by:

$$f_n = \left(\frac{10^5}{H}\right)^{1/2} \tag{a}$$

where f_n is the natural frequency in hertz (cycles per second) and H is the height of the stack in feet. To ensure that failure from resonance will not occur, good design practice limits the wake frequency to 80% of the natural frequency. What is the maximum height of stack that should be erected? Assume air at 32°F and 14.7 psia.

Solution

This example is solved by matching the natural frequency of the stack with 80% of the vortex shedding frequency.

(a) Convert velocity

$$v = \frac{50 \times 5280 \text{ ft/mile}}{3600 \text{ s/hr}} = 71.33 \text{ ft/s}$$

(b) Fluid properties- air
From Table D.2, $M = 28.97$ lbm/lbmole. From Table D.2 for 32°F, $\mu = 0.360 \times 10^{-6}$ lbf · s/ft^2:

$$\rho = \frac{p}{RT} = \frac{144 \times 14.7}{(1545/28.97) \times (32 + 460)} = 0.08067 \text{ lbm/ft}^3 \tag{1.39}$$

(c) Reynolds number

$$\text{Re}_D = \frac{\rho D v}{\mu g_c} = \frac{0.08067 \times 16 \times 73.33}{0.360 \times 10^{-6} \times 32.17} = 8,173,000$$

(d) Maximum Strouhal number
From Figure 7.10 at $\text{Re}_d = 8 \times 10^6$, $S_{\text{max}} = 0.3$.

(e) Minimum natural frequency

$$f_{n(\text{min})} = \frac{f_{w(\text{min})}}{0.8} = \frac{S_{\text{max}} v}{0.8D} = \frac{0.3 \times 73.3}{0.8 \times 16} = 1.718 \text{ Hz} \tag{7.18}$$

(f) Calculate maximum stack height

$$f_n = \left(\frac{10^5}{H}\right)^{1/2} = \left(\frac{10^5}{1.718}\right)^{1/2} = 241 \text{ ft} \tag{a}$$

7.9 RESISTANCE OF SHIPS

Dimensional and Physical Analysis

Consider a ship of length L moving on the surface of a liquid at a speed v. The liquid has a density ρ, viscosity μ, and a surface tension σ. The acceleration of gravity is g, and the movement of the ship produces a drag force of F_D. The dimensional analysis of ship resistance results in the same parameters as were obtained from analyses of other objects. Thus, the total drag of a ship is given by the following:

$$F_D = C_D \frac{\rho v^2(\text{wetted surface})}{2g_c} = C_D \frac{\rho v^2 S_w}{2g_c} \tag{7.19}$$

Surface tension forces are very weak for this analysis, and, therefore, these forces are not considered in the computation of ship resistance. Conventional practice is to divide the drag coefficient into three individual coefficients as follows:

$$C_D = C_r + C_f + \Delta C_f \tag{7.20}$$

where

C_r = residual coefficient, a function of Froude number (typical values of C_r for passenger and cargo ships are shown in Table 7.4)

C_f = skin friction coefficient for a smooth, flat plate, as computed from equation (7.11) or (7.12).

ΔC_f = allowance for surface roughness (for actual ships $\Delta C_f = 4 \times 10^{-4}$ and for models $\Delta C_f = 0$.)

The values of the residual coefficient given in Table 7.4 are typical. Actual values of C_r can be obtained only from model or actual ship testing.

Substituting equation (7.20) in equation (7.19) leads to equation (7.21):

$$F_D = \frac{(C_r + C_f + \Delta C_f)\rho v^2 S_w}{2g_c} = F_r + F_f + F_\Delta \tag{7.21}$$

Table 7.4 Typical Values of C_r for Passenger and Cargo Ships

Froude number	Residual coefficient (C_r)
0.15	5.3×10^{-4}
0.20	7.2×10^{-4}
0.25	8.7×10^{-4}
0.30	15.3×10^{-4}
0.35	20.0×10^{-4}

In equation (7.21), F_r is the residual drag, F_f is the skin friction drag, and F_Δ is the drag force allowance for roughness.

Example 7.9: *Ship's Propulsion Efficiency* An ocean liner 600 ft long has a wetted surface area of 90,000 ft^2. It requires 14,000 shaft horsepower to drive this ship at 16 knots through 68°F seawater ($\rho = 64$ lbm/ft^3, $\mu = 23 \times 10^{-6}$ lbf · s/ft^2. Estimate the propulsive efficiency of this ship.

Solution

This example is solved by computing the drag force of the ship and comparing it with the ideal drag.

(a) Unit conversions

$$v = 16 \text{ knots} \times 1.6878 \text{ ft/s/knot} = 27 \text{ ft/s}$$

$$P = 14{,}000 \text{ horsepower} \times 550 = 7.7 \times 10^6 \text{ ft lbf/s}$$

(b) Froude number

$$F = \frac{v}{\sqrt{Lg}} = \frac{27}{\sqrt{600 \times 32.17}} = 0.1943$$

(c) Reynolds number

$$\text{Re}_x = \frac{\rho x v}{\mu g_c} = \frac{64 \times 600 \times 27}{23 \times 10^{-6} \times 32.17} = 1.40 \times 10^9$$

(d) Residual coefficient
From Table 7.5 for $F = 0.1943$, $C_r = 6.98 \times 10^{-4}$ (by linear interpolation).

(e) Skin friction coefficient

$$C_f = \frac{0.455}{[\log_{10} \text{Re}_x]^{2.58}} = \frac{0.455}{[\log_{10}(1.40 \times 10^9)]^{2.58}} = 15.07 \times 10^{-4} \qquad (7.12)$$

(f) Drag force
Use Equation (7.21). For ships $\Delta C_f = 4 \times 10^4$:

$$F_D = \frac{(C_r + C_f + \Delta C_f)\rho v^2 S_w}{2g_c}$$

$$= \frac{(6.98 \times 10^{-4} + 15.07 \times 10^{-4} + 4 \times 10^{-4}) \times 64 \times 27^2 \times 90{,}000}{2 \times 32.17}$$

$$F_D = 170{,}000 \text{ lbf}$$

(g) Propulsive efficiency

$$P_D = F_D v = 170{,}000 \times 27 = 4.590 \times 10^6 \text{ ft} \cdot \text{lbf/s}$$

$$\text{efficiency} = \frac{\text{power required to overcome drag of ship}}{\text{power supplied}}$$

$$= \frac{P_D}{P} = \frac{4.590 \times 106}{7.7 \times 106} = 0.596, \text{ or } 59.60\%$$

7.10 LIFTING VANES

Definitions

In Figure 7.11 the following notation is used in studies of lifting vanes or airfoils:

c = chord length

s = span of vane

$\frac{s}{c}$ = aspect ratio

α = angle of attack

cs = characteristic area

Physical Analysis

When fluid flows about an object such as a lifting vane whose axis is not aligned with the flow, the velocities on either side of the object will have different magnitudes.

With the characteristic area being taken as cs, the product of the chord times the span, the drag and lift forces for lifting vanes can be written from equations (7.4) and (7.5) as follows:

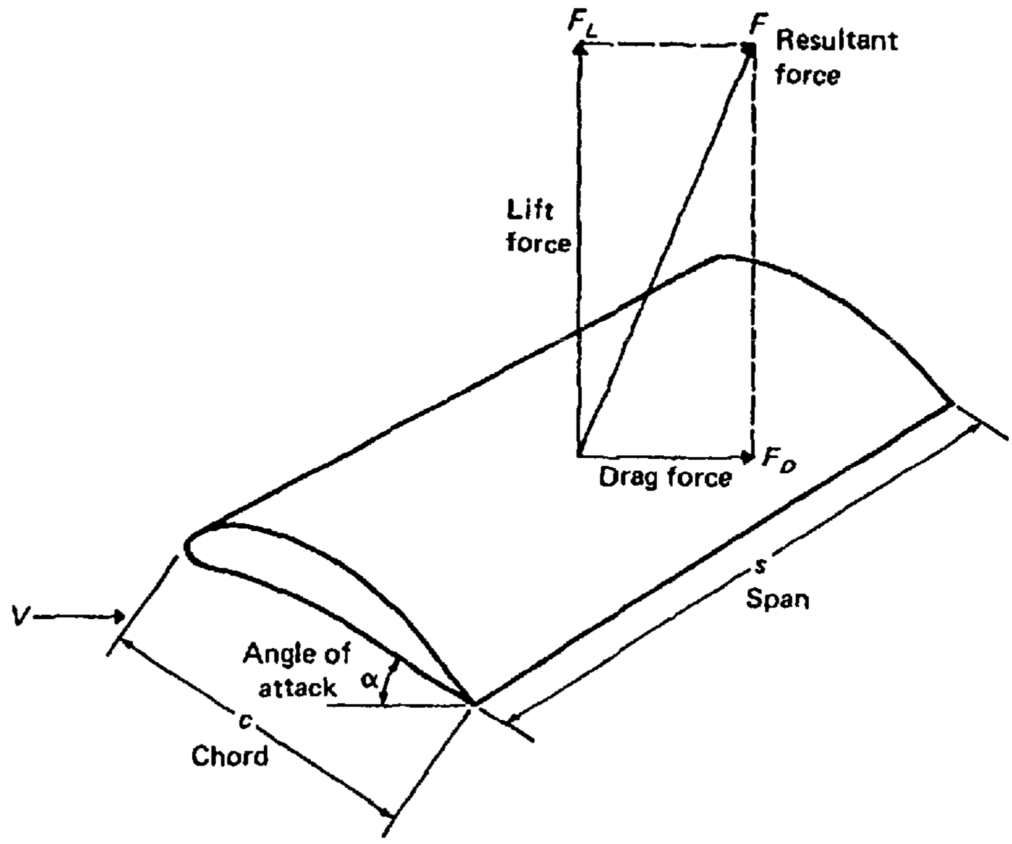


Figure 7.11 Notation for lifting vane study.

$$F_D = C_D \frac{\rho v^2 A}{2g_c} = C_D \frac{\rho v^2 cs}{2g_c} \tag{7.19}$$

$$F_L = C_L \frac{\rho v^2 A}{2g_c} = C_L \frac{\rho v^2 cs}{2g_c} \tag{7.20}$$

Although drag and lift coefficients are a function of Mach and Reynolds numbers, this section will be limited to the consideration of the "shape" effects on a typical lifting vane at constant Reynolds numbers in incompressible fluids.

Examination of Figure 7.12 indicates that the velocity is higher on the upper side and lower on the underside. The increased velocity of the upper side will result in decreased pressures, and the decreased velocities on the underside will result in increased pressures.

Figure 7.13 shows the pressure profile around a lifting vane. Lift is the result of these unbalanced pressures acting on the vane.

While it is possible to determine the lift characteristics by integration of the velocity profile for simple objects, actual practice is to develop the lift

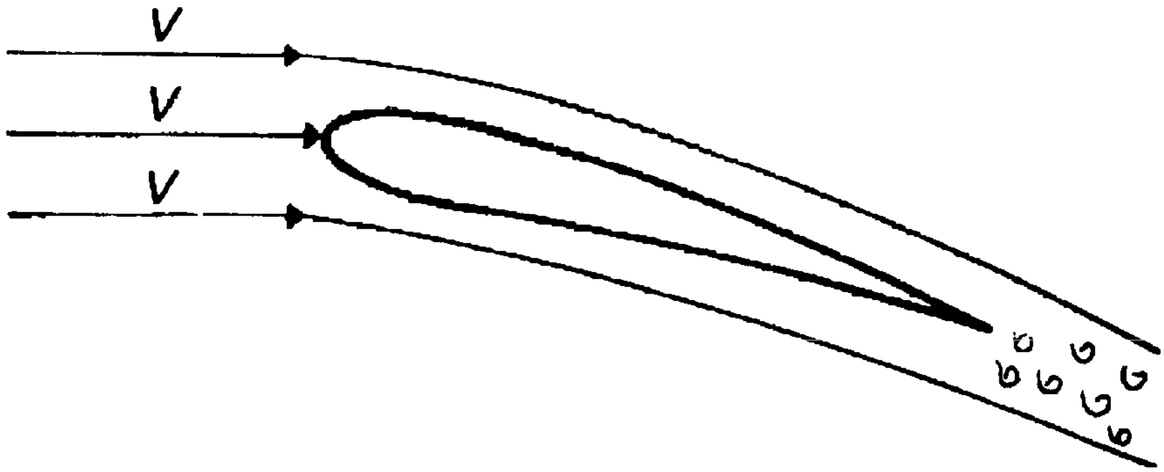


Figure 7.12 Flow around a lifting vane.

and drag coefficients from wind tunnel testing. Figure 7.14 shows the results of a typical test on a vane. The object is to design a vane to have the minimum drag for the maximum lift. *Stall* is the point where the coefficient of lift reaches its maximum and starts to decrease in value with increased angle of attack. When stall occurs, there is a marked turbulence in the wake and damage can result to machinery employing lifting-type vanes.

Example 7.10: *Power Required for Flight* A Boeing 727 airplane has a wing span of 145.9 ft and a chord length of 19.8 ft. It weighs 328,000 lbf when loaded and is designed to cruise at 500 knots at an altitude of 40,000 ft (density of air at 40,000 ft = 0.01889 lbm/ft^3). Estimate the power required to drive the airplane at this speed. Assume that the wing has the same characteristics as the lifting vane of Figure 7.14. Neglect lift and drag of the fuselage.

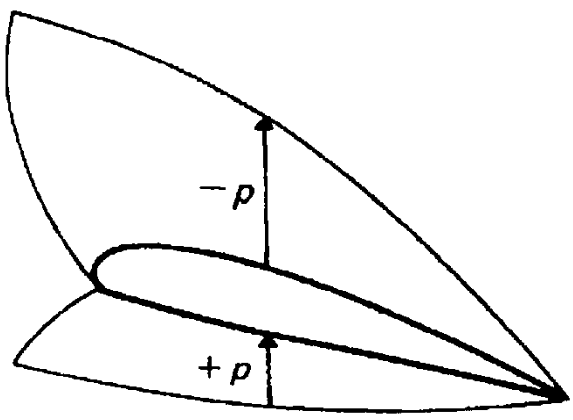
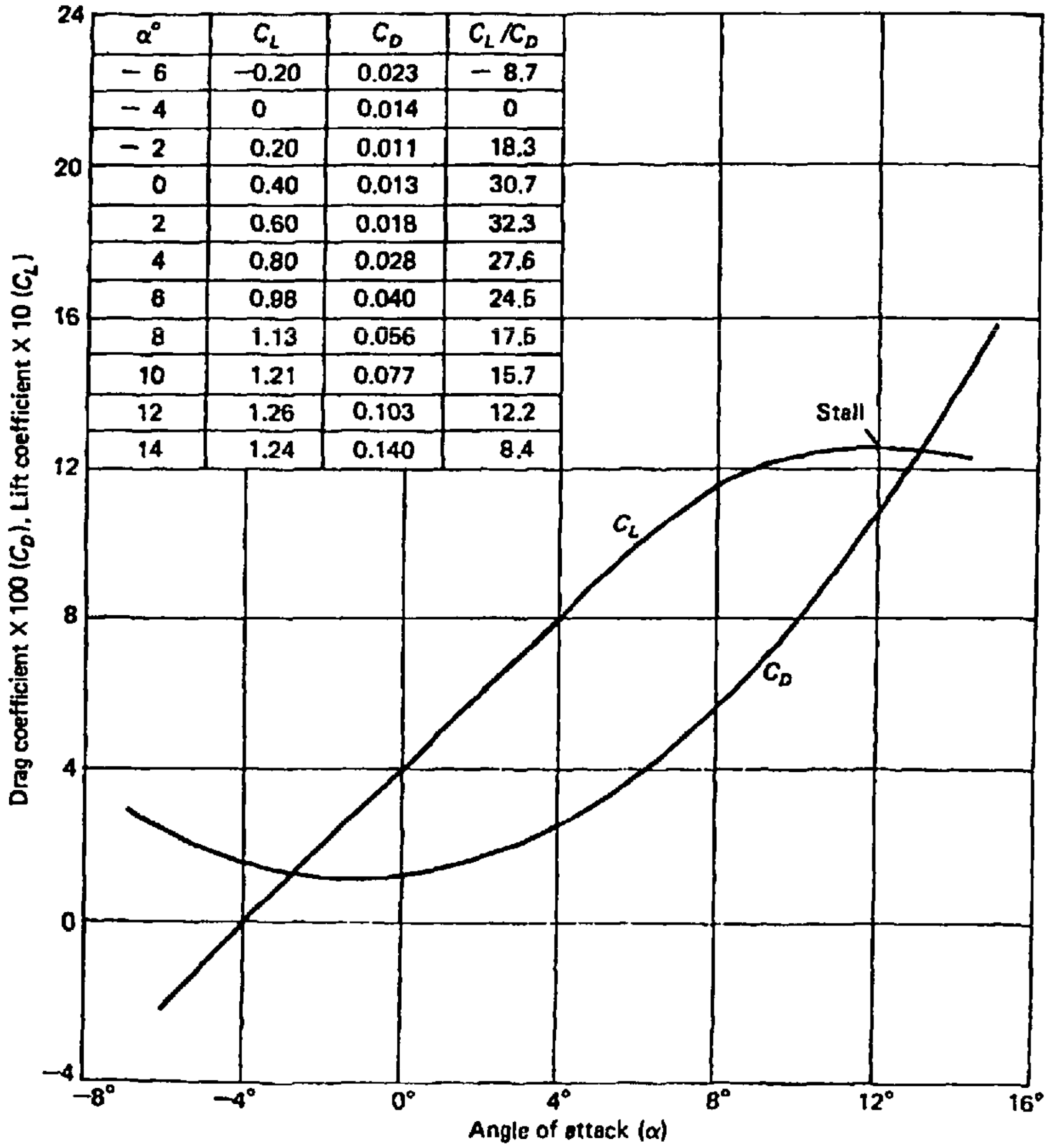


Figure 7.13 Pressure profile of a lifting vane.



$\alpha°$	C_L	C_D	C_L/C_D
−6	−0.20	0.023	−8.7
−4	0	0.014	0
−2	0.20	0.011	18.3
0	0.40	0.013	30.7
2	0.60	0.018	32.3
4	0.80	0.028	27.6
6	0.98	0.040	24.5
8	1.13	0.056	17.5
10	1.21	0.077	15.7
12	1.26	0.103	12.2
14	1.24	0.140	8.4

Figure 7.14 Characteristics of a typical lifting vane.

Solution

This example is solved by calculating the lift force. First, calculate the coefficient of lift and then calculate the drag force. Required power is computed from the drag force (the 727's power must overcome the drag force).

(a) Unit conversion

$v = 500 \text{ knots} \times 1.6878 \text{ ft/s/knot} = 844 \text{ ft/s}$

(b) Fluid properties

$\rho = 0.01889 \text{ lbm/ft}^3$

(c) Lift coefficient

If the buoyant force of the aircraft is neglected, the lift force is its weight, so the required lift coefficient becomes the following:

$$C_L = F_L \frac{2g_c}{\rho v^2 cs} = 328{,}000 \frac{2 \times 32.17}{0.01889 \times 844^2 \times 19.8 \times 145.9} = 0.5426 \tag{7.20}$$

(d) Drag coefficient

From Figure 7.14, $C_D = 0.0166$ (by linear interpolation).

(e) Drag force

Dividing equation (7.19) by equation (7.20) yields the following:

$$F_D = \frac{F_L C_D}{C_L} = \frac{328{,}000 \times 0.0166}{0.5426} = 10{,}035 \text{ lbf}$$

(f) Required power

$$P_D = F_D v = 10{,}035 \times 844 = 8{,}469{,}540 \text{ ft·lbf/s}$$

$$\text{horsepower} = \frac{8{,}469{,}540}{550} = 15{,}400 \text{ hp}$$

7.11 PROPELLER CHARACTERISTICS

Consider a propeller rotating in a fixed plane with fluid approaching at a velocity of v. The fluid has a density of ρ, a viscosity of μ, and a bulk modulus of elasticity of E. The propeller rotates at a speed of N, has a diameter of D, and produces a thrust of T for a power input of P.

An efficient propeller is one in which every cross section of each blade has the performance characteristics of a well-designed lifting vane. Figure 7.15 shows a cross section of a propeller blade at radius r. The study and analysis of propeller performance and design is called *blade element theory*.

The thrust force for a propeller is defined as follows:

$$T = C_T \frac{D^4 N^2 \rho}{g_c} \tag{7.21}$$

C_T in equation (7.21) is known as the thrust coefficient.

The power input to a propeller is expressed as follows:

$$P = C_P \frac{D^5 N^3 \rho}{g_c} \tag{7.22}$$

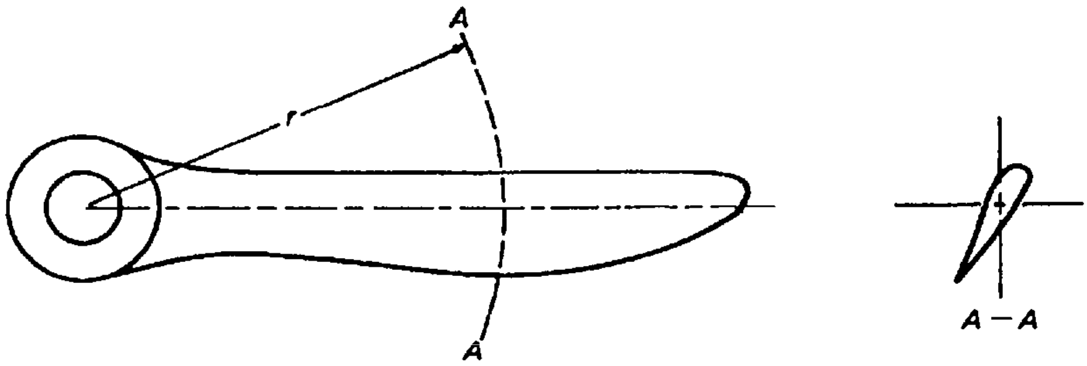


Figure 7.15 Propeller blade.

The velocity ratio V when applied to propellers is called the *advance-diameter ratio* because it is the distance (V/N) that the propeller could advance in the fluid per revolution divided by the diameter.

The efficiency η of a propeller is defined as the useful power divided by the power input. The useful power is defined as Tv, the thrust by equation (7.21) as $C_T D^4 N^2 \rho/g_c$, and the power input from equation (7.22) as $C_P D^5 N^3 \rho/g_c$. Therefore, the efficiency is expressed as follows:

$$\eta = \frac{\text{useful power}}{\text{power input}} = \frac{(C_T D^4 N^2 \rho/g_c)v}{C_P D^5 N^3 \rho/g_c} = \frac{C_T v}{C_P N D} = \frac{C_T V}{C_P} \tag{7.23}$$

Example 7.11: *Maximum Propeller Efficiency* A 9-ft-diameter propeller, which has the performance characteristics given in Figure 7.16, rotates at a speed of 22 rps at an altitude of 20,000 ft ($\rho = 0.04079$ lbm/ft^3). Determine for maximum propeller efficiency: (a) air speed, (b) available thrust, and (c) power required to drive propeller.

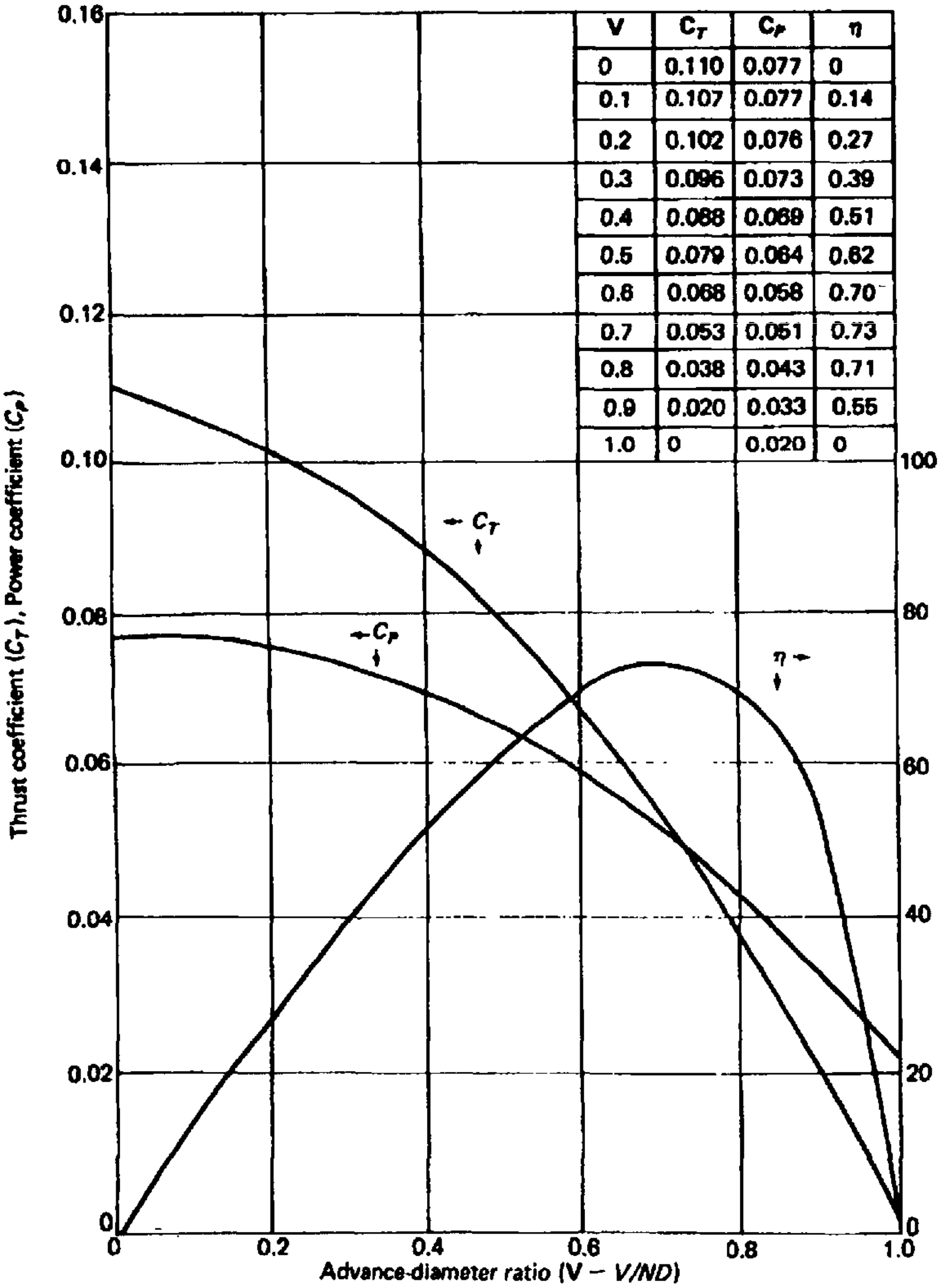
Solution

This example is solved by obtaining the advance diameter ratio from Figure 7.16 for maximum efficiency and calculating the required data.

(a) Fluid properties

$$\rho = 0.04079 \text{ lbm/ft}^3$$

(b) Advance diameter ratio for maximum efficiency
From Figure 7.16, maximum efficiency occurs at $V = 0.7$. At this point $C_T = 0.053$, $C_P = 0.051$, and $\eta = 0.73$.



V	C_T	C_P	η
0	0.110	0.077	0
0.1	0.107	0.077	0.14
0.2	0.102	0.076	0.27
0.3	0.096	0.073	0.39
0.4	0.088	0.069	0.51
0.5	0.079	0.064	0.62
0.6	0.068	0.058	0.70
0.7	0.053	0.051	0.73
0.8	0.038	0.043	0.71
0.9	0.020	0.033	0.55
1.0	0	0.020	0

Figure 7.16 Characteristics of a typical propeller.

(c) Air speed

$$v = VDN = 0.7 \times 9 \times 22 = 138.6 \text{ ft/s}$$

(d) Thrust

$$T = C_T \frac{D^4 N^2 \rho}{g_c} = 0.053 \frac{9^4 22^2 0.04079}{32.17} = 213.4 \text{ lbf} \quad (7.21)$$

(e) Power input

$$P = C_P \frac{D^5 N^3 \rho}{g_c} = 0.051 \frac{9^5 22^3 0.04079}{32.17} = 40{,}659 \text{ ft·lbf}$$

$$P = \frac{40{,}659}{550} = 73.93 \text{ hp} \tag{7.22}$$

8

Unsteady Flow

8.1 INTRODUCTION

This chapter considers unsteady-state phenomena applied to some important engineering applications, such as unsteady flow in closed conduits, velocity of pressure waves in elastic pipes, water hammer, and time to change tank levels.

Although examples are solved in USCS units only, the equations are structured so that SI units may also be used.

8.2 BACKGROUND

This chapter covers only a few facets of unsteady flow. For further information on this subject, the reader is referred to Rich (1963), a highly recommended source.

8.3 EQUATION OF MOTION

Velocity and Acceleration

In the most general case of fluid motion, the resultant fluid velocity, U, along a streamline is a function of both distance s and time t:

$U = f(s, t)$

The differential form of this relationship is written as equation (8.1):

$$dU = \frac{\partial U}{\partial s} ds + \frac{\partial U}{\partial t} dt \tag{8.1}$$

An expression for acceleration (α) may be obtained by dividing equation (8.1) by dt and noting that $U = ds/dt$:

$$\alpha = \frac{dU}{dt} = \frac{\partial U}{\partial s}\frac{ds}{dt} + \frac{\partial U}{\partial t}\frac{dt}{dt} = U\frac{\partial U}{\partial s} + \frac{\partial U}{\partial t} \tag{8.2}$$

For steady flow, $\partial U/\partial t = 0$.

Fluid Forces

Consider the fluid element flowing in the streamtube shown in Figure 8.1. This element has a length of dL, an area normal to the motion of dA, and a perimeter of dP. The elemental mass is $\rho\, dA\, dL$. The increase in elevation of this mass is dz, and the motion of the element is upward.

Forces tending to change the velocity U of this fluid mass are as follows:

1. *Pressure* forces on the ends of the element:

$$dF_p = p\, dA - \left(p + \frac{\partial p}{\partial L}\, dL\right) dA = -\frac{\partial p}{\partial L}\, dL\, dA \tag{8.3}$$

2. *Gravity* force due to the component of weight in the direction of motion:

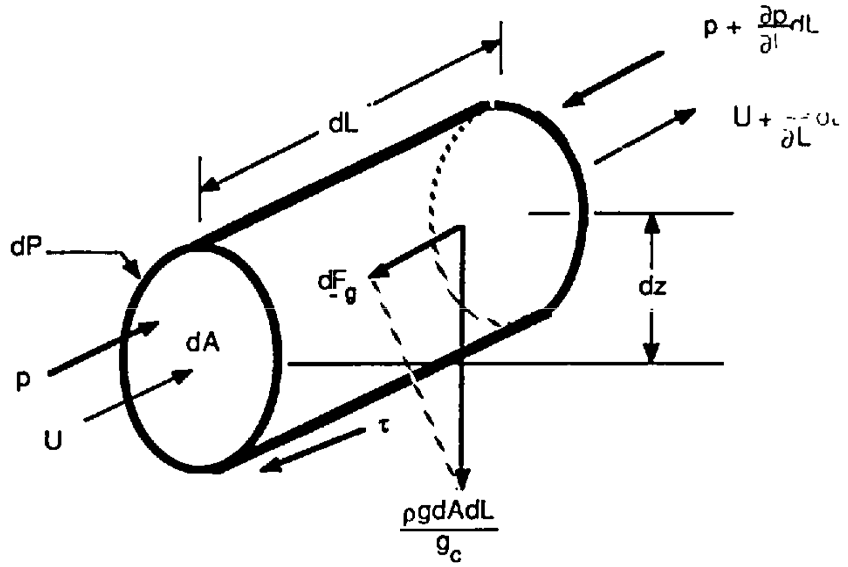


Figure 8.1 Elements of a streamtube.

$$dF_g = -\frac{\rho g\ dA\ dL}{g_c}\left(\frac{dz}{dL}\right) = \frac{\rho g\ dA\ dz}{g_c} \tag{8.4}$$

3. *Friction* force on the outer surface of the element:

$$dF_f = -\tau\ dP\ dL \tag{8.5}$$

4. The *combined* force becomes the following:

$$\begin{aligned}\sum dF &= \frac{\partial p}{\partial L}\ dA\ dL - \frac{\rho g\ dA\ dz}{g_c} - \tau\ dP\ dA\ dL \\ &= -dA\left(\frac{\partial p}{\partial L}\ dL + \frac{\rho g\ dz}{g_c} + \frac{\tau\ dP\ dL}{dA}\right)\end{aligned} \tag{8.6}$$

Newton's Second Law of Motion

Newton's second law of motion was first presented as equation (1.12). It is rewritten here as equation (8.7):

$$\sum dF = \frac{m\alpha}{g_c} = \frac{\rho\ dA\ dL}{g_c}\left(U\frac{\partial U}{\partial L} + \frac{\partial U}{\partial t}\right) \tag{8.7}$$

Equating equation (8.6) and equation (8.7) yields the following:

$$-dA\left(\frac{\partial p}{\partial L}dL + \frac{\rho g\ dz}{g_c} + \frac{\tau\ dP\ dL}{dA}\right) = \frac{\rho\ dA\ dL}{g_c}\left(U\frac{\partial U}{\partial L} + \frac{\partial U}{\partial t}\right) \tag{8.8}$$

Simplifying equation (8.8) results in equation (8.9):

$$\frac{g}{g_c}\ dz + \frac{dL}{\rho}\left(\frac{\partial p}{\partial L}\right) + \frac{dL}{g_c}\left(U\frac{\partial U}{\partial L} + \frac{\partial U}{\partial t}\right) + \frac{\tau\ dL}{\rho}\left(\frac{dP}{dA}\right) = 0 \tag{8.9}$$

The length of the element does not change with either time or distance, so $(\partial p/\partial L) = (dp/dL)$, $(\partial U/\partial L) = (dU/dL)$, and $(dA/dP) = R_h$, the hydraulic radius. Substituting these relations in equation (8.9) and simplifying results in equation (8.10):

$$\frac{g}{g_c}\ dz + \frac{dp}{\rho} + \frac{U\ dU}{g_c} + \frac{dL}{g_c}\left(\frac{\partial U}{\partial t}\right) + \frac{\tau\ dL}{\rho R_h} = 0 \tag{8.10}$$

8.4 ONE-DIMENSIONAL EQUATION OF MOTION

When the flow is one-dimensional, $v = U$. Substituting v for U in equation (8.10) gives the following expression:

$$\frac{g}{g_c}\,dz + \frac{dp}{\rho} + \frac{v\,dv}{g_c} + \frac{DL}{g_c}\left(\frac{\partial v}{\partial t}\right) + \frac{\tau\,dL}{\rho R_h} = 0 \tag{8.11}$$

Equation (8.11) may be arranged in Bernoulli form by multiplying it by g_c/g and noting from equation (1.14) that $\gamma = \rho g/g_c$:

$$dz + \frac{dp}{\gamma} + \frac{v\,dv}{g} + \frac{dL}{g}\left(\frac{\partial v}{\partial t}\right) + \frac{\tau\,dL}{\gamma R_h} = 0 \tag{8.12}$$

Integrating equation (8.12) for an incompressible fluid ($\gamma_1 = \gamma_2$) leads to the following:

$$z_2 - z_1 + \frac{p_2 - p_1}{\gamma} + \frac{v_2^2 - v_2^2}{2g} + \frac{1}{g}\int_1^2 \left(\frac{\partial v}{\partial t}\,dL\right) + \frac{1}{\gamma R_h}\int_1^2 \tau\,dL = 0 \tag{8.13}$$

Let

$$H_L = \frac{1}{\gamma R_h}\int_1^2 \tau\,dL \tag{8.14}$$

where H_L is the head "lost" due to friction. Substituting this expression into equation (8.13) yields the one-dimensional time-dependent equation of motion:

$$z_2 - z_1 + \frac{p_2 - p_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + \frac{1}{g}\int_1^2 \left(\frac{\partial v}{\partial t}\,dL\right) + H_L = 0 \tag{8.15}$$

Note that when $\partial v/\partial t = 0$ (steady flow), equation (8.15) reduces to equation (8.1).

8.5 UNSTEADY INCOMPRESSIBLE FLOW IN CLOSED CONDUITS

For flow in pipes the acceleration term of equation (8.15) may be integrated by noting that the length of the pipe, L, is not a function of either velocity or time and that $\partial v/\partial t$ becomes dv/dt. Integrating the acceleration term of equation (8.15), changing the notation, and rearranging the terms produces the following:

$$\left(\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2\right) + \frac{L}{g}\frac{dv}{dt} + H_L \tag{8.16}$$

For an incompressible fluid in a constant-area conduit, $v_1 = v_2$. From equation (4.3), the friction head loss term, $H_L = \sum Kv^2/2g$, so equation (8.16) may now be written as follows:

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + \frac{L}{g}\frac{dv}{dt} + \sum K \frac{v^2}{2g} \tag{8.17}$$

In equation (8.17), $\sum K$ is the sum of the resistance coefficients of the pipe.

8.6 ESTABLISHMENT OF STEADY FLOW

Figure 8.2 shows a pipe of length L connected to a reservoir or some other limitless source of an incompressible fluid supplying a constant head of H. At the start of the process, the valve at the end of the pipe is closed. The valve is then suddenly opened and liquid begins to flow. The frictional resistance of the pipe is $\sum K$. The time required to establish steady flow may be determined as follows.

Considering section 1 to be at the liquid surface and section 2 to be at the valve discharge (both under atmospheric pressure), then $p_1 = p_2$, and $H = z_1 - z_2$. For this situation, equation (8.17) reduces to the following:

$$\frac{L}{g}\frac{dv}{dt} = H - \sum K \frac{v^2}{2g} \qquad \text{or} \qquad \int dt = \frac{L}{g}\int \frac{dv}{H - \sum K \frac{v^2}{2g}} \tag{8.18}$$

For steady flow, $dv/dt = 0$. Rearranging equation (8.18) yields the following:

$$H - \sum K \frac{v_F^2}{2g} = 0 \qquad \text{or} \qquad \frac{\sum K}{2g} = \frac{H}{v_F^2} \tag{8.19}$$

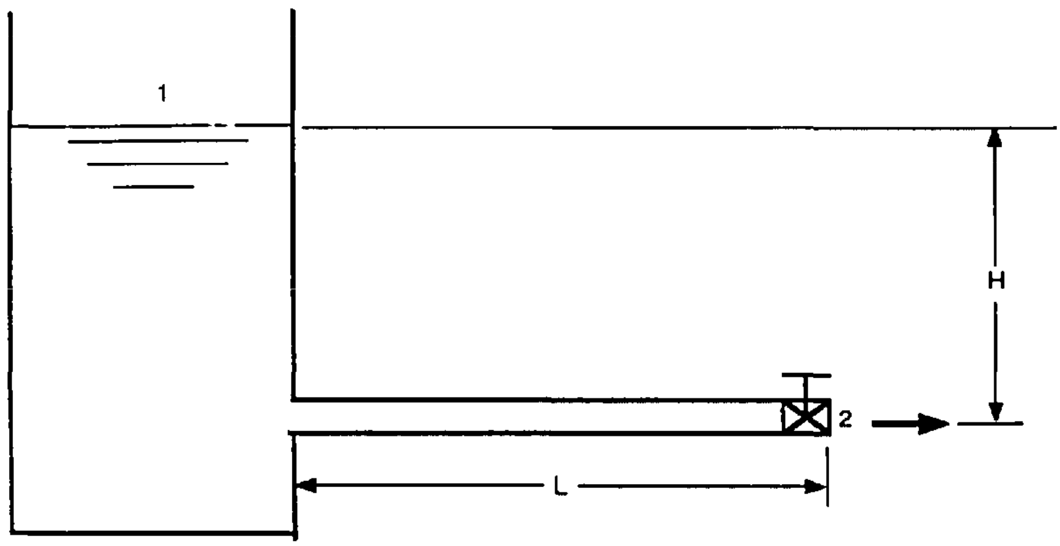


Figure 8.2 Notation for establishment of flow.

Substituting equation (8.19) in equation (8.18) and simplifying yields the following:

$$\int_0^t dt = \frac{L}{g}\int_0^v \frac{dv}{H - H\dfrac{v^2}{v_F^2}} = \frac{Lv_F^2}{gH}\int_0^v \frac{dv}{v_F^2 - v^2} = t = \frac{Lv_F}{2gH}\log_e\left(\frac{v_F + v}{v_F - v}\right) \tag{8.20}$$

Mathematically, the time required for v_F to equal v is infinity.

Example 8.1: *Time to Establish Flow* Water is to be discharged from a tank through a piping system into the atmosphere under a constant head of 20 feet. Valves, fittings, and pipe in the system have a combined resistance coefficient ($\sum K$) of 62. After a valve is suddenly opened to permit flow, how much time will it take to attain 90% of the final velocity?

Solution

This example is solved by calculating the final velocity using equation (8.19) and the time from equation (8.20).

(a) Final (steady-state) velocity:

$$v_F = \sqrt{\frac{2gH}{\sum K}} = \sqrt{\frac{2 \times 32.17 \times 20}{62}} = 4.56 \text{ ft/s} \tag{8.19}$$

(b) Time to establish 90% of flow:

$$t = \frac{Lv_F}{2gH}\log_e\left(\frac{v_F + v}{v_F - v}\right) = \frac{2000 \times 4.56}{2 \times 32.17 \times 20}\log_e\left(\frac{4.56 + 0.9 \times 4.46}{4.56 - 0.9 \times 4.46}\right) = 20.87 \text{ s} \tag{8.20}$$

8.7 VELOCITY OF PRESSURE WAVES IN PIPES

In Section 1.6.2, the acoustic velocity of a fluid was derived assuming the pipe to rigid. This analysis resulted in equation (1.57):

$$c = \sqrt{\frac{E_s g_c}{\rho}} \tag{1.57}$$

Pressure in an elastic pipe causes the pipe walls to stretch, and a combined modulus of elasticity E_c is needed to account for the volume change of both the fluid and the pipe. The change in volume is expressed in equation (8.21):

$$dv = dv_{\text{fluid}} + dv_{\text{pipe}} \tag{8.21}$$

The combined modulus may be expressed as follows:

$$\frac{1}{E_c} = -\frac{dv}{v\,dp} = \left(-\frac{dv}{v\,dp}\right)_{\text{fluid}} + \left(-\frac{dv}{v\,dp}\right)_{\text{pipe}} \tag{8.22}$$

The first right-hand term of equation (8.22) is $1/E_s$.

From the mechanics of materials, Young's modulus of elasticity is defined as E_Y = unit stress/unit strain. Applying this concept to the circumferential stress in a thin-walled cylinder gives a unit stress of $r\,dp/t_w$. If the circumference is stretched by the amount dl, the increment of unit deformation is $dl/2\pi r$. Further, $dl = 2\pi\,dr$. From these relations, the following can be written:

$$\begin{aligned} E_Y &= \frac{\text{unit stress}}{\text{unit strain}} \\ &= \frac{r\,dp/t_w}{dl/2\pi r} = \frac{r\,dp/t_w}{2\pi\,dr/2\pi r} = \frac{r^2\,dp}{t_w\,dr} \\ dp &= \frac{E_Y t_w\,dr}{r^2} \end{aligned} \tag{8.23}$$

The volume of the pipe is $V = \pi r^2 L$, and the increase in volume is $dV = 2\pi r\,dr\,L$. Substituting for dv, v, and dp from equation (8.23) in the second right-hand term of equation (8.22) results in the following:

$$\left(-\frac{dv}{v\,dp}\right)_{\text{pipe}} = -\frac{2\pi r\,dr\,L}{(\pi r^2 L)(E_Y t_w\,dr\,L/r^2)} = \frac{2r}{E_Y t_w} = \frac{D}{E_Y t_w} \tag{8.24}$$

Equation (8.22) may now be written as follows:

$$\frac{1}{E_c} = \frac{1}{E_s} + \frac{1}{E_Y}\left(\frac{D}{t_w}\right) \quad \text{or} \quad E_c = \frac{E_s}{1 + (E_s/E_Y)(D/t_w)} \tag{8.25}$$

Substituting E_c from equation (8.25) for E_s in equation (1.57) results in the following:

$$c_p = \sqrt{\frac{g_c}{\rho}\left(\frac{E_s}{1 + (E_s/E_Y)(D/t_w)}\right)} \tag{8.25}$$

In this equation c_p is the velocity of a pressure wave in an elastic pipe. Table 8.1 shows the values of E_Y and E_s for selected substances.

Table 8.1 Modulus of Elasticity for Selected Substances at 68°F (20°C)

Solid	Young's modulus of elasticity, E_Y lbf/in.^{2}	GPa
Aluminum	10,000,000	69
Brass	15,000,000	103
Bronze	16,000,000	110
Cast iron, gray	10,000,000	69
Cast iron, malleable	24,000,000	165
Concrete	4,000,000	28
Magnesium	7,000,000	48
Monel	26,000,000	179
Nylon	300,000	2
Polystyrene	450,000	3
Steel	29,000,000	200
Steel, stainless	28,000,000	193
Titanium	17,000,000	117
Liquid	**Bulk modulus of elasticity, E_v lbf/in.^{2}**	**GPa**
Benzene	223,000	1.54
Carbon tetrachloride	204,000	1.41
Ethanol	155,000	1.07
Glycerin	676,000	4.66
Mercury	4,150,000	28.61
Octane, normal	145,000	1.00
Water	318,000	2.19

Example 8.2: *Velocity of a Pressure Wave* Water at 68°F flows in a 10-in. schedule 40 steel pipe. Estimate the velocity of a sound wave, assuming (1) the pipe to be rigid, and (2) the pipe to be elastic.

Solution

This example is solved by using equation (1.57) to determine part (1) and equation (8.25) for part (2).

(a) Substance properties
Water at 68°F: From Table D.1, $\rho = 62.31$ lbf/ft^3; from Table 8.1, $E_v =$ 318,000 lbf/ft^2.

Steel at 68°F: From Table 8.1, $E_y = 29{,}000{,}000$ lbf/ft^2.

(b) Pipe properties
From Table C.3 for 10-in. schedule 40 steep pipe:

$$d = 0.8350 \times 12 = 10.02 \text{ in.}, \; t_w = 0.365 \text{ in.}$$

Part 1: *Rigid pipe*

$$c = \sqrt{\frac{E_s g_c}{\rho}} = \sqrt{\frac{(318{,}000 \times 1144) \times 32.17}{62.31}} = 4{,}862 \text{ ft/s}$$

Part 2: *Elastic pipe*

$$c_p = \sqrt{\frac{g_c}{\rho}\left(\frac{E_s}{1 + E_s/E_y)(D/t_w)}\right)}$$

$$c_p = \sqrt{\left(\frac{32.17}{62.31}\right)\left(\frac{318{,}000 \times 144}{1 + (318{,}000/29{,}000{,}000)(10.02/0.365)}\right)} \tag{8.25}$$

$$c_p = 4{,}263 \text{ ft/s}$$

8.8 WATER HAMMER

Water hammer is a series of shocks sounding like hammer blows that is produced by suddenly reducing the flow of a fluid in a pipe or conduit. Water hammer can be dangerous, causing pipes to burst.

Consider an incompressible fluid flowing with a velocity v in the elastic pipe shown in Figure 8.3. The valve, which is located at distance L from the pipe entrance, is suddenly closed, reducing the velocity from v to 0. The pressure wave produced by this sudden closure has a velocity of c_p traveling upstream in the opposite direction of v. In a short interval of time, dt, an element of fluid $c_p\ dt$ is brought to rest. Application of the impulse-momentum concept yields equation (8.26):

$$F\,dt = \frac{m\,dv}{g_c} = pA - [pA - (p + dp)A]dt = \frac{(\rho A c_p\,dt)\,dv}{g_c}$$

or

$$-dp = \frac{\rho c_p\,dv}{g_c} \tag{8.26}$$

Integrating equation (8.26) leads to an expression for the increase in pressure due to the sudden closing of the valve:

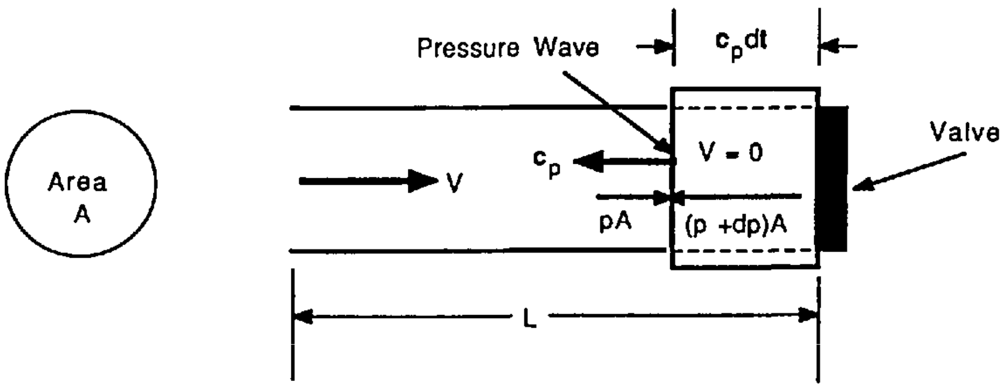


Figure 8.3 Notation for water hammer study.

$$\int_0^{p_{WH}} dp = -\frac{\rho c_p}{g_c}\int_v^0 dv = p_{WH} = \frac{\rho c_p v}{g_c} \tag{8.27}$$

p_{WH} is the increase in pressure due to the sudden closing of the valve.

Time of Valve Closure

The time for the pressure wave to travel the length of pipe L and return is given in equation (8.28):

$$t_c = \frac{2L}{c_p} \tag{8.28}$$

Here, t_c is the critical time for the valve to close. If the valve is closed in time t_c or less, it may be considered to be a *sudden closure* and produces an increase of pressure p_{WH} as calculated by equation (8.27). If the time of closure is greater than t_c, then it is called a *slow closure*. The approximate pressure rise due to slow closure may be estimated using the following:

$$p_{SC} \approx \frac{t_c}{t}p_{WH} = \frac{(2L/c_p)}{t}\left(\frac{\rho c_p v}{g_c}\right) = \frac{2L\rho v}{t g_c} \tag{8.29}$$

In this equation, p_{SC} is the increase in pressure due to the slow closing of the valve.

Example 8.3: *Pressure Rise Due to Water Hammer* Water flows at 68°F in a 3-in. schedule 40 steel pipe with a velocity of 10 ft/s. A valve located 200 ft downstream is closed in (1) 1/20 second, (2) 10 seconds. Estimate the pressure rise in the pipe.

Solution

This example is solved by calculating the critical time of closure using equation (8.28). The pressure rise is then calculated using equation (8.27) or equation (8.29) as appropriate.

(a) Substance properties
Water at 68°F: From Table D.1, $\rho = 62.31$ lbf/ft^3; from Table 8.1, $E_s =$ 318,000 lbf/ft^2.
Steel at 68°F: From Table 8.1, $E_y = 29{,}000{,}000$ lbf/ft^2.

(b) Pipe properties
From Table C.3 for 3-in. schedule 40 steel pipe:

$$D = 0.2557 \times 12 = 3.0684 \text{ in.}, \qquad t_w = 0.365 \text{ in.}$$

(c) Velocity of the pressure wave

$$c_p = \sqrt{\frac{g_c}{\rho}\left(\frac{E_s}{1 + E_s/E_y)(D/t_w)}\right)}$$

$$c_p = \sqrt{\left(\frac{32.17}{62.31}\right)\left(\frac{318{,}000 \times 144}{1 + (318{,}000/29{,}000{,}000)(3.068/0.216)}\right)} \tag{8.25}$$

$$c_p = 4{,}523 \text{ ft/s}$$

(d) Critical time for valve closure

$$t_c = \frac{2L}{c_p} = \frac{2 \times 200}{4523} = \frac{1}{11}\text{ s} \tag{8.28}$$

Part 1: Closure time is 1/20 second, which is less than the 1/11 second just calculated:

$$p_{WH} = \frac{\rho c_p v}{g_c} = \frac{62.31 \times 4523 \times 10}{32.17} = 87{,}606 \text{ lbf/ft}^2/144 = 608 \text{ psi} \tag{8.27}$$

Part 2: Closure time is 10 seconds, which is greater than the 1/11 second just calculated:

$$p_{SC} = \frac{2L\rho v}{tg_c} = \frac{2 \times 200 \times 62.31 \times 10}{10 \times 32.17} = \frac{775 \text{ lbf/ft}^2}{144} = 5.38 \text{ psi} \tag{8.29}$$

Example 8.4: *Forces on an Air Duct* Air at 77 F flows with a velocity of 20 ft/s through a 5-ft square $\frac{1}{16}$-in.-thick aluminum metal ventilation duct. The flow is suddenly stopped by control devices. Estimate the force produced on the 5-ft × 5-ft area of closure.

Solution

This example is solved by considering air to be incompressible at standard atmospheric pressure. The temperature of 77°F is close enough to 68°F to permit the use of Table 8.1 for Young's modulus E_{Y} for aluminum. The force is then calculated using equation (8.27).

(a) Substance properties
Air at 77°F and 14.696 psia: From Table D1, $M = 28.97$ lbm/lbmole:

$$\rho = \frac{p}{RT} = \frac{144 \times 14.696}{(1545/28.97) \times (77 + 460)} = 0.07398 \text{ lbm/ft}^3 \tag{1.39}$$

From Table D.2, $k = 1.401$:

$$E_s = kp = 1.401 \times (14.696 \times 144) = 2{,}965 \text{ lbf/in.}^2$$

Aluminum at 68°F: From Table 8.1, $E_{\text{Y}} = 10{,}000{,}000$ lbf/in.^{2}.

(b) Duct properties
From Table C.2 for a square duct flowing full:

$$A = D^2 = 5 \times 5 = 25 \text{ ft}^2$$

$$D_e = 4R_h = 5 \times 12 = 60 \text{ in.,} \qquad t_w = \frac{1}{16} \text{ in.}$$

(c) Velocity of the pressure wave

$$c_p = \sqrt{\frac{g_c}{\rho}\left(\frac{E_s}{1 + (E_s/E_{\text{Y}})(D/t_w)}\right)}$$

$$c_p = \sqrt{\left(\frac{32.17}{0.07398}\right)\left(\frac{2965}{1 + (2965/10{,}000{,}000)[60/(1/16)]}\right)} \tag{8.25}$$

$$c_p = 1{,}002 \text{ ft/s}$$

(d) Pressure increase

$$p_{WH} = \frac{\rho c_p v}{g_c} = \frac{0.07398 \times 1002 \times 20}{32.17} = 46 \text{ lbf/ft}^2 \tag{8.27}$$

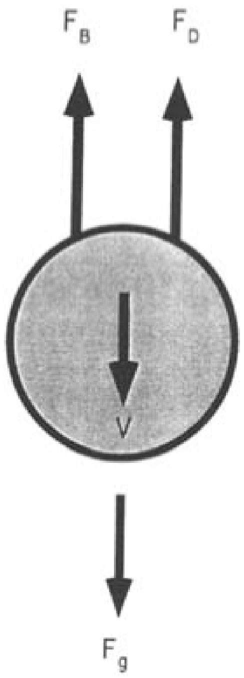


Figure 7.7 Notation for Example 7.4.

(b) Properties of the sphere

$$D = \frac{0.06}{12} = 0.005 \text{ ft}$$

$$V = \frac{\pi D^3}{6}$$

(c) Drag force

$$F_D = 3\pi\mu D v$$

(d) Buoyant force

$$F_B = \gamma_o V$$

(e) Gravity force

$$F_g = \gamma_s V$$

(f) Free body

$$\sum Fz = 0 = F_B - F_g - F_L = F_B - F_g + F_D$$

Substituting the force equations into the free-body equation and solving for μ results in the following:

$$\rho_1 A_1 v_1 = \rho A\left(\frac{dz}{dt}\right) + \rho_2 A_2 v_1$$

For a liquid, $\rho_1 = \rho = \rho_2$ and $Q = Av$; the conservation of mass statement becomes:

$$Q_{\text{in}} = A\,\frac{dz}{dt} + Q_{\text{out}} \qquad \text{or} \qquad \int_{t_1}^{t_2} dt = \int_{z_1}^{z_2} \frac{A\,dz}{Q_{\text{in}} - Q_{\text{out}}} \tag{8.30}$$

Tanks of Uniform Cross Section

Figure 8.5 shows a tank of uniform cross section A being supplied at the rate of Q_{in} and drained by a piping system whose internal diameter is D. The piping system has a total resistance coefficient of $\sum K$.

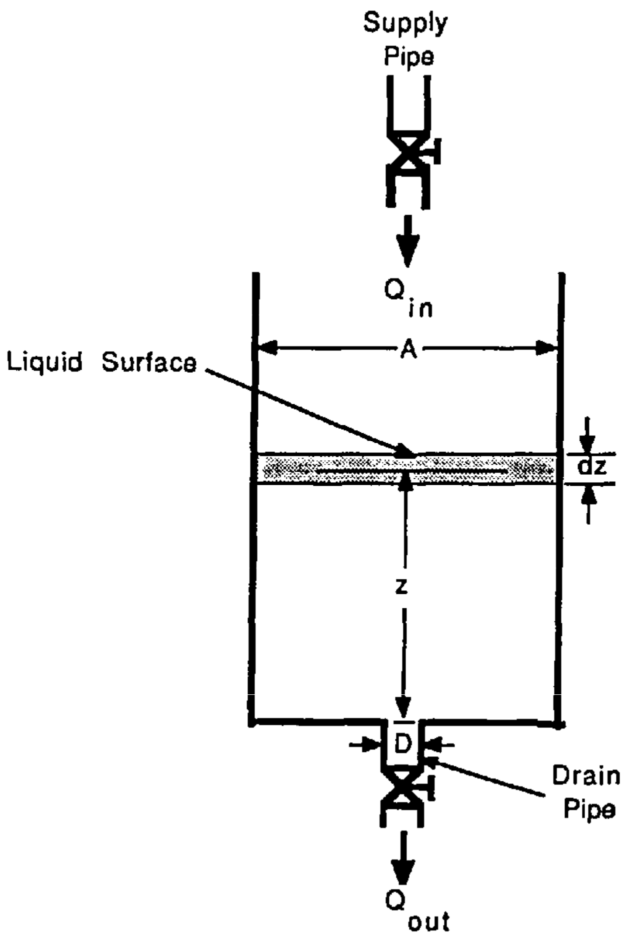


Figure 8.5 Tank of uniform cross section.

Application of pressure-height relations, equation (2.6), with energy loss, equation (4.1), and the definition of resistance coefficient, equation (4.3), results in the following:

$$\frac{\Delta p}{\rho} = H_f - \sum K \frac{v^2}{2g_c} = \frac{gz}{g_c} \qquad \text{or} \qquad v = \sqrt{\frac{2gz}{\sum K}} \tag{8.31}$$

The volumetric flow rate Q through the pipe from the continuity equation $= Av$; substituting this into equation (8.31) produces the following:

$$Q_{\text{out}} = Av = \frac{\pi D^2}{4}\sqrt{\frac{2gz}{\sum K}} \tag{8.32}$$

Solving equation (8.32) for z yields the following:

$$z = 8\left(\frac{\sum K}{g\pi^2 D^4}\right) Q_{\text{out}}^2 \tag{8.33}$$

Differentiating equation (8.33) yields the following:

$$dz = 16\left(\frac{\sum K}{g\pi^2 D^4}\right) Q_{\text{out}}\, dQ_{\text{out}} \tag{8.34}$$

Substituting equation (8.34) in equation (8.30) gives the following expression:

$$\int_{t_1}^{t_2} dt = \frac{16\sum K}{g\pi^2 D^4} \int_{Q_{\text{out1}}}^{Q_{\text{out2}}} \frac{Q_{\text{out}}\, dQ_{\text{out}}}{Q_{\text{in}} - Q_{\text{out}}} \tag{8.35}$$

Integrating equation (8.35) yields the desired equation for time to lower the level in a tank:

$$t_2 - t_1 = \frac{16\sum K}{g\pi^2 D^4}\left[Q_{\text{in}} \log_e\left(\frac{Q_{\text{in}} - Q_{\text{out1}}}{Q_{\text{in}} - Q_{\text{out2}}}\right) + Q_{\text{out1}} - Q_{\text{out2}}\right] \tag{8.36}$$

Example 8.5: *Time to Lower the Level in Tank with Inflow* An open cylindrical tank 6 ft in diameter is filled with water to a depth of 10 ft. A 4-in. internal diameter piping system is located at the bottom of the tank. The combined resistance coefficient of the piping system is $\sum K = 2.69$. A pipe at the top of the tank supplies water at the rate of 1 ft^3/s. Determine (1) the time to lower the level by 2 ft and (2) the minimum height of the tank required to prevent overflow.

Solution

This example is solved by application of equation (8.36) for the time to change tank levels. The level to prevent overflow is achieved once steady-state conditions are achieved.
Part 1: Time to lower level from 10 ft to 8 ft:

$$Q_{\text{out}} = \frac{\pi D^2}{4}\sqrt{\frac{2gz}{\sum K}} = \frac{\pi(4/12)^2}{4}\sqrt{\frac{2 \times 32.17}{2.69}}z^{1/2} = 0.4268z^{1/2} \tag{8.32}$$

$$Q_{\text{out1}} = 0.4268 \times (10)^{1/2} = 1.350 \text{ ft}^3/s$$
$$Q_{\text{out2}} = 0.4268 \times (8)^{1/2} = 1.207 \text{ ft}^3/\text{s}$$

$$t_2 - t_1 = \frac{16\sum K}{g\pi^2 D^4}\left[Q_{\text{in}} \log_e\left(\frac{Qin - Q_{\text{out1}}}{Q_{\text{in}} - Q_{\text{out2}}}\right) + Q_{\text{out1}} - Q_{\text{out2}}\right]$$
$$t_2 - t_1 = \frac{16 \times 2.69 \times 28.97}{32.17 \times \pi^2 \times (4/12)^4}\left[1 \times \log_e\left(\frac{1 - 1.350}{1 - 1.207}\right) + 1.350 - 1.207\right]$$
$$= 207 \text{ s} \tag{8.36}$$

Part (2): Minimum tank height to prevent overflow:

$$Q_{\text{out}} = Q_{\text{in}} = 1 = 0.4268z^{1/2}, \qquad z = 5.49 \text{ ft}$$

8.10 TIME TO CHANGE TANK LEVELS WITHOUT INFLOW

If there is no inflow, then $Q_{\text{in}} = 0$ and equation (8.36) reduces to the following:

$$t_2 - t_1 = \frac{16\sum K}{g\pi^2 D^4}(Q_{\text{out1}} - Q_{\text{out2}}) \tag{8.37}$$

Substituting equation (8.32) in equation (8.37) yields the following:

$$t_2 - t_2 = \frac{8A}{\pi D^2}\sqrt{\frac{\sum K}{2g}}(z_1^{1/2} - z_2^{1/2}) \tag{8.38}$$

Example 8.6: *Time to Discharge a Quantity of Liquid from a Tank* The tank shown in Figure 8.6 has a uniform internal diameter of 20 ft and is filled with 200,000 gallons of a liquid. The drain piping is located on a centerline 3 ft above the bottom of the tank and consists of 4-in. schedule 40 steel pipe containing a sharp-edged inlet, one flanged globe valve ($K_v = 6.45$), and one flanged 90° standard elbow ($K_F = 0.49$). The length of straight pipe is negligible. Estimate the time required to drain 50,000 gallons from the tank.

Solution

This example is solved by the application equation (8.38) and the theory presented in Chapter 4.

(a) Pipe properties
From Table C.3 for schedule 40 steel pipe:

$$D = 0.3355 \text{ ft}$$

From Table 4.4 for sharp-edged inlet: $K_i = 0.5$:

$$Exit: \quad K_e = 1.0$$

The total pipe resistance coefficient is:

$$\sum K = K_i + K_v + K_F + K_e = 0.5 + 6.45 + 0.49 + 1.0 = 8.44$$

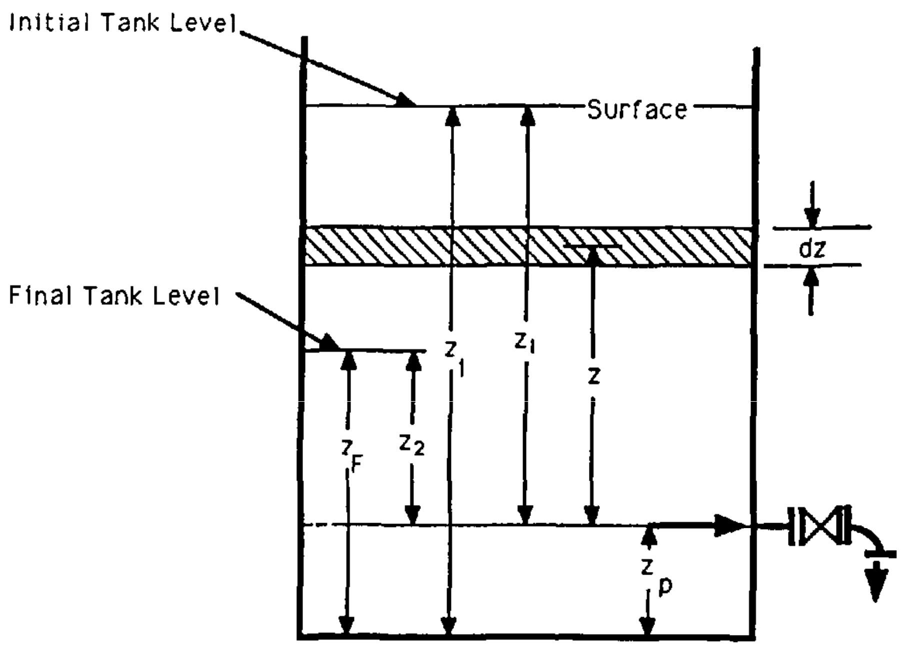


Figure 8.6 Notation for Example 8.6.

(b) Tank properties

$$A = \frac{\pi D_T^3}{4} = \pi \times \frac{20^2}{4} = 14.16 \text{ ft}^3$$
$$V_t = 200{,}000 \text{ gallons} \times 0.133680 \text{ cubic feet/gallon} = 26{,}736 \text{ ft}^3$$

Initial depth of liquid in tank:

$$Z_i = \frac{V_i}{A_t} = \frac{26{,}736}{314.16} = 85.10 \text{ ft}$$
$$z_1 = z_i - z_p = 85.10 - 3.00 = 82.10 \text{ ft}$$

Final depth of liquid in tank:

$$z_F = \frac{200{,}000 - 50{,}000}{200{,}000} \times 85.10 = 63.83 \text{ ft}$$
$$z_2 = z_i - z_p = 63.83 - 3.00 = 60.83 \text{ ft}$$

(c) Time to remove 50,000 gallons:

$$t_2 - t_1 = \frac{8A}{\pi D^2}\sqrt{\frac{\sum K}{2g}}\left(z_1^{1/2} - z_2^{1/2}\right) \tag{8.38}$$
$$t_2 - t_1 = \frac{8 \times 314.16}{\pi \times 0.3355^2}\sqrt{\frac{8.44}{2 \times 32.17}}\left(82.10^{1/2} - 60.83^{1/2}\right) = 3{,}274 \text{ s}$$

REFERENCE

Rich, G. R., *Hydraulics Transients*, Dover Publications, New York, N.Y., 1963.

Appendix A
Conversion Factors

Universal gas constant	R_u	= 1545 ft·lbf/(lb·mol)(R) = 8314 J/(kg mol)(K)
		= 0.08205 L·atm/(g·mol)(K) = 0.08314 bar·m^3/(kg·mol)(K)
		= 1.986 Btu/(lb·mol)(R) = 0.730 atm·ft^3/(lb·mol)(R)
		= 10.73 psia·ft^3/(lb·mol)(R)
Length	1m	= 100 cm = 1000 mm = 10^6 microns (μm)
		= 39.37 in. = 3.2808 ft = 0.0006214 mi
	1 km	= 0.6215 mi = 3281 ft
	1 ft	= 12 in. = 30.48 cm
	1 mi	= 5280 ft
Mass	1 kg	= 1000 g = 0.001 metric ton = 2.205 lbm
	1 lbm	= 453.593 g = 0.453593 kg = 5 × 10^{-4} ton
Volume	1 m^3	= 1000 liters = 10^6 cm^3 = 10^6 ml
		= 35.3145 ft^3 = 220.83 imperial gallons = 264.17 gallons
	1 ft^3	= 1728 in^3 = 7.4805 gallons = 0.028317 m^3
		= 28.317 liters
	1 in.^{3}	= 16.387 cm^3
Force	1 N	= 1 kg m/s^2 = 0.22481 lbf
	1 lbf	= 32.174 lbm·ft/s^2 = 4.4482 N
Pressure	1 lbf/in.^{2}	= 2.036 in. Hg at 32 F
	1 in. Hg	= 0.0334 atm = 0.491 lbf/in^2.
	1 atm	= 14.696 lbf/in.^{2} = 760 mm Hg at 32 F = 101.325 kPa
	1 bar	= 0.9869 atm = 105 N/m^2 = 100 kPa
	1 torr	= 1 mm Hg at 0 C = 1.933 × 10 − 2 psi

Density	1 lbm/ft^3	= 0.01602 g/cm^3
	1 g/cm^3	= 1 kg/l = 62.4 lbm/ft^3 = 10^3 kg/m^3
Energy	1 Btu	= 778.16 ft·lbf = 252.16 cal = 1055 J
	1 J	= 1 N·m = 0.7375 ft·lbf
	1 Btu/lbm	= 2.32 kJ/kg
	1 kJ/kg	= 0.431·Btu/lbm
Power	1 W	= 1 J/s = 860.42 cal/hr = 3.413 Btu/hr
	1 kW	= 1.3405 hp = 737.3 ft·lbf/s
	1 hp	= 746 W = 550 ft·lbf/s = 2545 Btu/hr
Velocity	1 m/s	= 2.237 mi/hr = 3.60 km/hr = 3.281 ft/s

Appendix B

Pipe Schedules

Recommended Piping Design Equations

The American National Standard (ANSI/ASME B.31.1) Code for Pressure Piping recommends the following equation to determine pipe thickness as a function of allowable stress and fluid pressure:

$$\frac{\bar{p}}{\bar{S}_a} = \frac{2(t_m - A_t)}{D_o - 2y(t_m - A_t)} \tag{B.1}$$

where

$\bar{p}$ = difference between internal and external pressures

$\bar{S}_a$ = allowable stress

D_o = pipe outer diameter

t_m = minimum wall thickness

A_t = additional wall thickness required to compensate for material removed in threading, grooving, etc.

y = correction factor for material and temperature

For service at and below 900°F (482°C), the standard recommends $y = 0.4$. Substituting this value of y into equation (B.1) yields the following:

$$\frac{\bar{p}}{\bar{S}_a} = \frac{2(t_m - A_t)}{D_o - 0.8(t_m - A_t} \qquad T \leq 900°\text{F} \tag{B.2}$$

Piping Schedules

ANSI B36.10—1970 assigned schedule numbers to pipe sizes in use. Schedule numbers are based on an allowance of 0.1 for A_t, $y = 0$, and $t_m = 7t_s/8$, where t_s is the schedule thickness (the 7/8 factor allows for a 12.5% variation in wall thickness). Substituting these values into equation (B.2) results in the following:

$$\frac{\bar{p}}{\bar{S}_a} = \frac{2(7t_s/8 - 0.1)}{D_o - 0(7t_s/8 - 0.1)} \approx \frac{N_s}{1000}$$

or

$$N_s \approx \frac{1000\bar{p}}{\bar{S}_a} \tag{B.3}$$

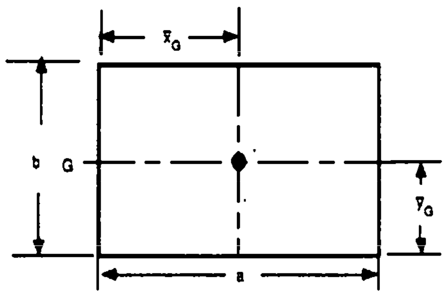
In equation (B.3), N_s is the schedule number.

The relationship of equation (B.3) is very approximate due to the rounding off of existing sizes. The schedule number calculated with equation (B.3) gives a conservative value. Before pipe design thickness calculations are performed, the appropriate ANSI standard should *always* be consulted.

Appendix C
Properties of Areas, Pipes, and Tubing

Table C.1 Properties of Areas

RECTANGLE



$A = ab$

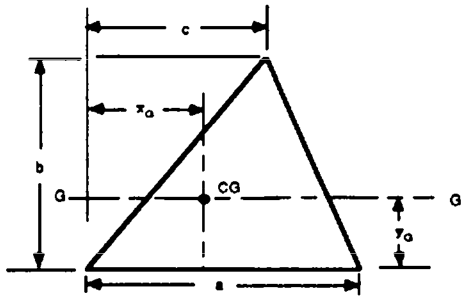
$\bar{x}_G = \frac{a}{2}$

$\bar{y}_G = \frac{b}{2}$

$I_G = \frac{ab^2}{12}$

$\frac{I_G}{A} = \frac{b^2}{12}$

TRIANGLE



$A = \frac{ab}{2}$

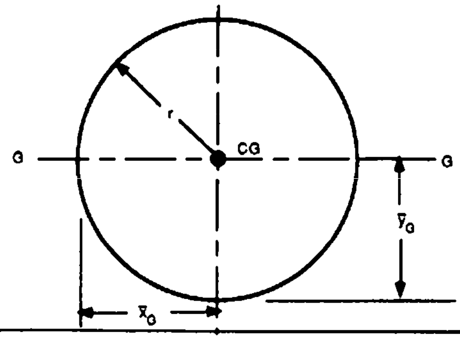
$\bar{x}_G = \frac{a+c}{3}$

$\bar{y}_G = \frac{b}{3}$

$I_G = \frac{ab^3}{36}$

$\frac{I_G}{A} = \frac{b^2}{18}$

CIRCLE



$r^2 = x^2 + y^2$

$A = \pi r^2$

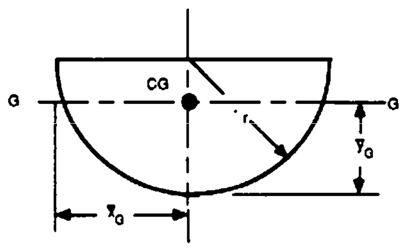
$\bar{x}_G = r$

$\bar{y}_G = r$

$I_G = \frac{\pi r^4}{4}$

$\frac{I_G}{A} = \frac{r^2}{4}$

HALF CIRCLE



$A = \frac{\pi r^2}{2}$

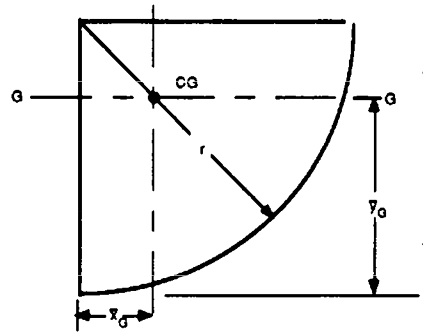
$\bar{x}_G = r$

$\bar{y}_G = r\left(1 - \frac{4}{3\pi}\right)$

$I_G = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$

$\frac{I_G}{A} = \left(\frac{r}{6\pi}\right)^2(9\pi^2 - 64)$

QUARTER CIRCLE



$A = \frac{\pi r^2}{4}$

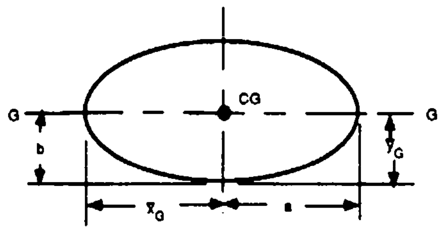
$\bar{x}_G = \frac{4r}{3\pi}$

$\bar{y}_G = r\left(1 - \frac{4}{3\pi}\right)$

$I_G = \left(\frac{r^4}{2}\right)\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$

$\frac{I_G}{A} = \left(\frac{r}{6\pi}\right)^2(9\pi^2 - 64)$

ELLIPSE



$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$A = \pi ab$

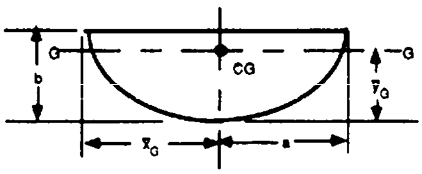
$\bar{x}_G = a$

$\bar{y}_G = b$

$I_G = \frac{\pi ab^3}{4}$

$\frac{I_G}{A} = \frac{b^2}{4}$

HALF ELLIPSE



$A = \frac{\pi ab}{2}$

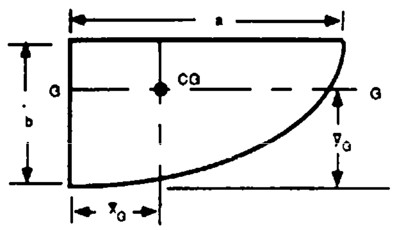
$\bar{x}_G = a$

$\bar{y}_G = b\left(1 - \frac{4}{3\pi}\right)$

$I_G = ab^3\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$

$\frac{I_G}{A} = \left(\frac{b}{6\pi}\right)^2(9\pi^2 - 64)$

QUARTER CIRCLE



$A = \frac{\pi ab}{4}$

$\bar{x}_G = \frac{4a}{3\pi}$

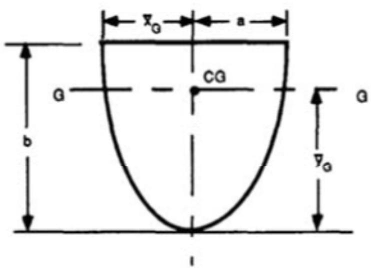
$\bar{y}_G = b\left(1 - \frac{4}{3\pi}\right)$

$I_G = \left(\frac{ab^3}{2}\right)\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$

$\frac{I_G}{A} = \left(\frac{b}{6\pi}\right)^2(9\pi^2 - 64)$

Table C.1 Properties of Areas (*Continued*)

PARABOLA



$$x^2 = \frac{a^2}{b}y$$

$$A = \frac{4ab}{3}$$

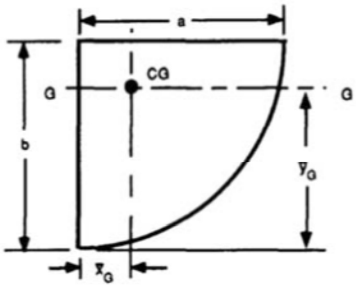
$$\bar{x}_G = a$$

$$\bar{y}_G = \frac{3b}{5}$$

$$I_G = \frac{16ab^3}{175}$$

$$\frac{I_G}{A} = \frac{4b^2}{55}$$

HALF PARABOLA



$$A = \frac{2ab}{3}$$

$$\bar{x}_G = \frac{3a}{8}$$

$$\bar{y}_G = \frac{3b}{5}$$

$$I_G = \frac{8ab^3}{175}$$

$$\frac{I_G}{A} = \frac{4b^3}{55}$$

Table C.2 Values of Flow Areas A and Hydraulic Radius R_h for Various Cross Sections

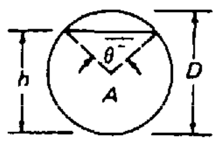
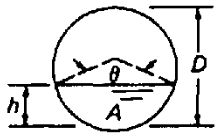
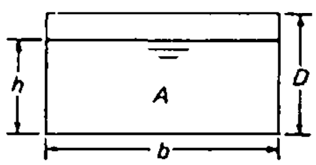
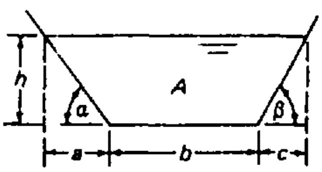
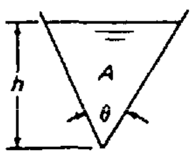
CROSS SECTIONS	CONDITION		EQUATIONS
	Flowing Full	$h/D = 1$	$A = \pi D^2/4 \quad R_h = D/4$
	Upper Half Partly Full	$0.5 < h/D < 1$	$\cos^{-1}(\theta/2) = (2h/D - 1)$ $A = [\pi(360 - \theta) + 180 \sin\theta](D^2/1440)$ $R_h = [1 + (180 \sin\theta)/(\pi\theta)](D/4)$
		$h/D = 0.8128$	$A = 0.6839\, D^2 \quad R_h \max = 0.30430$
	Lower Half	$h/D = 0.5$	$A = \pi D^2/8 \quad R_h \max = h/2$
	Partly Full	$0 < h/D < 0.5$	$\cos^{-1}(\theta/2) = (1 - 2h/D)$ $A = [\pi\theta - 180 \sin\theta](D^2/1440)$ $R_h = [1 - (180 \sin\theta)/(\pi\theta)](D/4)$
	Flowing Full	$h/D = 1$	$A = bD \quad R_h = bD/2(b + D)$
		Square $b = D$	$A = D^2 \quad R_h = D/4$
	Partly Full	$h/D < 1$ $h/b = 0.5$ $b \to \infty,\ h \to 0$	$A = bh \quad R_h = bh/(2h + b)$ $A = b^2/2 \quad R_h \max = h/2$ $R_h \to h$ (wide shallow stream)
	$\alpha \neq \beta$		$R_h \max = h/2$ $A = [b + \frac{1}{2}h\,(\cot\alpha + \cot\beta)]h$ $R_h = A/[b + h(\operatorname{cosec}\alpha + \operatorname{cosec}\beta)]$
	$\alpha = \beta$, $\frac{h}{a} = \frac{1}{2}$	$\alpha = 26°\,34'$	$A = (b + 2h)h$ $R_h = (b + 2h)h/(b + 4.472h)$
	$\alpha = \beta$, $\frac{h}{a} = \frac{1}{\sqrt{3}}$	$\alpha = 30°$	$A = (b + 1.732h)h$ $R_h = (b + 1.732h)h/(b + 4h)$
	$\alpha = \beta$, $\frac{h}{a} = \frac{2}{3}$	$\alpha = 33°\,41'$	$A = (b + 1.5h)h$ $R_h = (b + 1.5h)h/(b + 3.606h)$
	$\alpha = \beta$, $\frac{h}{a} = 1$	$\alpha = 45°$	$A = (b + h)h$ $R_h = (b + h)h/(b + 2.828h)$
	$\alpha = \beta$, $\frac{h}{a} = \frac{3}{2}$	$\alpha = 56°\,19'$	$A = (b + 0.6667h)h$ $R_h = (b + 0.6667h)h/(b + 2.404/h)$
	$\alpha = \beta$, $\frac{h}{a} = \sqrt{3}$	$\alpha = 60°$	$A = (b + 0.5774h)h$ $R_h = (b + 0.5774h)h/(b + 2.309h)$
	θ = any angle		$A = \tan(\theta/2)h^2 \quad R_h = \sin(\theta/2)h/2$
	$\theta = 30$		$A = 0.2679h^2 \quad R_h = 0.1294h$
	$\theta = 45$		$A = 0.4142h^2 \quad R_h = 0.1913h$
	$\theta = 60$		$A = 0.5774h^2 \quad R_h = 0.2500h$
	$\theta = 90$		$A = h^2 \quad R_h = 0.3536h$

Table C.3 Properties of Wrought Steel and Stainless Steel Pipe

Nominal Pipe	Outside Diameter		Schedule*			Wall Thickness		Internal Diameter		Flow Area		Relative Roughness	For Complete Turbulence	
Size in.	in.	mm	a	b	c	in.	mm	feet	mm	square feet	square meters	ε/D	Friction Factor	Reynolds Number
				10S		0.049	1.24	0.02558	7.798	5.140E-04	4.776E-05	5.863E-03	0.03189	546,000
1/8	0.405	10.29	40	40S	STD	0.068	1.73	0.02242	6.833	3.947E-04	3.667E-05	6.691E-03	0.03323	478,000
			80	80S	XS	0.095	2.41	0.01792	5.461	2.521E-04	2.342E-05	8.372E-03	0.03572	382,000
				10S		0.065	1.65	0.03417	10.414	9.168E-04	8.518E-05	4.390E-03	0.02921	729,000
1/4	0.540	13.72	40	40S	STD	0.088	2.24	0.03033	9.246	7.227E-04	6.714E-05	4.945E-03	0.03027	647,000
			80	80S	XS	0.119	3.02	0.02517	7.671	4.974E-04	4.621E-05	5.960E-03	0.03205	537,000
				10S		0.065	1.65	0.04542	13.84	1.620E-03	1.505E-04	3.303E-03	0.02689	969,000
3/8	0.675	17.15	40	40S	STD	0.091	2.31	0.04108	12.52	1.326E-03	1.232E-04	3.651E-03	0.02767	876,000
			80	80S	XS	0.126	3.20	0.03525	10.74	9.759E-04	9.066E-05	4.255E-03	0.02894	752,000
				5S		0.065	1.65	0.05917	18.03	0.002749	0.0002554	2.535E-03	0 02497	1,262,000
				10S		0.083	2.11	0.05617	17.12	0.002478	0.0002302	2.671E-03	0.02533	1,198,000
1/2	0.840	21.34	40	40S	STD	0.109	2.77	0.05183	15.80	0.002110	0.0001960	2.894E-03	0.02590	1,106,000
			80	80S	XS	0.147	3.73	0.04550	13.87	0.001626	0.0001511	3.297E-03	0.02687	971,000
			160			0.188	4.78	0.03867	11.79	0.001174	0.0001091	3.879E-03	0.02816	825,000
					XXS	0.294	7.47	0.02100	6.401	0.0003464	0.00003218	7.143E-03	0.03393	448,000
				5S		0.065	1.65	0.07667	23.37	0.004616	0.0004289	1.957E-03	0.02328	1,636,000
				10S		0.083	2.11	0.07367	22.45	0.004262	0.0003960	2.036E-03	0.02353	1,572,000
3/4	1.050	26.67	40	40S	STD	0.113	2.87	0.06867	20.93	0.003703	0.0003440	2.184E-03	0.02398	1,465,000
			80	80S	XS	0.154	3.91	0.06183	18.85	0.003003	0.0002790	2.426E-03	0.02467	1,319,000
			160			0.219	5.56	0.05100	15.54	0.002043	0.0001898	2.941E-03	0.02602	1,088,000
					XXS	0.308	7.82	0.03617	11.02	0.001027	0.0000954	4.147E-03	0.02872	772,000
				5S		0.065	1.65	0.09875	30.10	0.007659	0.0007115	1.519E-03	0.02180	2,107,000
				10S		0.109	2.77	0.09142	27.86	0.006564	0.0006098	1.641E-03	0.02224	1,950,000
1	1.315	33.41	40	40S	STD.	0.133	3.38	0.08742	26.64	0.006002	0.0005576	1.716E-03	0.02249	1,865,000
			80	80S	XS	0.179	4.55	0.07975	24.31	0.004995	0.0004641	1.881E-03	0.02304	1,701,000
			160			0.250	6.35	0.06792	20.70	0.003623	0.0003366	2.209E-03	0.02405	1,449,000
					XXS	0.358	9.09	0.04992	15.21	0.001957	0.0001818	3.005E-03	0.02618	1,065,000

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
1-1/4	1.660	42.16		5S		0.065	1.65	0.1275	38.86	0.01277	0.001186	1.176E-03	0.02044	2,720,000
				10S		0.109	2.77	0.1202	36.63	0.01134	0.001054	1.248E-03	0.02074	2,564,000
			40	40S	STD	0.140	3.56	0.1150	35.05	0.01039	0.0009650	1.304E-03	0.02097	2,453,000
			80	80S	XS	0.191	4.85	0.1065	32.46	0.008908	0.0008276	1.408E-03	0.02138	2,272,000
			160			0.250	6.35	0.0967	29.46	0.007339	0.0006818	1.552E-03	0.02192	2,062,000
					XXS	0.382	9.70	0.0747	22.76	0.004379	0.0004068	2.009E-03	0.02345	1,593,000
1- 1/2	1.900	48.26		5S		0.065	1.65	0.1475	44.96	0.01709	0.001587	1.017E-03	0.01972	3,147,000
				10S		0.109	2.77	0.1402	42.72	0.01543	0.001434	1.070E-03	0.01996	2,990,000
			40	40S	STD	0.145	3.68	0.1342	40.89	0.01414	0.001313	1.118E-03	0.02018	2,862,000
			80	80S	XS	0.200	5.08	0.1250	38.10	0.01227	0.001140	1.200E-03	0.02054	2,667,000
			160			0.281	7.14	0.1115	33.99	0.009764	0.0009071	1.345E-03	0.02113	2,379,000
					XXS	0.400	10.16	0.0917	27.94	0.006600	0.0006131	1.636E-03	0.02222	1,956,000
2	2.375	60.33		5S		0.065	1.65	0.1871	57.02	0.02749	0.002554	8.018E-04	0.01862	3,991,000
				10S		0.109	2.77	0.1798	54.79	0.02538	0.002358	8.345E-04	0.01880	3,835,000
			40	40S	STD	0.154	3.91	0.1723	52.50	0.02330	0.002165	8.708E-04	0.01899	3,675,000
			80	80S	XS	0.218	5.54	0.1616	49.25	0.02051	0.001905	9.283E-04	0.01928	3,447,000
			160			0.344	8.74	0.1406	42.85	0.01552	0.001442	1.067E-03	0.01995	2,999,000
					XXS	0.436	11.07	0.1253	38.18	0.01232	0.001145	1.198E-03	0.02053	2,672,000
2- 1/2	2.875	73.03		5S		0.083	2.11	0.2258	68.81	0.04003	0.003719	6.645E-04	0.01782	4,816,000
				10S		0.120	3.05	0.2196	66.93	0.03787	0.003518	6.831E-04	0.01793	4,684,000
			40	40S	STD	0.203	5.16	0.2058	62.71	0.03325	0.003089	7.290E-04	0.01821	4,389,000
			80	80S	XS	0.276	7.01	0.1936	59.00	0.02943	0.002734	7.749E-04	0.01847	4,130,000
			160			0.375	9.53	0.1771	53.98	0.02463	0.002288	8.471E-04	0.01887	3,778,000
					XXS	0.552	14.02	0.1476	44.98	0.01711	0.001589	1.016E-03	0.01971	3,148,000

Table C.3 Properties of Wrought Steel and Stainless Steel Pipe *(Continued)*

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S		0.083	2.11	0.2778	84.68	0.06063	0.005632	5.399E-04	0.01699	5,927,000
				10S		0.120	3.05	0.2717	82.80	0.05796	0.005385	5.521E-04	0.01708	5,796,000
3	3.500	88.90	40	40S	STD	0.216	5.49	0.2557	77.93	0.05134	0.004769	5.867E-04	0.01731	5,454,000
			80	80S	XS	0.300	7.62	0.2417	73.66	0.04587	0.004261	6.207E-04	0.01754	5,156,000
			160			0.438	11.13	0.2187	66.65	0.03755	0.003489	6.860E-04	0.01795	4,665,000
					XXS	0.600	15.24	0.1917	58.42	0.02885	0.002680	7.826E-04	0.01851	4,089,000
				5S		0.083	2.11	0.3195	97.38	0.08017	0.007448	4.695E-04	0.01647	6,816,000
3-1/2	4.000	101.6		10S		0.120	3.05	0.3133	95.50	0.07711	0.007164	4.787E-04	0.01654	6,684,000
			40	40S	STD	0.226	5.74	0.2957	90.12	0.06866	0.006379	5.073E-04	0.01675	6,308,000
			80	80S	XS	0.318	8.08	0.2803	85.45	0.06172	0.005734	5.351E-04	0.01696	5,980,000
					XXS	0.616	15.65	0.2307	70.31	0.04179	0.003882	6.503E-04	0.01773	4,921,000
				5S	5S	0.083	2.11	0.3612	110.1	0.1024	0.009518	4.153E-04	0.01602	7,705,000
				10S		0.120	3.05	0.3550	108.2	0.09898	0.009196	4.225E-04	0.01609	7,573,000
			40	40S	STD	0.237	6.02	0.3355	102.3	0.08840	0.008213	4.471E-04	0.01629	7,157,000
4	4.500	114.3	80	80S	XS	0.337	8.56	0.3188	97.18	0.07984	0.007417	4.705E-04	0.01647	6,802,000
			120			0.438	11.13	0.3020	92.05	0.07163	0.006655	4.967E-04	0.01667	6,443,000
			160			0.531	13.49	0.2865	87.33	0.06447	0.005989	5.236E-04	0.01687	6,112,000
					XXS	0.674	17.12	0.2627	80.06	0.05419	0.005034	5.711E-04	0.01721	5,604,000
				5S	5S	0.109	2.77	0.4454	135.8	0.1558	0.01448	3.368E-04	0.01531	9,502,000
				10S		0.134	3.40	0.4413	134.5	0.1529	0.01421	3.399E-04	0.01534	9,413,000
			40	40S	STD	0.258	6.55	0.4206	128.2	0.1389	0.01291	3.566E-04	0.01550	8,972,000
5	5.563	141.3	80	80S	XS	0.375	9.53	0.4011	122.3	0.1263	0.01174	3.740E-04	0.01566	8,556,000
			120			0.500	12.70	0.3803	115.9	0.1136	0.01055	3.945E-04	0.01584	8,112,000
			160			0.625	15.88	0.3594	109.6	0.1015	0.009426	4.173E-04	0.01604	7,668,000
					XXS	0.750	19.05	0.3386	103.2	0.09004	0.008365	4.430E-04	0.01625	7,223,000

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S	5S	0.109	2.77	0.5339	162.7	0.2239	0.02080	2.809E-04	0.01473	11,390,000
				10S		0.134	3.40	0.5298	161.5	0.2204	0.02048	2.832E-04	0.01476	11,300,000
			40	40S	STD	0.280	7.11	0.5054	154.1	0.2006	0.01864	2.968E-04	0.01490	10,780,000
6	6.625	168.3	80	80S	XS	0.432	10.97	0.4801	146.3	0.1810	0.01682	3.124E-04	0.01507	10,240,000
			120			0.562	14.27	0.4584	139.7	0.1650	0.01533	3.272E-04	0.01522	9,780,000
			160			0.719	18.26	0.4323	131.7	0.1467	0.01363	3.470E-04	0.01541	9,221,000
					XXS	0.864	21.95	0.4081	124.4	0.1308	0.01215	3.676E-04	0.01560	8,706,000
				5S	5S	0.109	2.77	0.7006	213.5	0.3855	0.03581	2.141E-04	0.01392	14,950,000
				10S	10S	0.148	3.76	0.6941	211.6	0.3784	0.03515	2.161E-04	0.01395	14,810,000
			20			0.250	6.35	0.6771	206.4	0.3601	0.03345	2.215E-04	0.01402	14,440,000
			30			0.277	7.04	0.6726	205.0	0.3553	0.03301	2.230E-04	0.01404	14,350,000
			40	40S.	STD	0.322	8.18	0.6651	202.7	0.3474	0.03228	2.255E-04	0.01407	14,190,000
8	8.625	219.1	60			0.406	10.31	0.6511	198.5	0.3329	0.03093	2.304E-04	0.01413	13,890,000
			80	80S	XS	0.500	12.70	0.6354	193.7	0.3171	0.02946	2.361E-04	0.01420	13,560,000
			100			0.594	15.09	0.6198	188.9	0.3017	0.02803	2.420E-04	0.01428	13,220,000
			120			0.719	18.26	0.5989	182.5	0.2817	0.02617	2.505E-04	0.01438	12,780,000
			140			0.812	20.62	0.5834	177.8	0.2673	0.02484	2.571E-04	0.01446	12,450,000
					XXS	0.875	22.23	0.5729	174.6	0.2578	0.02395	2.618E-04	0.01451	12,220,000
			160			0.906	23.01	0.5678	173.1	0.2532	0 02352	2.642E-04	0.01454	12,110,000

Table C.3 Properties of Wrought Steel and Stainless Steel Pipe (*Continued*)

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S	5S	0.134	3.40	0.8735	266.2	0.5993	0.05567	1.717E-04	0.01331	18,630,000
				10S	10S	0.165	4.19	0.8683	264.7	0.5922	0.05502	1.727E-04	0.01333	18,520,000
			20			0.250	6.35	0.8542	260.4	0.5730	0.05324	1.756E-04	0.01337	18,220,000
			30			0.307	7.80	0.8447	257.5	0.5604	0.05206	1.776E-04	0.01340	18,020,000
			40	40S.	STD	0.365	9.27	0.8350	254.5	0.5476	0.05087	1.796E-04	0.01343	17,810,000
10	10.750	273.1	60			0.500	12.70	0.8125	247.7	0.5185	0.04817	1.846E-04	0.01351	17,330,000
			80	80S	XS	0.500	12.70	0.8125	247.7	0.5185	0.04817	1.846E-04	0.01351	17,330,000
			100			0.593	15.06	0.7970	242.9	0.4989	0.04635	1.882E-04	0.01356	17,000,000
			120			0.718	18.24	0.7762	236.6	0.4732	0.04396	1.933E-04	0.01363	16,560,000
			140			0.843	21.41	0.7553	230.2	0.4481	0.04163	1.986E-04	0.01371	16,110,000
					XXS	1.000	25.40	0.7292	222.3	0.4176	0.03879	2.057E-04	0.01381	15,560,000
			160			1.125	28.58	0.7083	215.9	0.3941	0.03661	2.118E-04	0.01389	15,110,000
				5S		0.134	3.40	1.0402	317.0	0.8498	0.07895	1.442E-04	0.01286	22,190,000
				10S		0.165	4.19	1.0350	315.5	0.8413	0.07816	1.449E-04	0.01287	22,080,000
			20			0.250	6.35	1.0208	311.2	0.8185	0.07604	1.469E-04	0.01291	21,780,000
			30			0.307	7.80	1.0113	308.3	0.8033	0.07463	1.483E-04	0.01293	21,580,000
				40S	(STD)	0.375	9.53	1.0000	304.8	0.7854	0.07297	1.500E-04	0.01296	21,330,000
			40			0.406	10.31	0.9948	303.2	0.7773	0.07221	1.508E-04	0.01297	21,220,000
12	12.750	323.9		80S	XS	0.500	12.70	0.9792	298.5	0.7530	0.06996	1.532E-04	0.01301	20,890,000
			60			0.562	14.27	0.9688	295.3	0.7372	0.06849	1.548E-04	0.01304	20,670,000
			80			0.688	17.48	0.9478	288.9	0.7056	0.06555	1.583E-04	0.01310	20,220,000
			100			0.844	21.44	0.9218	281.0	0.6674	0.06200	1.627E-04	0.01317	19,670,000
			120		XXS	1.000	25.40	0.8958	273.1	0.6303	0.05856	1.674E-04	0.01325	19,110,000
			140			1.125	28.58	0.8750	266.7	0.6013	0.05586	1.714E-04	0.01331	18,670,000
					160	1.312	33.32	0.8438	257.2	0.5592	0.05196	1.778E-04	0.01341	18,000,000

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S		0.156	3.96	1.1407	347.7	1.0219	0.09494	1.315E-04	0.01263	24,330,000
				10S		0.188	4.78	1.1353	346.0	1.0124	0.09405	1.321E-04	0.01264	24,220,000
			10			0.250	6.35	1.1250	342.9	0.9940	0.09235	1.333E-04	0.01266	24,000,000
			20			0.312	7.92	1.1147	339.8	0.9758	0.09066	1.346E-04	0.01269	23,780,000
			30		STD	0.375	9.53	1.1042	336.6	0.9575	0.08896	1.358E-04	0.01271	23,560,000
			40			0.437	11.10	1.0938	333.4	0.9397	0.08730	1.371E-04	0.01273	23,340,000
14	14.000	355.6			XS	0.500	12.70	1.0833	330.2	0.9218	0.08563	1.385E-04	0.01276	23,110,000
			60			0.593	15.06	1.0678	325.5	0.8956	0.08320	1.405E-04	0.01279	22,780,000
					XXS	0.625	15.88	1.0625	323.9	0.8866	0.08237	1.412E-04	0.01281	22,670,000
			80			0.750	19.05	1.0417	317.5	0.8522	0.07917	1.440E-04	0.01286	22,220,000
			100			0.938	23.83	1.0103	307.9	0.8017	0.07448	1.485E-04	0.01293	21,550,000
			120			1.094	27.79	0.9843	300.0	0.7610	0.07070	1.524E-04	0.01300	21,000,000
			140			1.250	31.75	0.9583	292.1	0.7213	0.06701	1.565E-04	0.01307	20,440,000
			160			1.406	35.71	0.9323	284.2	0.6827	0.06343	1.609E-04	0.01314	19,890,000
				5S		0.165	4.19	1.306	398.0	1.339	0.1244	1.149E-04	0.01230	27,860,000
				10S		0.188	4.78	1.302	396.8	1.331	0.1237	1.152E-04	0.01231	27,780,000
			10			0.250	6.35	1.292	393.7	1.310	0.1217	1.161E-04	0.01233	27,560,000
			20			0.312	7.92	1.281	390.6	1.289	0.1198	1.171E-04	0.01235	27,340,000
			30		STD	0.375	9.53	1.271	387.4	1.268	0.1178	1.180E-04	0.01237	27,110,000
			40		XS	0.500	12.70	1.250	381.0	1.227	0.1140	1.200E-04	0.01241	26,670,000
16	16.000	406.4	60			0.656	16.66	1.224	373.1	1.177	0.1093	1.225E-04	0.01246	26,110,000
			80			0.843	21.41	1.193	363.6	1.118	0.1038	1.258E-04	0.01252	25,450,000
			100			1.031	26.19	1.162	354.0	1.060	0.09844	1.291E-04	0.01258	24,780,000
			120			1.219	30.96	1.130	344.5	1.003	0.09320	1.327E-04	0.01265	24,110,000
			140			1.437	36.50	1.094	333.4	0.9397	0.08730	1.371E-04	0.01273	23,340,000
			160			1.594	40.49	1.068	325.4	0.8953	0.08317	1.405E-04	0.01279	22,780,000

Table C.3 Properties of Wrought Steel and Stainless Steel Pipe (*Continued*)

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S		0.165	4.19	1.473	448.8	1.703	0.1582	1.019E-04	0.01202	31,410,000
				10S		0.188	4.78	1.469	447.6	1.694	0.1574	1.021E-04	0.01203	31,330,000
			10			0.250	6.35	1.458	444.5	1.670	0.1552	1.029E-04	0.01204	31,110,000
			20			0.312	7.92	1.448	441.4	1.647	0.1530	1.036E-04	0.01206	30,890,000
					STD	0.375	9.53	1.438	438.2	1.623	0.1508	1.043E-04	0.01208	30,670,000
			30			0.437	11.10	1.427	435.0	1 600	0.1486	1.051E-04	0.01209	30,450,000
					XS	0.500	12.70	1.417	431.8	1.576	0.1464	1.059E-04	0.01211	30,220,000
18	18.000	457.2	40			0.562	14.27	1.406	428.7	1.553	0.1443	1.067E-04	0.01213	30,000,000
			60			0.750	19.05	1.375	419.1	1.485	0.1380	1.091E-04	0.01218	29,330,000
			80			0.938	23.83	1.344	409.5	1.418	0.1317	1.116E-04	0.01223	28,660,000
			100			1.156	29.36	1.307	398.5	1.342	0.1247	1.147E-04	0.01230	27,890,000
			120			1.375	34.93	1.271	387.4	1.268	0.1178	1.180E-04	0.01237	27,110,000
			140			1.562	39.67	1.240	377.9	1.207	0.1121	1.210E-04	0.01243	26,450,000
			160			1.781	45.24	1.203	366.7	1.137	0.1056	1.247E-04	0.01250	25,670,000
				5S		0.188	4.78	1.635	498.4	2.100	0.1951	9.172E-05	0.01179	34,890,000
				10S		0.218	5.54	1.630	496.9	2.088	0.1939	9.201E-05	0.01179	34,780,000
			10			0.250	6.35	1.625	495.3	2.074	0.1927	9.231E-05	0.01180	34,670,000
			20		STD	0.375	9.53	1.604	489.0	2.021	0.1878	9.351E-05	0.01183	34,220,000
			30		XS	0.500	12.70	1.583	482.6	1.969	0.1829	9.474E-05	0.01186	33,780,000
20	20.000	508.0	40			0.593	15.06	1.568	477.9	1.931	0.1794	9.567E-05	0.01188	33,450,000
			60			0.812	20.62	1.531	466.8	1.842	0.1711	9.795E-05	0.01193	32,670,000
			80			1.031	26.19	1.495	455.6	1.755	0.1630	1.003E-04	0.01199	31,890,000
			100			1.281	32.54	1.453	442.9	1.659	0.1541	1.032E-04	0.01205	31,000,000
			120			1.500	38.10	1.417	431.8	1.576	0.1464	1.059E-04	0.01211	30,220,000
			140			1.750	44.45	1.375	419.1	1.485	0.1380	1.091E-04	0.01218	29,330,000
			160			1.968	49.99	1.339	408.0	1.407	0.1308	1.121E-04	0.01224	28,560,000

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
				5S		0.188	4.78	1.802	549.2	2.550	0.2369	8.324E-05	0.01157	38,440,000
				10S		0.218	5.54	1.797	547.7	2.536	0.2356	8.347E-05	0.01158	38,340,000
			10			0.250	6.35	1.792	546.1	2.521	0.2342	8.372E-05	0.01159	38,220,000
			20		STD	0.375	9.53	1.771	539.8	2.463	0.2288	8.471E-05	0.01161	37,780,000
			30		XS	0.500	12.70	1.750	533.4	2.405	0.2235	8.571E-05	0.01164	37,330,000
22	22.000	558.8	40			0.593	15.06	1.735	528.7	2.363	0.2195	8.648E-05	0.01166	37,000,000
			60			0.875	22.23	1.688	514.4	2.237	0.2078	8.889E-05	0.01172	36,000,000
			80			1.125	28.58	1.646	501.7	2.127	0.1976	9.114E-05	0.01177	35,110,000
			100			1.375	34.93	1.604	489.0	2.021	0.1878	9.351E-05	0.01183	34,220,000
			120			1.625	41.28	1.563	476.3	1.917	0.1781	9.600E-05	0.01189	33,330,000
			140			1.875	47.63	1.521	463.6	1.817	0.1688	9.863E-05	0.01195	32,440,000
			160			2.125	53.98	1.479	450.9	1.718	0.1596	1.014E-04	0.01201	31,560,000
				5S		0.218	5.54	1.964	598.5	3.028	0.2814	7.639E-05	0.01139	41,890,000
			10			0.250	6.35	1.958	596.9	3.012	0.2798	7.660E-05	0.01139	41,780,000
			20		STD	0.375	9.53	1.938	590.6	2.948	0.2739	7.742E-05	0.01142	41,330,000
					XS	0.500	12.70	1.917	584.2	2.885	0.2680	7.826E-05	0.01144	40,890,000
			30			0.562	14.27	1.906	581.1	2.854	0.2652	7.869E-05	0.01145	40,670,000
24	24.000	609.6	40			0.688	17.48	1.885	574.6	2.792	0.2594	7.956E-05	0.01148	40,220,000
			60			0.968	24.59	1.839	560.4	2.655	0.2467	8.158E-05	0.01153	39,220,000
			80			1.218	30.94	1.797	547.7	2.536	0.2356	8.347E-05	0.01158	38,340,000
			100			1.531	38.89	1.745	531.8	2.391	0.2221	8.597E-05	0.01164	37,220,000
			120			1.812	46.02	1.698	517.6	2.264	0.2104	8.834E-05	0.01170	36,220,000
			140			2.062	52.37	1.656	504.9	2.155	0.2002	9.056E-05	0.01176	35,340,000
			160			2.343	59.51	1.610	490.6	2.035	0.1890	9.320E-05	0.01182	34,340,000

Table C.3 Properties of Wrought Steel and Stainless Steel Pipe (*Continued*)

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
			10			0.312	7.92	2.115	644.6	3.512	0.3263	7.093E-05	0.01123	45,110,000
26	26.000	660.4			STD	0.375	9.53	2.104	641.4	3.477	0.3231	7.129E-05	0.01124	44,890,000
			20		XS	0.500	12.70	2.083	635.0	3.409	0.3167	7.200E-05	0.01127	44,440,000
			10			0.312	7.92	2.281	695.4	4.088	0.3797	6.575E-05	0.01108	48,670,000
28	28.000	711.2			STD	0.375	9.53	2.271	692.2	4.050	0.3763	6.606E-05	0.01109	48,440,000
			20		XS	0.500	12.70	2.250	685.8	3.976	0.3694	6.667E-05	0.01111	48,000,000
			30			0.625	15.88	2.229	679.5	3.903	0.3626	6.729E-05	0.01113	47,560,000
				5S		0.250	6.35	2.458	749.3	4.746	0.4410	6.102E-05	0.01093	52,440,000
			10	10S		0.312	7.92	2.448	746.2	4.707	0.4373	6.127E-05	0.01094	52,220,000
30	30.000	762.0			STD	0.375	9.53	2.438	743.0	4.666	0.4335	6.154E-05	0.01095	52,000,000
			20		XS	0.500	12.70	2.417	736.6	4.587	0.4261	6.207E-05	0.01096	51,560,000
			30			0.625	15.88	2.396	730.3	4.508	0.4188	6.261E-05	0.01098	51,110,000
			40			0.750	19.05	2.375	723.9	4.430	0.4116	6.316E-05	0.01100	50,670,000
			10			0.312	7.92	2.615	797.0	5.369	0.4988	5.737E-05	0.01081	55,780,000
					STD	0.375	9.53	2.604	793.8	5.326	0.4948	5.760E-05	0.01082	55,560,000
32	32.000	812.8	20		XS	0.500	12.70	2.583	787.4	5.241	0.4869	5.806E-05	0.01083	55,110,000
			30			0.625	15.88	2.563	781.1	5.157	0.4791	5.854E-05	0.01085	54,670,000
			40			0.688	17.48	2.552	777.8	5.115	0.4752	5.878E-05	0.01086	54,440,000
			10			0.312	7.92	2.781	847.8	6.076	0.5645	5.393E-05	0.01069	59,340,000
					STD	0.375	9.53	2.771	844.6	6.030	0.5602	5.414E-05	0.01070	59,110,000
34	34.000	863.6	20		XS	0.500	12.70	2.750	838.2	5.940	0.5518	5.455E-05	0.01071	58,670,000
			30			0.625	15.88	2.729	831.9	5.850	0.5435	5.496E-05	0.01072	58,220,000
			40			0.688	17.48	2.719	828.6	5.805	0.5393	5.517E-05	0.01073	58,000,000

Nominal Pipe Size in.	Outside Diameter in.	Outside Diameter mm	Schedule* a	Schedule* b	Schedule* c	Wall Thickness in.	Wall Thickness mm	Internal Diameter feet	Internal Diameter mm	Flow Area square feet	Flow Area square meters	Relative Roughness ε/D	For Complete Turbulence Friction Factor	For Complete Turbulence Reynolds Number
36	36.000	914.4	10			0.312	7.92	2.948	898.6	6.826	0.6341	5.088E-05	0.01058	62,890,000
					STD	0.375	9.53	2.938	895.4	6.777	0.6296	5.106E-05	0.01058	62,670,000
			20		XS	0.500	12.70	2.917	889.0	6.681	0.6207	5.143E-05	0.01060	62,220,000
			30			0.625	15.88	2.896	882.7	6.586	0.6119	5.180E-05	0.01061	61,780,000
			40			0.750	19.05	2.875	876.3	6.492	0.6031	5.217E-05	0.01062	61,330,000
42	42.000	1066.8			STD	0.375	9.53	3.438	1047.8	9.281	0.8622	4.364E-05	0.01029	73,330,000
			20		XS	0.500	12.70	3.417	1041.4	9.168	0.8518	4.390E-05	0.01030	72,890,000
			30			0.625	15.88	3.396	1035.1	9.057	0.8414	4.417E-05	0.01032	72,440,000
			40			0.750	19.05	3.375	1028.7	8.946	0.8311	4.444E-05	0.01033	72,000,000

*Schedule numbers:

1. Standard-weight pipe (STD) and schedule 40 are the same in all sizes through 10 inch. For 12 inch and above, standard-weight pipe has a wall thickness of 3/8 inch.
2. Extrastrong-weight pipe (XS) and schedule 80 are the same in all sizes through 8 inch. For 10 inch and above, extrastrong-weight pipe has a wall thickness of 1/2 inch.
3. Double extrastrong-weight pipe (XXS) has no corresponding schedule number.
4. a. ANSI B36:10 steel pipe schedule numbers
   b. ANSI B36.19 stainless steel pipe schedule numbers
   c. ANSI B36.10 steel pipe nominal wall designation.

Relative roughness: ε/D was calculated using an absolute roughness value of $\varepsilon = 150 \times 10^{-6}$ feet.

Table C.4 Properties of 250-psi Cast Iron Pipe

Pipe size, in.	Outside diameter		Thickness class	Wall thickness		Internal diameter		Flow area		$10^6\epsilon/D$
	in.	mm		in.	mm	ft	mm	ft²	mm²	
3	3.96	100.6	22	0.32	8.1	0.2767	84.8	0.06012	5 595	3,072
4	4.80	121.9	22	0.35	8.9	0.3417	104.1	0.09168	8 511	2,488
6	6.90	175.3	22	0.38	9.7	0.5117	155.9	0.2056	19 090	1,661
8	9.05	229.9	22	0.41	10.4	0.6858	209.11	0.3694	34 340	1,239
10	11.10	281.9	22	0.44	11.2	0.8517	259.5	0.5696	52 890	998.0
12	13.20	335.3	23	0.52	13.2	1.013	308.9	0.8065	74 940	838.8
14	15.30	388.6	24	0.59	15.0	1.177	358.6	1.087	101 000	722.4
16	17.40	442.0	24	0.63	16.0	1.345	410.0	1.421	132 000	632.0
18	19.50	495.3	24	0.68	17.3	1.512	460.7	1.795	166 700	562.3
20	21.60	548.6	24	0.72	18.3	1.680	512.0	2.217	205 900	506.0
24	25.80	655.3	24	0.79	20.1	2.018	615.1	3.199	297 200	421.1
30	32.00	812.8	25	0.99	25.1	2.501	762.6	4.915	456 800	399.8
36	38.30	972.9	25	1.10	27.9	3.008	917.1	7.108	660 600	282.5
42	44.50	1 130	25	1.22	31.0	3.505	1 068	9.649	895 800	242.5
48	50.80	1 290	25	1.33	33.8	4.012	1 222	12.64	1 174 000	211.9

NOTES

1. Outside diameter and wall thickness extracted from *American National Standard Cast Iron Pipe Centrifugally Cast in Metal Molds for Water or Other Liquids*, ANSI A21.6– 1970
2. Metrication by author.
3. Values of relative roughness, ϵ/D, were computed using $\epsilon = 850 \times 10^{-6}$ ft (260 μm).

Table C.5 Properties of Seamless Copper Water Tube

Standard size, in.	Outside diameter		Type	Wall thickness		Internal diameter		Flow area		$10^6\epsilon/D$
	in.	mm		in.	mm	ft	mm	ft²	mm²	
1/4	0.375	9.53	K	0.035	0.89	0.02542	7.75	0.0005074	47.17	196.7
			L	0.030	0.76	0.02625	8.01	0.0005412	50.39	190.5
3/8	0.500	12.70	K	0.049	1.25	0.03350	10.22	0.0008814	82.03	149.3
			L	0.035	0.89	0.03583	10.92	0.001008	93.66	141.6
			M	0.025	0.64	0.03750	11.42	0.001104	102.4	133.3
1/2	0.625	15.88	K	0.049	1.24	0.04392	13.40	0.001515	141.0	113.8
			L	0.040	1.02	0.04542	13.84	0.001620	150.5	110.1
			M	0.028	0.71	0.04742	14.46	0.001766	164.2	103.4
5/8	0.750	19.05	K	0.049	1.24	0.05433	16.57	0.002319	215.6	92.02
			L	0.042	1.07	0.05550	16.91	0.002419	224.6	90.09
3/4	0.875	22.22	K	0.065	1.65	0.06208	18.92	0.003027	281.1	80.54
			L	0.045	1.14	0.06542	19.94	0.003361	312.3	76.43
			M	0.032	0.81	0.06758	20.60	0.003587	333.3	73.98
1	1.125	28.58	K	0.065	1.65	0.08292	25.28	0.005400	501.9	60.30
			L	0.050	1.27	0.08542	26.04	0.005730	532.6	58.54
			M	0.035	0.89	0.08792	26.80	0.006071	564.1	56.87
1 1/4	1.375	34.93	K	0.065	1.65	0.1038	31.63	0.008454	785.8	48.19
			L	0.055	1.40	0.1054	32.13	0.008728	810.8	47.93
			M	0.042	1.07	0.1076	32.79	0.009090	844.4	46.48

Table C.5 Properties of Seamless Copper Water Tube (*Continued*)

Standard size, in.	Outside diameter		Type	Wall thickness		Internal diameter		Flow area		$10^6\epsilon/D$
	in.	mm		in.	mm	ft	mm	ft²	mm²	
$1\frac{1}{2}$	1.625	41.28	K	0.072	1.83	0.1234	37.62	0.01196	1 112	40.51
			L	0.060	1.52	0.1254	38.24	0.01235	1 148	39.87
			M	0.049	1.24	0.1273	38.80	0.01272	1 182	39.29
2	2.125	53.98	K	0.083	2.11	0.1633	49.76	0.02093	1 195	30.62
			L	0.070	1.78	0.1654	50.42	0.02149	1 997	30.22
			M	0.058	1.48	0.1674	51.02	0.02701	2 044	29.87
$2\frac{1}{2}$	2.625	66.68	K	0.095	2.41	0.2029	61.86	0.03234	3 005	24.64
			L	0.080	2.03	0.2054	62.62	0.03314	3 080	24.34
			M	0.065	1.65	0.2079	63.38	0.03395	4 017	24.05
3	3.125	79.38	K	0.109	2.77	0.2423	73.84	0.04609	4 282	20.64
			L	0.090	2.29	0.2454	74.80	0.04730	4 394	20.37
			M	0.072	1.83	0.2484	75.72	0.04847	4 503	20.13
$3\frac{1}{2}$	3.625	92.08	K	0.120	3.05	0.2821	85.98	0.06249	5 806	17.72
			L	0.100	2.54	0.2854	87.00	0.06398	5 945	17.52
			M	0.083	2.11	0.2883	87.86	0.06523	6 063	17.35
4	4.125	104.8	K	0.134	3.40	0.3214	98.00	0.08114	7 543	15.56
			L	0.110	2.79	0.3254	99.22	0.08317	7 732	15.36
			M	0.095	2.41	0.3279	99.98	0.08445	7 851	15.25

Standard size, in.	Outside diameter		Type	Wall thickness		Internal diameter		Flow area		$10^6\varepsilon/D$
	in.	mm		in.	mm	ft	mm	ft^2	mm^2	
5	5.125	130.2	K	0.160	4.06	0.4004	122.1	0.1259	11 710	12.49
			L	0.125	3.18	0.4063	123.8	0.1296	12 050	12.31
			M	0.109	2.77	0.4089	124.7	0.1313	12 210	12.23
6	6.125	155.6	K	0.192	4.88	0.4784	145.8	0.1798	16 700	10.45
			L	0.140	3.56	0.4871	148.5	0.1863	17 320	10.27
			M	0.122	3.10	0.4901	143.9	0.1886	17 530	10.20
8	8.125	206.4	K	0.271	6.88	0.6319	192.6	0.3136	29 150	7.912
			L	0.200	5.08	0.6438	196.2	0.3255	30 250	7.767
			M	0.170	4.32	0.6488	197.8	0.3306	30 720	7.707
10	10.125	257.2	K	0.388	8.59	0.7874	240.0	0.4870	45 250	6.350
			L	0.250	6.35	0.8021	244.5	0.5053	46 950	6.234
			M	0.212	5.38	0.8084	246.4	0.5133	47 680	6.185
12	12.125	308.0	K	0.405	10.29	0.9429	287.4	0.6983	64 880	5.303
			L	0.280	7.11	0.9638	293.8	0.7295	67 790	5.188
			M	0.254	6.45	0.9681	295.1	0.7361	68 400	5.165

NOTES

1. Outside diameter and wall thickness in inches extracted from *American National Standard Seamless Copper Water Tube*, ANSI H 23.1– 1970.
2. Metrication by author.
3. Types: K, for underground service and general plumbing: L, for interior plumbing: M, for use with soldered fittings only.
4. Values of relative roughness, ε/D, were computed using $\varepsilon = 5 \times 10^{-6}$ ft(1.5 μm).

Table C.6 Typical Maximum Allowable Stress Values for Selected Piping Materials [a]

Material	Grade	Temperature, °F	Stress, psi	Temperature, °C	Stress, Mpa
		Carbon steel—specification A-106			
	A	−20 to 600	12,000	−29 to 316	82.7
		700	11,600	371	80.0
		800	9,000	427	62.0
	B	−20 to 600	15,000	−29 to 316	103.4
		700	14,300	371	98.6
		800	10,800	427	74.5
		Low and intermediate alloy steel—specification A-335			
C-½Mo	P1	−20 to 800	13,700	−29 to 427	94.4
½Cr-½Mo	P2	−20 to 800	13,700	−20 to 427	94.4
		800	13,400	427	92.4
		900	12,500	482	86.2
		1000	6,200	538	42.7
1¼Cr-½Mo-Si	P11	−20 to 800	15,000	−29 to 427	103.4
		900	13,100	482	90.3
		1000	6,500	538	44.8
		1100	3,000	593	20.7
1Cr-½Mo	P12	−20 to 700	15,000	−29 to 371	103.4
		800	14,700	427	101.3
		900	13,100	482	90.3
		1000	6,500	538	44.8
		1100	2,800	593	19.3
2¼Cr-1Mo	P22	−20 to 800	15,000	−29 to 427	103.4
		900	13,000	482	89.6
		1000	7,800	538	53.8
		1100	4,200	593	29.0
		Stainless steel—specification A-213			
18Cr-10Ni-Cb	TP 347	−20 to 100	18,700	−29 to 38	128.9
		200	17,200	93	118.6
		300	16,000	149	110.3
		400	15,000	204	103.4
		500	14,000	260	96.5
		600	13,400	316	92.4
		700	12,900	371	88.9
		800	12,700	427	87.6
		900	12,600	482	86.9
		1000	12,500	538	86.2
		1100	9,100	593	62.7
		1200	6,100	649	42.1

[a] Stress values are for the solution of text problems only. For actual piping design ANSI B31.1 "Power Piping" values must be used.

Appendix D
Fluid Properties

Table D.1 Critical and Saturated Properties of Selected Fluids

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Acetic Acid $C_2H_4O_2$					**M = 60.052**				**Zc = 0.201**						
t p	16.66	61.99	1.289E+03	1.870E-01	1.250E-01	4.101E-01	1051.5	65.64	1271	26.54	27.81	1.906	6.37E-02	3.98E-03	7.47	0.156
	20.00	68.00	1.572E+03	2.280E-01	1.529E-01	5.017E-01	1048.1	65.43	1207	25.21	27.48	1.883	7.64E-02	4.77E-03	7.59	0.159
	40.00	104.00	4.674E+03	6.780E-01	4.640E-01	1.522E+00	1027.3	64.13	907	18.95	25.52	1.749	2.01E-01	1.26E-02	8.29	0.173
	60.00	140.00	1.203E+04	1.745E+00	1.219E+00	4.001E+00	1005.8	62.79	703	14.68	23.57	1.615	4.62E-01	2.88E-02	8.98	0.188
	80.00	176.00	2.740E+04	3.975E+00	2.841E+00	9.321E+00	983.5	61.40	559	11.67	21.64	1.483	9.59E-01	5.99E-02	9.68	0.202
	100.00	212.00	5.655E+04	8.202E+00	6.004E+00	1.970E+01	960.4	59.96	453	9.47	19.71	1.350	1.83E+00	1.14E-01	10.39	0.217
n b p	117.90	244.22	1.013E+05	1.470E+01	1.100E+01	3.611E+01	938.9	58.61	382	7.97	17.99	1.233	3.10E+00	1.93E-01	11.02	0.230
	120.00	248.00	1.076E+05	1.561E+01	1.172E+01	3.845E+01	936.3	58.45	374	7.82	17.79	1.219	3.27E+00	2.04E-01	11.09	0.232
	140.00	284.00	1.914E+05	2.775E+01	2.142E+01	7.028E+01	911.0	56.87	313	6.54	15.89	1.089	5.52E+00	3.44E-01	11.81	0.247
	160.00	320.00	3.216E+05	4.664E+01	3.708E+01	1.217E+02	884.4	55.21	265	5.54	14.00	0.960	8.87E+00	5.54E-01	12.54	0.262
	180.00	356.00	5.152E+05	7.472E+01	6.136E+01	2.013E+02	856.1	53.44	227	4.74	12.13	0.831	1.37E+01	8.55E-01	13.28	0.277
	200.00	392.00	7.921E+05	1.149E+02	9.783E+01	3.210E+02	825.7	51.54	195	4.08	10.28	0.705	2.05E+01	1.28E+00	14.06	0.294
	220.00	428.00	1.176E+06	1.705E+02	1.513E+02	4.963E+02	792.6	49.48	169	3.53	8.45	0.579	3.02E+01	1.89E+00	14.89	0.311
	240.00	464.00	1.693E+06	2.455E+02	2.284E+02	7.492E+02	755.8	47.19	146	3.05	6.65	0.456	4.23E+01	2.64E+00	15.77	0.329
	260.00	500.00	2.373E+06	3.442E+02	3.390E+02	1.112E+03	713.8	44.56	126	2.64	4.88	0.334	6.17E+01	3.85E+00	16.84	0.352
	280.00	536.00	3.251E+06	4.714E+02	4.998E+02	1.640E+03	663.2	41.40	106	2.28	3.15	0.216	8.83E+01	5.51E+00	18.17	0.380
	300.00	572.00	4.361E+06	6.326E+02	7.480E+02	2.454E+03	594.6	37.12	90	1.87	1.48	0.101	1.33E+02	8.31E+00	20.31	0.424
c p	319.56	607.21	5.786E+06	8.391E+02	1.677E+03	5.503E+03	351.8	21.96	49	1.03	0.00	0.000	3.51E+02	2.19E+01	49.46	1.033
	Acetone C_3H_6O					**M = 58.08**				**Zc = 0.233**						
t p	-94.70	-138.46	2.594E+00	3.762E-04	2.897E-04	9.504E-04	913.1	57.00	2002	41.81	36.25	2.621	1.01E-04	6.29E-06	2.00	0.042
	-80.00	-112.00	1.695E+01	2.458E-03	1.924E-03	6.311E-03	898.4	56.09	1455	30.39	36.34	2.490	6.18E-04	3.86E-05	3.04	0.063
	-60.00	-76.00	1.368E+02	1.985E-02	1.589E-02	5.213E-02	878.2	54.82	965	20.15	33.76	2.313	4.56E-03	2.86E-04	4.58	0.096
	-40.00	-40.00	7.378E+02	1.070E-01	8.775E-02	2.879E-01	857.4	53.52	686	14.34	31.20	2.138	2.26E-02	1.41E-03	6.07	0.127
	-20.00	-4.00	2.941E+03	4.265E-01	3.587E-01	1.177E+00	836.0	52.19	515	10.77	28.66	1.964	8.29E-02	5.17E-03	7.32	0.153
	0.00	32.00	9.303E+03	1.349E+00	1.165E+00	3.823E+00	814.1	50.82	404	8.43	26.15	1.792	2.41E-01	1.51E-02	8.27	0.173
	15.56	60.00	2.008E+04	2.912E+00	2.570E+00	8.433E+00	796.5	49.72	342	7.13	24.21	1.659	4.90E-01	3.06E-02	8.81	0.184
	20.00	68.00	2.458E+04	3.566E+00	3.166E+00	1.039E+01	791.4	49.41	327	6.82	23.66	1.621	5.90E-01	3.68E-02	8.93	0.186
	40.00	104.00	5.637E+04	8.176E+00	7.486E+00	2.456E+01	767.9	47.94	272	5.68	21.20	1.453	1.26E+00	7.86E-02	9.34	0.195
n b p	56.29	133.32	1.013E+05	1.470E+01	1.361E+01	4.532E+01	748.0	46.70	238	4.97	19.22	1.317	2.15E+00	1.34E-01	9.50	0.198
	60.00	140.00	1.154E+05	1.673E+01	1.582E+01	5.192E+01	743.4	46.41	231	4.83	18.77	1.286	2.42E+00	1.51E-01	9.56	0.200
	80.00	176.00	2.154E+05	3.124E+01	3.060E+01	1.004E+02	717.7	44.81	192	4.02	16.38	1.122	4.29E+00	2.68E-01	9.74	0.203
	100.00	212.00	3.731E+05	5.411E+01	5.509E+01	1.807E+02	690.6	43.11	165	3.45	14.02	0.961	7.15E+00	4.46E-01	9.88	0.206
	120.00	248.00	6.077E+05	8.813E+01	9.363E+01	3.072E+02	661.7	41.31	143	2.98	11.71	0.803	1.14E+01	7.12E-01	10.08	0.210
	140.00	284.00	9.405E+05	1.364E+02	1.521E+02	4.990E+02	630.6	39.36	124	2.59	9.45	0.648	1.76E+01	1.10E+00	10.42	0.218
	160.00	320.00	1.395E+06	2.024E+02	2.387E+02	7.830E+02	596.2	37.22	108	2.25	7.25	0.497	2.68E+01	1.68E+00	11.00	0.230
	180.00	356.00	1.996E+06	2.898E+02	3.657E+02	1.200E+03	557.1	34.78	93	1.94	5.11	0.350	4.09E+01	2.55E+00	12.00	0.251
	200.00	392.00	2.778E+06	4.029E+02	5.553E+02	1.822E+03	510.1	31.85	79	1.66	3.08	0.211	6.52E+01	4.07E+00	14.11	0.295
	220.00	428.00	3.768E+06	5.465E+02	8.630E+02	2.832E+03	445.2	27.79	65	1.36	1.19	0.082	1.09E+02	6.80E+00	17.79	0.371
c p	235.05	455.09	4.702E+06	6.819E+02	1.725E+03	5.660E+03	277.9	17.35	39	0.81	0.00	0.000	2.78E+02	1.73E+01	38.64	0.807

t p = triple point n b p = normal boiling point c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_l		Liquid Viscosity μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Acetonitrile C_2H_3N				**M = 41.062**				**Zc = 0.2184**							
t p	-43.83	-46.89	1.757E+02	2.548E-02	2.113E-02	6.928E-02	848.5	52.97	690	14.41	37.52	2.571	3.80E-03	2.37E-04	4.97	0.104
	-40.00	-40.00	2.408E+02	3.492E-02	2.908E-02	9.536E-02	844.8	52.74	657	13.73	37.02	2.537	5.13E-03	3.20E-04	5.21	0.109
	-20.00	-4.00	1.041E+03	1.510E-01	1.287E-01	4.221E-01	825.2	51.52	523	10.92	34.43	2.359	2.06E-02	1.28E-03	6.36	0.133
	0.00	32.00	3.507E+03	5.087E-01	4.444E-01	1.457E+00	805.2	50.27	428	8.93	31.85	2.183	6.48E-02	4.05E-03	7.27	0.152
	15.56	60.00	7.876E+03	1.142E+00	1.018E+00	3.339E+00	789.2	49.27	372	7.77	29.86	2.046	1.39E-01	8.67E-03	7.80	0.163
	20.00	68.00	9.743E+03	1.413E+00	1.267E+00	4.155E+00	784.6	48.98	358	7.48	29.30	2.008	1.70E-01	1.06E-02	7.92	0.165
	40.00	104.00	2.326E+04	3.376E+00	3.111E+00	1.020E+01	783.3	47.65	305	6.37	26.76	1.834	3.85E-01	2.40E-02	8.37	0.175
	60.00	140.00	4.937E+04	7.160E+00	6.793E+00	2.228E+01	741.3	48.28	264	5.51	24.24	1.661	7.82E-01	4.88E-02	8.67	0.181
	80.00	176.00	9.516E+04	1.380E+01	1.351E+01	4.432E+01	718.5	44.86	231	4.82	21.75	1.490	1.45E+00	9.07E-02	8.88	0.186
n b p	81.60	178.88	1.013E+05	1.470E+01	1.442E+01	4.730E+01	716.7	44.74	228	4.77	21.55	1.477	1.55E+00	9.65E-02	9.03	0.189
	100.00	212.00	1.700E+05	2.465E+01	2.496E+01	8.185E+01	694.7	43.37	203	4.25	19.28	1.321	2.52E+00	1.58E-01	9.06	0.189
	120.00	248.00	2.853E+05	4.139E+01	4.347E+01	1.425E+02	669.7	41.81	180	3.77	16.84	1.154	4.16E+00	2.59E-01	9.25	0.193
	140.00	284.00	4.555E+05	6.807E+01	7.224E+01	2.369E+02	643.3	40.16	161	3.36	14.43	0.989	6.57E+00	4.10E-01	9.49	0.198
	160.00	320.00	6.981E+05	1.013E+02	1.158E+02	3.798E+02	615.0	38.40	144	3.00	12.06	0.826	1.01E+01	6.30E-01	9.84	0.206
	180.00	356.00	1.035E+06	1.501E+02	1.807E+02	5.925E+02	584.4	36.48	128	2.68	9.73	0.666	1.52E+01	9.47E-01	10.35	0.216
	200.00	392.00	1.494E+06	2.166E+02	2.768E+02	9.077E+02	550.5	34.37	114	2.38	7.44	0.510	2.25E+01	1.41E+00	11.09	0.232
	220.00	428.00	2.109E+06	3.058E+02	4.205E+02	1.379E+03	511.6	31.94	101	2.10	5.22	0.358	3.24E+01	2.02E+00	11.80	0.246
	240.00	464.00	2.926E+06	4.243E+02	6.430E+02	2.108E+03	464.2	28.98	87	1.82	3.08	0.211	5.27E+01	3.29E+00	14.55	0.304
	260.00	500.00	4.003E+06	5.806E+02	1.032E+03	3.383E+03	395.9	24.71	71	1.48	1.07	0.073	8.93E+01	5.57E+00	19.07	0.398
c p	272.35	522.23	4.833E+06	7.010E+02	2.078E+03	6.814E+03	237.3	14.81	42	0.87	0.00	0.000	2.37E+02	1.48E+01	41.50	0.867
	Acetylene C_2H_2				**M = 26.038**				**Zc = 0.271**							
n b p	-84.15	-119.47	1.013E+05	1.470E+01	*solid*	*solid*	*solid*	*solid*	*solid*	*solid*	*solid*	*solid*	1.73E+00	1.08E-01	6.61	0.138
t p	-80.75	-113.35	1.260E+05	1.828E+01	2.083E+01	6.833E+01	617.1	38.52	196	4.10	18.75	1.285	2.12E+00	1.32E-01	6.91	0.144
	-80.00	-112.00	1.316E+05	1.912E+01	2.183E+01	7.159E+01	616.0	38.45	195	4.07	18.62	1.276	2.21E+00	1.38E-01	6.96	0.145
	-60.00	-76.00	3.536E+05	5.128E+01	6.171E+01	2.024E+02	584.5	36.49	162	3.39	14.99	1.027	5.60E+00	3.50E-01	7.83	0.159
	-40.00	-40.00	7.816E+06	1.134E+02	1.450E+02	4.755E+02	549.9	34.33	137	2.86	11.47	0.786	1.21E+01	7.55E-01	8.23	0.172
	-20.00	-4.00	1.509E+06	2.189E+02	3.015E+02	9.888E+02	510.7	31.88	116	2.42	8.08	0.553	2.37E+01	1.48E+00	9.00	0.188
	0.00	32.00	2.650E+06	3.843E+02	5.830E+02	1.912E+03	463.6	28.94	97	2.03	4.85	0.332	4.43E+01	2.77E+00	10.23	0.214
	15.56	60.00	3.914E+06	5.676E+02	9.594E+02	3.147E+03	416.1	25.98	83	1.73	2.50	0.171	7.22E+01	4.50E+00	11.87	0.248
	20.00	68.00	4.349E+06	6.308E+02	1.112E+03	3.647E+03	399.0	24.91	78	1.63	1.87	0.128	8.33E+01	5.20E+00	12.52	0.261
c p	35.17	95.31	6.139E+06	8.904E+02	2.717E+03	8.910E+03	230.5	14.39	43	0.90	0.00	0.000	2.31E+02	1.44E+01	43.17	0.902
	Air (R 729)				**M = 28.97**				**Zc = 0.230**							
t p	-213.15	-351.67	6.552E+03	9.503E-01	7.056E-01	2.314E+00	947.4	59.14	325	6.79	10.68	0.732	1.45E-01	9.08E-03	2.64	0.055
	-200.00	-328.00	5.196E+04	7.536E+00	5.937E+00	1.947E+01	892.8	55.74	199	4.16	8.13	0.557	1.53E+00	9.54E-02	4.82	0.101
n b p	-194.35	-317.83	1.013E+05	1.470E+01	1.191E+01	3.907E+01	867.6	54.16	170	3.54	7.09	0.486	3.33E+00	2.08E-01	5.72	0.119
	-180.00	-292.00	4.001E+05	5.804E+01	5.117E+01	1.678E+02	797.7	49.80	124	2.59	4.56	0.313	1.32E+01	8.21E-01	7.63	0.159
	-160.00	-256.00	1.512E+06	2.193E+02	2.284E+02	7.491E+02	675.4	42.16	91	1.90	1.51	0.103	5.51E+01	3.44E+00	10.49	0.219
c p	-140.70	-221.26	3.774E+06	5.474E+02	1.058E+03	3.469E+03	364.0	22.72	21	0.43	0.00	0.000	2.88E+02	1.80E+01	20.70	0.432

t p = triple point, n b p = normal boiling point, c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature T		Vapor Pressure-P_v		Vapor Head-$h_v = P/\gamma$		Liquid Density ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
		Ammonia (R 717) NH_3				M = 17.030				Zc = 0.242						
t p	-77.74	-107.93	6.060E+03	8.789E-01	8.425E-01	2.763E+00	733.9	45.81	519	10.85	44.18	3.027	6.37E-02	3.98E-03	6.23	0.130
	-60.00	-76.00	2.185E+04	3.169E+00	3.122E+00	1.024E+01	713.9	44.57	409	8.53	40.09	2.747	2.12E-01	1.32E-02	7.60	0.159
	-40.00	-40.00	7.154E+04	1.038E+01	1.058E+01	3.469E+01	689.9	43.07	311	6.49	35.52	2.434	6.42E-01	4.01E-02	8.80	0.184
n b p	-33.43	-28.17	1.013E+05	1.470E+01	1.517E+01	4.974E+01	681.6	42.55	284	5.93	34.03	2.332	8.89E-01	5.55E-02	9.13	0.191
	-20.00	-4.00	1.896E+05	2.750E+01	2.909E+01	9.540E+01	664.9	41.51	237	4.95	31.01	2.125	1.60E+00	9.98E-02	9.73	0.203
	0.00	32.00	4.281E+05	6.209E+01	6.839E+01	2.243E+02	638.6	39.87	164	3.84	26.55	1.819	3.45E+00	2.15E-01	10.50	0.219
	15.56	60.00	7.399E+05	1.073E+02	1.224E+02	4.013E+02	616.9	38.51	153	3.20	23.14	1.585	5.81E+00	3.63E-01	11.05	0.231
	20.00	68.00	8.550E+05	1.240E+02	1.429E+02	4.686E+02	610.4	38.11	146	3.05	22.17	1.519	6.68E+00	4.17E-01	11.20	0.234
	40.00	104.00	1.551E+06	2.249E+02	2.730E+02	8.953E+02	579.6	36.18	120	2.51	17.86	1.224	1.20E+01	7.49E-01	11.93	0.249
	60.00	140.00	2.608E+06	3.782E+02	4.880E+02	1.600E+03	545.2	34.03	102	2.13	13.66	0.936	2.04E+01	1.27E+00	12.80	0.267
	100.00	212.00	6.235E+06	9.043E+02	1.382E+03	4.566E+03	456.9	28.52	70	1.46	5.64	0.386	5.57E+01	3.48E+00	15.43	0.322
	120.00	248.00	9.082E+06	1.317E+03	2.402E+03	7.875E+03	385.8	24.08	48	1.00	1.97	0.135	9.88E+01	6.15E+00	17.78	0.371
c p	132.50	270.50	1.130E+07	1.640E+03	4.907E+03	1.609E+04	235.0	14.67	25	0.52	0.00	0.000	2.35E+02	1.47E+01	24.90	0.520
		Argon (R 740) Ar				M = 39.948				Zc = 0.291						
t p	-189.28	-308.70	6.895E+04	1.000E+01	4.971E+00	1.630E+01	1415.0	88.34	290	6.05	13.37	0.916	4.04E+00	2.52E-01	6.99	0.146
n b p	-185.88	-302.55	1.013E+05	1.470E+01	7.416E+00	2.432E+01	1393.8	87.01	261	5.45	12.49	0.856	5.75E+00	3.59E-01	7.39	0.154
	-180.00	-292.00	1.806E+05	2.620E+01	1.358E+01	4.453E+01	1357.0	84.72	220	4.59	11.02	0.755	9.79E+00	6.11E-01	7.99	0.167
	-160.00	-256.00	8.140E+05	1.181E+02	6.825E+01	2.238E+02	1216.6	75.95	137	2.85	6.35	0.435	4.04E+01	2.52E+00	9.83	0.205
	-140.00	-220.00	2.350E+06	3.408E+02	2.325E+02	7.624E+02	1031.1	64.37	94	1.96	2.38	0.163	1.22E+02	7.63E+00	12.89	0.269
c p	-122.42	-188.36	4.865E+06	7.056E+02	9.276E+02	3.042E+03	536.0	33.40	28	0.58	0.00	0.000	5.35E+02	3.34E+01	27.90	0.583
		Benzene C_6H_6				M = 78.113				Zc = 0.271						
t p	5.53	41.95	4.796E+03	6.956E-01	5.487E-01	1.799E+00	891.7	55.67	804	16.79	30.81	2.111	1.68E-01	1.05E-02	7.12	0.149
	15.56	60.00	8.063E+03	1.169E+00	9.326E-01	3.058E+00	882.0	55.06	694	14.48	29.46	2.019	2.68E-01	1.68E-02	7.46	0.156
	20.00	68.00	1.002E+04	1.453E+00	1.165E+00	3.819E+00	877.7	54.79	651	13.60	28.87	1.978	3.27E-01	2.04E-02	7.60	0.159
	40.00	104.00	2.433E+04	3.526E+00	2.894E+00	9.489E+00	857.7	53.55	496	10.36	26.24	1.798	7.39E-01	4.62E-02	8.21	0.171
	60.00	140.00	5.210E+04	7.556E+00	6.349E+00	2.082E+01	837.1	52.26	386	8.07	23.65	1.621	1.49E+00	9.32E-02	8.77	0.183
	80.00	176.00	1.008E+05	1.462E+01	1.261E+01	4.135E+01	815.6	50.92	307	6.42	21.12	1.447	2.75E+00	1.72E-01	9.30	0.194
n b p	80.09	176.16	1.013E+05	1.470E+01	1.268E+01	4.157E+01	815.5	50.91	307	6.41	21.11	1.446	2.76E+00	1.72E-01	9.30	0.194
	100.00	212.00	1.797E+05	2.606E+01	2.311E+01	7.578E+01	793.2	49.52	249	5.21	18.65	1.278	4.71E+00	2.94E-01	9.84	0.206
	120.00	248.00	2.993E+05	4.341E+01	3.967E+01	1.301E+02	769.7	48.05	206	4.31	16.23	1.112	7.61E+00	4.75E-01	10.41	0.217
	140.00	284.00	4.715E+05	6.839E+01	6.457E+01	2.116E+02	744.9	48.50	174	3.64	13.89	0.952	1.18E+01	7.37E-01	11.04	0.230
	160.00	320.00	7.090E+05	1.028E+02	1.007E+02	3.301E+02	718.5	44.85	150	3.14	11.62	0.796	1.74E+01	1.08E+00	11.75	0.245
	180.00	356.00	1.025E+06	1.487E+02	1.516E+02	4.972E+02	690.0	43.07	132	2.75	9.43	0.646	2.51E+01	1.57E+00	12.57	0.262
	200.00	392.00	1.436E+06	2.082E+02	2.223E+02	7.291E+02	658.8	41.13	118	2.46	7.33	0.503	3.59E+01	2.24E+00	13.53	0.283
	220.00	428.00	1.956E+06	2.836E+02	3.199E+02	1.049E+03	623.8	38.94	108	2.25	5.35	0.367	5.10E+01	3.18E+00	14.65	0.306
	240.00	464.00	2.604E+06	3.776E+02	4.557E+02	1.494E+03	582.9	36.39	100	2.09	3.50	0.240	7.26E+01	4.53E+00	15.97	0.333
	260.00	500.00	3.401E+06	4.933E+02	6.527E+02	2.140E+03	531.6	33.19	95	1.98	1.83	0.125	1.05E+02	6.53E+00	17.50	0.365
	280.00	536.00	4.375E+06	6.345E+02	9.887E+02	3.242E+03	451.4	28.18	91	1.91	0.43	0.029	1.66E+02	1.04E+01	19.28	0.403
c p	289.01	552.22	4.898E+06	7.104E+02	1.656E+03	5.432E+03	301.7	18.83	90	1.89	0.00	0.000	3.02E+02	1.88E+01	90.34	1.887

t p = triple point, n b p = normal boiling point, c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\rho$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Bromine Br_2				**M = 159.808**				**Zc = 0.287**							
t p	-7.30	18.66	5.848E+03	8.482E-01	1.863E-01	6.108E-01	3203.3	199.96	1283	26.79	46.38	3.178	4.25E-01	2.65E-02	14.85	0.310
	0.00	32.00	8.662E+03	1.256E+00	2.777E-01	9.108E-01	3181.7	198.63	1196	24.98	45.14	3.093	6.13E-01	3.83E-02	15.19	0.317
	15.56	60.00	1.857E+04	2.693E+00	6.042E-01	1.981E+00	3134.9	195.71	1033	21.58	42.53	2.914	1.25E+00	7.77E-02	15.91	0.332
	20.00	68.00	2.270E+04	3.292E+00	7.420E-01	2.433E+00	3121.4	194.86	992	20.71	41.79	2.863	1.50E+00	9.37E-02	16.12	0.337
	40.00	104.00	5.181E+04	7.514E+00	1.728E+00	5.666E+00	3059.2	190.96	831	17.35	38.47	2.636	3.21E+00	2.01E-01	17.07	0.357
n b p	58.75	137.75	1.013E+05	1.470E+01	3.447E+00	1.130E+01	2999.1	187.23	712	14.87	35.39	2.425	5.95E+00	3.72E-01	17.97	0.375
	60.00	140.00	1.058E+05	1.534E+01	3.603E+00	1.182E+01	2995.1	186.96	705	14.73	35.19	2.411	6.19E+00	3.87E-01	18.03	0.377
	80.00	176.00	1.973E+05	2.861E+01	6.872E+00	2.254E+01	2928.6	182.83	607	12.67	31.95	2.190	1.10E+01	6.84E-01	19.02	0.397
	100.00	212.00	3.415E+05	4.953E+01	1.218E+01	3.995E+01	2859.5	178.51	529	11.04	28.76	1.971	1.81E+01	1.13E+00	20.03	0.418
	120.00	248.00	5.557E+05	8.060E+01	2.034E+01	6.670E+01	2787.4	174.01	466	9.74	25.62	1.755	2.83E+01	1.76E+00	21.07	0.440
	140.00	284.00	8.588E+05	1.246E+02	3.231E+01	1.060E+02	2711.7	169.28	416	8.69	22.53	1.544	4.22E+01	2.63E+00	22.16	0.463
	160.00	320.00	1.271E+06	1.844E+02	4.928E+01	1.616E+02	2631.7	164.29	375	7.84	19.50	1.336	6.07E+01	3.79E+00	23.31	0.487
	180.00	356.00	1.814E+06	2.630E+02	7.266E+01	2.383E+02	2546.4	158.97	341	7.13	16.53	1.133	8.47E+01	5.29E+00	24.54	0.513
	200.00	392.00	2.509E+06	3.638E+02	1.043E+02	3.419E+02	2454.6	153.23	313	6.54	13.64	0.934	1.15E+02	7.21E+00	25.88	0.540
	220.00	428.00	3.379E+06	4.901E+02	1.465E+02	4.803E+02	2354.0	146.95	289	6.04	10.82	0.742	1.55E+02	9.66E+00	27.38	0.572
	240.00	464.00	4.450E+06	6.455E+02	2.026E+02	6.643E+02	2241.4	139.92	269	5.62	8.11	0.556	2.05E+02	1.28E+01	29.11	0.608
	260.00	500.00	5.748E+06	8.337E+02	2.779E+02	9.113E+02	2110.4	131.75	252	5.26	5.52	0.378	2.68E+02	1.67E+01	31.21	0.652
	280.00	536.00	7.303E+06	1.059E+03	3.826E+02	1.255E+03	1947.2	121.56	237	4.95	3.10	0.212	3.59E+02	2.24E+01	34.09	0.712
	300.00	572.00	9.147E+06	1.327E+03	5.485E+02	1.799E+03	1701.5	106.22	224	4.68	0.93	0.064	5.16E+02	3.22E+01	39.30	0.821
c p	311.00	591.80	1.034E+07	1.499E+03	8.907E+02	2.921E+03	1183.8	73.90	72	1.50	0.00	0.000	1.18E+03	7.39E+01	71.66	1.497
	Butane-iso (R 600A) C_4H_{10}				**M = 58.123**				**Zc = 0.282**							
t p	-159.61	-255.30	1.422E-02	2.082E-06	1.956E-06	6.415E-06	741.4	46.28	17133	357.83	34.45	2.361	1.16E-06	7.26E-08	2.49	0.052
	-140.00	-220.00	1.273E+00	1.846E-04	1.797E-04	5.894E-04	722.6	45.11	5006	104.56	31.56	2.163	7.53E-05	4.70E-06	3.13	0.065
	-120.00	-184.00	3.298E+01	4.783E-03	4.783E-03	1.569E-02	703.4	43.91	2127	44.43	28.67	1.965	1.55E-03	9.70E-05	3.77	0.079
	-100.00	-148.00	3.586E+02	5.201E-02	5.348E-02	1.754E-01	664.1	42.71	1135	23.71	25.84	1.771	1.45E-02	9.05E-04	4.40	0.092
	-80.00	-112.00	2.183E+03	3.166E-01	3.351E-01	1.099E+00	664.5	41.48	698	14.59	23.08	1.581	7.93E-02	4.95E-03	5.02	0.105
	-60.00	-76.00	9.042E+03	1.312E+00	1.431E+00	4.693E+00	644.6	40.24	473	9.86	20.38	1.397	2.99E-01	1.87E-02	5.61	0.117
	-40.00	-40.00	2.836E+04	4.114E+00	4.637E+00	1.521E+01	624.0	38.96	343	7.16	17.76	1.217	8.65E-01	5.40E-02	6.18	0.129
	-20.00	-4.00	7.235E+04	1.049E+01	1.225E+01	4.016E+01	602.8	37.63	261	5.46	15.22	1.043	2.06E+00	1.26E-01	6.73	0.141
n b p	-11.72	10.90	1.013E+05	1.470E+01	1.741E+01	5.709E+01	593.8	37.07	237	4.94	14.19	0.972	2.81E+00	1.76E-01	6.96	0.145
	0.00	32.00	1.578E+05	2.289E+01	2.773E+01	9.095E+01	580.5	36.24	207	4.33	12.77	0.875	4.25E+00	2.65E-01	7.02	0.147
	15.56	60.00	2.659E+05	3.856E+01	4.824E+01	1.582E+02	562.3	35.10	177	3.70	10.92	0.749	6.95E+00	4.34E-01	7.64	0.160
	20.00	68.00	3.050E+05	4.424E+01	5.589E+01	1.833E+02	556.8	34.76	170	3.54	10.41	0.713	7.92E+00	4.94E-01	7.78	0.162
	40.00	104.00	5.364E+05	7.780E+01	1.030E+02	3.378E+02	531.2	33.16	141	2.94	8.16	0.559	1.37E+01	8.54E-01	8.26	0.173
	60.00	140.00	8.756E+05	1.270E+02	1.777E+02	5.827E+02	502.7	31.38	122	2.55	6.04	0.414	2.24E+01	1.40E+00	8.80	0.184
	80.00	176.00	1.348E+06	1.955E+02	2.926E+02	9.596E+02	469.9	29.34	103	2.16	4.07	0.279	3.57E+01	2.23E+00	9.71	0.203
	100.00	212.00	1.985E+06	2.878E+02	4.710E+02	1.545E+03	429.8	26.83	81	1.70	2.29	0.157	5.64E+01	3.52E+00	11.33	0.237
	120.00	248.00	2.832E+06	4.108E+02	7.750E+02	2.541E+03	372.9	23.26	59	1.22	0.78	0.053	9.53E+01	5.95E+00	13.96	0.292
c p	134.99	274.98	3.655E+06	5.301E+02	1.663E+03	5.452E+03	224.3	14.00	23	0.49	0.00	0.000	2.24E+02	1.40E+01	23.30	0.487

t p = triple point, n b p = normal boiling point, c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_f		Liquid Viscosity - μ_f		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Butane-n (R 600) C_4H_{10}				**M = 58.123**				**Zc = 0.274**							
t p	-138.29	-216.92	6.757E-01	9.800E-05	9.375E-05	3.074E-04	735.3	45.90	2144	44.78	32.90	2.254	3.49E-05	2.18E-06	1.99	0.041
	-120.00	-184.00	1.372E+01	1.990E-03	1.949E-03	6.391E-03	718.2	44.84	1271	26.55	30.36	2.080	6.30E-04	3.93E-05	2.67	0.056
	-100.00	-148.00	1.659E+02	2.406E-02	2.420E-02	7.935E-02	699.4	43.67	793	16.55	27.63	1.894	6.70E-03	4.18E-04	3.43	0.072
	-80.00	-112.00	1.118E+03	1.622E-01	1.676E-01	5.496E-01	680.5	42.48	540	11.27	24.96	1.710	4.05E-02	2.53E-03	4.19	0.087
	-60.00	-76.00	5.017E+03	7.277E-01	7.740E-01	2.538E+00	661.3	41.28	395	8.26	22.33	1.530	1.65E-01	1.03E-02	4.92	0.103
	-40.00	-40.00	1.679E+04	2.435E+00	2.669E+00	8.754E+00	641.7	40.06	306	6.40	19.76	1.354	5.10E-01	3.18E-02	5.63	0.118
	-20.00	-4.00	4.522E+04	6.558E+00	7.422E+00	2.434E+01	621.5	38.80	248	5.17	17.26	1.182	1.28E+00	7.99E-02	6.30	0.132
n b p	-0.50	31.10	1.013E+05	1.470E+01	1.720E+01	5.640E+01	601.1	37.53	206	4.31	14.88	1.019	2.71E+00	1.69E-01	6.98	0.146
	0.00	32.00	1.033E+05	1.499E+01	1.755E+01	5.758E+01	600.5	37.49	205	4.29	14.82	1.015	2.76E+00	1.72E-01	7.00	0.146
	15.56	60.00	1.797E+05	2.607E+01	3.142E+01	1.030E+02	583.5	36.43	179	3.73	12.97	0.869	4.63E+00	2.89E-01	7.46	0.156
	20.00	68.00	2.080E+05	3.017E+01	3.668E+01	1.203E+02	578.5	36.11	172	3.59	12.45	0.853	5.32E+00	3.32E-01	7.58	0.158
	40.00	104.00	3.793E+05	5.502E+01	6.974E+01	2.287E+02	554.9	34.64	141	2.94	10.17	0.697	9.43E+00	5.89E-01	8.13	0.170
	60.00	140.00	6.397E+05	9.277E+01	1.233E+02	4.043E+02	529.3	33.04	116	2.42	7.98	0.547	1.57E+01	9.82E-01	8.72	0.182
	80.00	176.00	1.013E+06	1.489E+02	2.064E+02	6.769E+02	500.7	31.26	96	1.99	5.90	0.404	2.52E+01	1.57E+00	9.46	0.198
	100.00	212.00	1.526E+06	2.214E+02	3.331E+02	1.092E+03	467.5	29.18	77	1.61	3.95	0.271	3.94E+01	2.46E+00	10.49	0.219
	120.00	248.00	2.211E+06	3.207E+02	5.294E+02	1.736E+03	426.1	26.60	59	1.23	2.17	0.149	6.17E+01	3.85E+00	11.96	0.250
	140.00	284.00	3.120E+06	4.525E+02	8.729E+02	2.862E+03	364.6	22.76	42	0.88	0.55	0.044	1.04E+02	6.48E+00	15.02	0.314
c p	152.03	305.65	3.796E+06	5.505E+02	1.707E+03	5.599E+03	226.8	14.16	24	0.50	0.00	0.000	2.27E+02	1.42E+01	24.00	0.501
	Carbon Dioxide (R 744) CO_2				**M = 44.010**		**ω = 0.2276**		**Zc = 0.274**							
n b p	-78.48	-109.26	1.013E+05	1.470E+01	*Solid*	*Solid*	*Solid*	*Solid*	*Solid*	*Solid*	*Solid*		2.80E+00	1.75E-01	9.75	0.204
t p	-56.57	-69.83	5.180E+05	7.512E+01	4.481E+01	1.470E+02	1178.6	73.58	251	5.24	16.63	1.139	1.38E+01	8.60E-01	10.92	0.228
	-40.00	-40.00	1.005E+06	1.458E+02	9.170E+01	3.009E+02	1117.5	69.76	202	4.21	12.76	0.875	2.61E+01	1.63E+00	12.40	0.259
	-20.00	-4.00	1.970E+06	2.858E+02	1.948E+03	6.391E+03	103.1	6.44	150	3.13	8.41	0.576	5.17E+01	3.23E+00	13.51	0.282
	0.00	32.00	3.485E+06	5.054E+02	3.836E+02	1.259E+03	926.4	57.83	106	2.22	4.49	0.308	9.76E+01	6.09E+00	15.61	0.326
	15.56	60.00	5.153E+06	7.474E+02	6.443E+02	2.114E+03	815.5	50.91	78	1.62	1.87	0.128	1.64E+02	1.02E+01	18.77	0.392
	20.00	68.00	5.726E+06	8.305E+02	7.555E+02	2.479E+03	772.7	48.24	70	1.47	1.22	0.083	1.94E+02	1.21E+01	20.04	0.419
c p	31.04	87.87	7.383E+06	1.071E+03	1.615E+03	5.299E+03	466.1	29.10	32	0.66	0.00	0.000	4.66E+02	2.91E+01	31.60	0.660
	Carbon Disulfide CS_2				**M = 76.131**				**Zc = 0.276**							
t p	-112.04	-169.67	1.528E+00	2.216E-04	1.088E-04	3.567E-04	1433.0	89.46	3489	72.86	52.77	3.616	8.68E-05	5.42E-06	6.66	0.143
	-100.00	-148.00	7.870E+00	1.141E-03	5.659E-04	1.856E-03	1418.6	68.56	2469	51.58	50.84	3.483	4.16E-04	2.60E-05	7.11	0.148
	-80.00	-112.00	7.346E+01	1.065E-02	5.376E-03	1.763E-02	1394.2	87.04	1530	31.95	47.66	3.266	3.48E-03	2.17E-04	7.55	0.158
	-60.00	-76.00	4.351E+02	6.310E-02	3.241E-02	1.063E-01	1369.3	85.48	1037	21.66	44.51	3.050	1.87E-02	1.17E-03	8.02	0.167
	-40.00	-40.00	1.848E+03	2.680E-01	1.403E-01	4.601E-01	1343.8	83.89	751	15.69	41.40	2.837	7.26E-02	4.53E-03	8.51	0.178
	-20.00	-4.00	6.119E+03	8.874E-01	4.738E-01	1.554E+00	1317.6	82.26	573	11.96	38.33	2.626	2.21E-01	1.38E-02	9.04	0.189
	0.00	32.00	1.673E+04	2.427E+00	1.323E+00	4.337E+00	1290.7	80.58	454	9.49	35.29	2.418	5.66E-01	3.53E-02	9.57	0.200
	15.56	60.00	3.295E+04	4.779E+00	2.649E+00	8.666E+00	1269.2	79.23	388	8.10	32.96	2.258	1.05E+00	6.57E-02	10.12	0.211
	20.00	68.00	3.942E+04	5.717E+00	3.184E+00	1.044E+01	1263.0	78.85	372	7.77	32.30	2.213	1.24E+00	7.73E-02	10.28	0.215
	40.00	104.00	8.248E+04	1.196E+01	6.818E+00	2.236E+01	1234.3	77.05	312	6.53	29.35	2.011	2.44E+00	1.52E-01	11.00	0.230

Continued on next page

t p = triple point, n b p = normal boiling point, c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - h_v		Liquid Density - ρ		Liquid Viscosity - μ		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Carbon Disulfide CS_2 (continued)															
n.b.p.	46.22	115.20	1.013E+05	1.470E+01	8.437E+00	2.767E+01	1225.2	76.49	297	6.21	28.44	1.949	2.95E+00	1.84E-01	11.22	0.234
	50.00	140.00	1.569E+05	2.275E+01	1.329E+01	4.357E+01	1204.6	75.20	268	5.60	26.45	1.812	4.44E+00	2.77E-01	11.72	0.245
	80.00	176.00	2.761E+05	4.004E+01	2.400E+01	7.870E+01	1173.7	73.27	234	4.89	23.59	1.617	7.40E+00	4.62E-01	12.46	0.260
	100.00	212.00	4.557E+05	6.610E+01	4.074E+01	1.336E+02	1141.3	71.25	207	4.33	20.80	1.425	1.18E+01	7.34E-01	13.21	0.276
	120.00	248.00	7.133E+05	1.035E+02	6.572E+01	2.155E+02	1107.3	69.13	166	3.88	18.08	1.237	1.79E+01	1.12E+00	13.99	0.292
	140.00	284.00	1.068E+06	1.549E+02	1.017E+02	3.335E+02	1071.2	66.87	168	3.51	15.38	1.054	2.64E+01	1.65E+00	14.80	0.309
	160.00	320.00	1.540E+06	2.233E+02	1.521E+02	4.988E+02	1032.5	64.46	154	3.21	12.78	0.876	3.81E+01	2.38E+00	15.68	0.328
	180.00	356.00	2.150E+06	3.116E+02	2.214E+02	7.262E+02	990.5	61.83	142	2.96	10.26	0.703	5.50E+01	3.43E+00	16.68	0.348
	200.00	392.00	2.921E+06	4.237E+02	3.157E+02	1.035E+03	944.1	58.94	132	2.75	7.84	0.537	7.92E+01	4.94E+00	17.88	0.373
	220.00	428.00	3.877E+06	5.624E+02	4.439E+02	1.456E+03	891.2	55.64	123	2.57	5.53	0.379	1.15E+02	7.16E+00	19.48	0.407
	240.00	464.00	5.043E+06	7.314E+02	6.217E+02	2.038E+03	827.6	51.67	115	2.41	3.38	0.231	1.69E+02	1.05E+01	21.94	0.458
	260.00	500.00	6.444E+06	9.347E+02	8.887E+02	2.908E+03	741.5	46.29	109	2.27	1.43	0.098	2.53E+02	1.58E+01	26.44	0.552
c.p.	278.85	533.93	7.903E+06	1.146E+03	1.695E+03	5.558E+03	475.8	29.70	43	0.91	0.00	0.000	4.76E+02	2.97E+01	43.39	0.906
	Carbon Monoxide CO				**M = 28.010**				**Zc = 0.296**							
t.p.	-205.00	-337.00	1.521E+04	2.206E+00	1.833E+00	6.012E+00	846.2	52.83	269	5.61	12.38	0.848	7.58E-01	4.73E-02	4.68	0.098
	-200.00	-328.00	3.386E+04	4.911E+00	4.182E+00	1.371E+01	826.1	51.57	225	4.71	11.30	0.775	1.59E+00	9.90E-02	5.04	0.105
n.b.p.	-191.45	-312.61	1.013E+05	1.470E+01	1.309E+01	4.291E+01	790.0	49.32	173	3.62	9.49	0.650	4.35E+00	2.72E-01	5.68	0.119
	-180.00	-292.00	3.030E+05	4.395E+01	4.193E+01	1.375E+02	737.3	46.03	129	2.70	7.13	0.488	1.21E+01	7.53E-01	6.55	0.137
	-160.00	-256.00	1.206E+06	1.749E+02	1.971E+02	6.463E+02	624.3	38.97	87	1.81	3.23	0.221	4.78E+01	2.98E+00	8.37	0.175
c.p.	-140.23	-220.41	3.499E+06	5.075E+02	1.185E+03	3.888E+03	301.2	18.80	20	0.42	0.00	0.000	3.01E+02	1.88E+01	19.69	0.415
	Carbon Tetrachloride CCl_4				**M = 153.823**				**Zc = 0.272**							
t.p.	-22.82	-9.08	1.122E+03	1.627E-01	6.864E-02	2.251E-01	1667.8	104.12	2043	42.67	32.31	2.214	8.30E-02	5.18E-03	8.96	0.187
	-20.00	-4.00	1.352E+03	1.961E-01	8.293E-02	2.720E-01	1663.0	103.82	1915	40.00	31.95	2.189	9.90E-02	6.18E-03	9.06	0.189
	0.00	32.00	4.454E+03	6.461E-01	2.791E-01	9.152E-01	1628.2	101.65	1276	26.65	29.41	2.015	3.03E-01	1.89E-02	9.70	0.203
	15.56	60.00	9.850E+03	1.429E+00	6.279E-01	2.059E+00	1600.5	99.92	980	20.47	27.46	1.661	6.36E-01	3.97E-02	10.20	0.213
	20.00	68.00	1.213E+04	1.760E+00	7.773E-01	2.549E+00	1592.5	99.42	916	19.12	26.90	1.844	7.72E-01	4.82E-02	10.35	0.216
	40.00	104.00	2.845E+04	4.126E+00	1.866E+00	6.118E+00	1555.6	97.11	696	14.53	24.44	1.675	1.71E+00	1.07E-01	10.99	0.230
	60.00	140.00	5.917E+04	8.582E+00	3.978E+00	1.305E+01	1517.4	94.73	552	11.53	22.02	1.509	3.37E+00	2.11E-01	11.64	0.243
n.b.p.	76.84	169.95	1.013E+05	1.470E+01	6.963E+00	2.283E+01	1484.5	92.68	467	9.76	20.05	1.374	5.58E+00	3.48E-01	12.18	0.254
	80.00	176.00	1.117E+05	1.621E+01	7.714E+00	2.530E+01	1477.8	92.25	453	9.46	19.65	1.347	6.08E+00	3.80E-01	12.29	0.257
	100.00	212.00	1.950E+05	2.829E+01	1.385E+01	4.543E+01	1436.4	89.67	382	7.97	17.33	1.187	1.03E+01	6.41E-01	12.95	0.270
	120.00	248.00	3.192E+05	4.629E+01	2.338E+01	7.666E+01	1393.0	86.96	328	6.84	15.06	1.032	1.64E+01	1.02E+00	13.62	0.284
	140.00	284.00	4.953E+05	7.184E+01	3.751E+01	1.230E+02	1347.1	84.09	285	5.96	12.85	0.861	2.50E+01	1.56E+00	14.32	0.299
	160.00	320.00	7.355E+05	1.067E+02	5.780E+01	1.895E+02	1298.1	81.04	251	5.24	10.71	0.734	3.65E+01	2.28E+00	15.07	0.315
	180.00	356.00	1.053E+06	1.527E+02	8.625E+01	2.828E+02	1245.3	77.74	222	4.64	8.83	0.592	5.25E+01	3.28E+00	15.88	0.332
	200.00	392.00	1.461E+06	2.120E+02	1.256E+02	4.118E+02	1187.2	74.12	197	4.12	6.65	0.455	7.42E+01	4.63E+00	16.82	0.351
	220.00	428.00	1.978E+06	2.868E+02	1.799E+02	5.899E+02	1121.7	70.03	175	3.67	4.76	0.326	1.04E+02	6.49E+00	17.96	0.375
	240.00	464.00	2.620E+06	3.800E+02	2.559E+02	8.392E+02	1044.4	65.20	156	3.26	3.00	0.206	1.46E+02	9.14E+00	19.50	0.407
	260.00	500.00	3.409E+06	4.944E+02	3.684E+02	1.208E+03	944.0	58.93	138	2.89	1.41	0.097	2.15E+02	1.34E+01	22.07	0.461
	280.00	536.00	4.371E+06	6.339E+02	5.907E+02	1.937E+03	754.9	47.13	122	2.55	0.13	0.009	3.60E+02	2.25E+01	28.90	0.604
c.p.	283.20	541.76	4.560E+06	6.613E+02	8.346E+02	2.737E+03	557.3	34.79	42	0.88	0.00	0.000	5.57E+02	3.48E+01	41.97	0.876

t.p. = triple point n.b.p. = normal boiling point c.p. = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_f		Liquid Viscosity - μ_f		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Chlorine Cl_2				**M = 70.906**				**Zc = 0.275**							
t p	-101.03	-149.85	1.366E+03	1.981E-01	8.087E-02	2.652E-01	1722.9	107.55	962	20.09	37.93	2.599	6.79E-02	4.24E-03	8.09	0.169
	-80.00	-112.00	7.605E+03	1.103E+00	4.633E-01	1.519E+00	1674.6	104.54	747	15.60	34.41	2.358	3.38E-01	2.11E-02	9.10	0.190
	-60.00	-76.00	2.738E+04	3.971E+00	1.717E+00	5.630E+00	1627.0	101.57	612	12.78	31.09	2.130	1.11E+00	6.91E-02	10.06	0.210
	-40.00	-40.00	7.709E+04	1.118E+01	4.986E+00	1.635E+01	1577.4	98.48	516	10.78	27.80	1.905	2.87E+00	1.79E-01	11.02	0.230
n b p	-34.03	-29.25	1.013E+05	1.470E+01	6.617E+00	2.170E+01	1562.2	97.52	493	10.29	26.82	1.838	3.69E+00	2.30E-01	11.30	0.236
	-20.00	-4.00	1.809E+05	2.624E+01	1.210E+01	3.967E+01	1525.5	95.24	446	9.30	24.53	1.681	6.29E+00	3.92E-01	11.98	0.250
	0.00	32.00	3.697E+05	5.363E+01	2.565E+01	8.410E+01	1470.9	91.82	392	8.18	21.31	1.460	1.28E+01	7.99E-01	12.96	0.271
	15.56	60.00	5.983E+05	8.677E+01	4.280E+01	1.404E+02	1426.0	89.02	356	7.47	18.82	1.290	1.88E+01	1.16E+00	13.73	0.287
	20.00	68.00	6.795E+05	9.855E+01	4.907E+01	1.609E+02	1412.8	88.20	349	7.29	18.11	1.241	2.26E+01	1.41E+00	13.97	0.292
	40.00	104.00	1.149E+06	1.667E+02	8.681E+01	2.847E+02	1350.3	84.30	315	6.58	14.97	1.026	3.84E+01	2.40E+00	15.06	0.315
	60.00	140.00	1.819E+06	2.638E+02	1.447E+02	4.747E+02	1282.0	80.03	287	5.99	11.87	0.813	6.00E+01	3.75E+00	16.25	0.339
	80.00	176.00	2.731E+06	3.961E+02	2.311E+02	7.579E+02	1205.3	75.25	264	5.51	8.84	0.606	9.10E+01	5.68E+00	17.65	0.369
	100.00	212.00	3.927E+06	5.696E+02	3.592E+02	1.176E+03	1115.6	69.64	244	5.10	5.89	0.403	1.36E+02	8.49E+00	19.45	0.406
	120.00	248.00	5.454E+05	7.911E+02	5.561E+02	1.824E+03	1000.6	62.46	227	4.74	3.05	0.209	2.08E+02	1.29E+01	22.21	0.464
	140.00	284.00	7.360E+06	1.067E+03	9.444E+02	3.097E+03	795.1	49.64	213	4.44	0.44	0.030	4.05E+02	2.53E+01	31.77	0.663
c p	144.00	291.20	7.711E+06	1.118E+03	1.373E+03	4.502E+03	573.0	35.77	43	0.90	0.00	0.000	5.73E+02	3.58E+01	43.14	0.901
	Chlorodifluoromethane (R 22) $CHClF_2$				**M = 86.469**				**Zc = 0.274**							
t p	-160.00	-256.00	1.975E-01	2.864E-05	1.181E-05	3.872E-05	1706.2	106.52	2779	58.04	39.97	2.739	1.81E-05	1.13E-06	1.43	0.030
	-140.00	-220.00	1.250E+01	1.814E-03	7.680E-04	2.516E-03	1661.1	103.70	1650	34.47	36.09	2.473	9.77E-04	6.10E-05	3.76	0.078
	-120.00	-184.00	2.333E+02	3.384E-02	1.474E-02	4.835E-02	1614.6	100.79	1058	22.11	32.29	2.212	1.58E-02	9.89E-04	5.61	0.117
	-100.00	-148.00	2.073E+03	3.007E-01	1.346E-01	4.415E-01	1570.8	96.06	727	15.19	28.58	1.958	1.25E-01	7.79E-03	7.09	0.148
	-80.00	-112.00	1.031E+04	1.495E+00	6.933E-01	2.274E+00	1516.8	94.69	531	11.09	24.97	1.711	5.59E-01	3.49E-02	8.27	0.173
	-60.00	-76.00	3.749E+04	5.438E+00	2.614E+00	8.573E+00	1463.1	91.34	409	8.53	21.46	1.470	1.66E+00	1.16E-01	9.25	0.193
n b p	-40.82	-41.48	1.013E+05	1.470E+01	7.337E+00	2.406E+01	1408.9	87.95	331	6.92	18.20	1.247	4.71E+00	2.94E-01	10.10	0.211
	-40.00	-40.00	1.053E+05	1.527E+01	7.636E+00	2.504E+01	1406.5	87.60	329	6.87	18.06	1.238	4.88E+00	3.05E-01	10.14	0.212
	-20.00	-4.00	2.453E+05	3.558E+01	1.859E+01	6.095E+01	1346.4	84.05	275	5.73	14.79	1.014	1.08E+01	6.75E-01	11.01	0.230
	0.00	32.00	4.979E+05	7.222E+01	3.964E+01	1.300E+02	1261.6	80.01	236	4.93	11.67	0.799	2.13E+01	1.33E+00	11.96	0.250
	15.56	60.00	8.021E+05	1.163E+02	6.672E+01	2.188E+02	1226.6	76.57	213	4.44	9.35	0.640	3.40E+01	2.12E+00	12.82	0.268
	20.00	68.00	9.101E+05	1.320E+02	7.673E+01	2.516E+02	1210.0	75.54	207	4.32	8.70	0.596	3.88E+01	2.41E+00	13.04	0.272
	40.00	104.00	1.534E+06	2.225E+02	1.387E+02	4.547E+02	1128.6	70.46	184	3.84	5.94	0.407	6.64E+01	4.15E+00	14.33	0.299
	80.00	140.00	2.426E+06	3.521E+02	2.404E+02	7.884E+02	1030.3	84.32	164	3.42	3.41	0.234	1.12E+02	6.99E+00	15.92	0.333
	80.00	176.00	3.663E+06	5.313E+02	4.181E+02	1.371E+03	893.9	55.80	123	2.57	1.24	0.085	1.97E+02	1.23E+01	17.93	0.375
c p	96.15	205.07	4.988E+06	7.234E+02	9.920E+02	3.253E+03	513.0	32.03	31	0.64	0.00	0.000	5.13E+02	3.20E+01	30.50	0.637
	Chloroform $CHCl_3$				**M = 119.378**				**Zc = 0.293**							
t p	-63.52	-82.34	6.932E+01	1.005E-02	4.308E-03	1.413E-02	1641.5	102.48	1786	37.30	38.68	2.650	4.75E-03	2.96E-04	7.54	0.157
	-60.00	-76.00	9.882E+01	1.433E-02	6.164E-03	2.021E-02	1635.5	102.10	1867	34.82	38.19	2.617	6.66E-03	4.16E-04	7.66	0.160
	-40.00	-40.00	5.797E+02	8.407E-02	3.695E-02	1.212E-01	1600.5	99.91	1174	24.51	35.43	2.427	3.57E-02	2.23E-03	8.37	0.175
	-20.00	-4.00	2.427E+03	3.520E-01	1.583E-01	5.190E-01	1564.6	97.67	873	18.23	32.69	2.240	1.38E-01	8.61E-03	9.07	0.189
	0.00	32.00	7.884E+03	1.143E+00	5.265E-01	1.727E+00	1527.7	96.37	677	14.14	30.00	2.055	4.17E-01	2.60E-02	9.77	0.204

Continued on next page

t p = triple point, n b p = normal boiling point, c p = critical point

	Temperature - T		Vapor Pressure-P_v		Vapor Head-$h_v = P/\gamma$		Liquid Density - ρ_f		Liquid Viscosity - μ_f		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Chloroform $CHCl_3$ (continued)															
	15.55	60.00	1.719E+04	2.493E+00	1.171E+00	3.839E+00	1498.2	93.53	569	11.88	27.92	1.913	8.63E-01	5.39E-02	10.31	0.215
	20.00	68.00	2.108E+04	3.058E+00	1.444E+00	4.735E+00	1489.7	93.00	543	11.34	27.33	1.873	1.04E+00	6.52E-02	10.47	0.219
	40.00	104.00	4.847E+04	7.030E+00	3.410E+00	1.118E+01	1450.4	90.55	447	9.33	24.71	1.693	2.27E+00	1.42E-01	11.17	0.233
	60.00	140.00	9.893E+04	1.435E+01	7.159E+00	2.348E+01	1409.7	88.00	375	7.83	22.13	1.516	4.41E+00	2.75E-01	11.87	0.248
n b p	61.18	142.12	1.013E+05	1.470E+01	7.346E+00	2.409E+01	1407.2	87.85	371	7.76	21.98	1.506	4.50E+00	2.81E-01	11.91	0.249
	80.00	176.00	1.836E+05	2.664E+01	1.370E+01	4.493E+01	1367.3	85.36	320	6.69	19.59	1.342	7.85E+00	4.90E-01	12.59	0.263
	100.00	212.00	3.159E+05	4.582E+01	2.436E+01	7.989E+01	1322.9	82.58	277	5.79	17.10	1.172	1.32E+01	8.22E-01	13.33	0.278
	120.00	248.00	5.110E+05	7.411E+01	4.085E+01	1.340E+02	1276.1	79.66	242	5.06	14.66	1.005	2.28E+01	1.42E+00	14.12	0.295
	140.00	284.00	7.860E+05	1.140E+02	6.539E+01	2.144E+02	1226.2	76.55	214	4.47	12.29	0.842	3.52E+01	2.20E+00	14.96	0.313
	160.00	320.00	1.160E+06	1.683E+02	1.010E+02	3.311E+02	1172.6	73.20	190	3.97	9.98	0.684	5.12E+01	3.20E+00	15.88	0.332
	180.00	356.00	1.656E+06	2.402E+02	1.517E+02	4.974E+02	1113.8	69.53	170	3.55	7.75	0.531	6.79E+01	4.24E+00	16.85	0.352
	200.00	392.00	2.298E+06	3.333E+02	2.238E+02	7.339E+02	1047.7	65.41	153	3.19	5.61	0.384	9.28E+01	5.79E+00	18.03	0.376
	220.00	428.00	3.117E+06	4.521E+02	3.278E+02	1.075E+03	970.1	60.56	138	2.88	3.59	0.246	1.24E+02	7.76E+00	19.46	0.407
	240.00	464.00	4.147E+06	6.015E+02	4.863E+02	1.595E+03	889.9	54.31	124	2.60	1.73	0.118	1.81E+02	1.13E+01	21.90	0.457
	260.00	500.00	5.430E+06	7.876E+02	8.104E+02	2.658E+03	683.6	42.68	113	2.35	0.17	0.012	3.29E+02	2.06E+01	29.64	0.619
c p	263.25	505.85	5.472E+06	7.936E+02	1.116E+03	3.665E+03	499.5	31.18	42	0.88	0.00	0.000	4.99E+02	3.12E+01	42.32	0.884
	Chloropentafluoroethane (R 115) C_2ClF_5				M = 154.467				Zc = 0.277							
t p	-106.15	-159.07	1.172E+03	1.700E-01	6.790E-02	2.228E-01	1760.1	109.88	1745	36.45	23.74	1.627	1.31E-01	8.16E-03	5.93	0.124
	-100.00	-148.00	2.066E+03	2.996E-01	1.209E-01	3.967E-01	1742.2	108.76	1465	30.59	22.81	1.563	2.22E-01	1.39E-02	6.49	0.135
	-80.00	-112.00	1.023E+04	1.484E+00	6.202E-01	2.035E+00	1682.2	105.02	895	18.70	19.85	1.360	9.96E-01	6.22E-02	8.02	0.167
	-60.00	-76.00	3.543E+04	5.139E+00	2.232E+00	7.322E+00	1619.0	101.07	600	12.54	16.96	1.162	3.18E+00	1.98E-01	9.21	0.192
	-40.00	-40.00	9.847E+04	1.426E+01	6.474E+00	2.124E+01	1550.9	96.82	431	9.00	14.14	0.969	8.20E+00	5.12E-01	10.18	0.213
n b p	-39.11	-38.40	1.013E+05	1.470E+01	6.672E+00	2.189E+01	1548.5	96.67	425	8.88	14.03	0.961	8.51E+00	5.32E-01	10.22	0.213
	-20.00	-4.00	2.218E+05	3.217E+01	1.530E+01	5.019E+01	1478.7	92.31	326	6.81	11.41	0.782	1.79E+01	1.12E+00	11.07	0.231
	0.00	32.00	4.420E+05	6.411E+01	3.226E+01	1.058E+02	1397.4	87.24	257	5.37	8.72	0.597	3.51E+01	2.19E+00	12.01	0.251
	15.56	60.00	7.041E+05	1.021E+02	5.398E+01	1.771E+02	1329.9	83.03	220	4.58	6.72	0.461	5.61E+01	3.50E+00	12.86	0.269
	20.00	68.00	7.966E+05	1.155E+02	6.206E+01	2.036E+02	1308.9	81.71	208	4.35	6.17	0.423	6.36E+01	3.97E+00	13.13	0.274
	40.00	104.00	1.329E+06	1.928E+02	1.126E+02	3.694E+02	1204.1	75.17	184	3.43	3.76	0.257	1.11E+02	6.95E+00	14.56	0.304
	60.00	140.00	2.091E+06	3.032E+02	1.991E+02	6.533E+02	1070.6	66.84	115	2.41	1.61	0.111	1.96E+02	1.22E+01	16.90	0.353
c p	79.94	175.89	3.155E+06	4.576E+02	5.248E+02	1.722E+03	613.1	38.28	29	0.60	0.00	0.000	6.13E+02	3.83E+01	28.70	0.599
	Chlorotrifluoromethane (R-13) $CClF_3$				M = 104.459				Zc = 0.278							
t p	-181.15	-294.07	4.088E-01	5.929E-05	2.236E-05	7.336E-05	1864.5	116.39	7803	162.97	31.66	2.169	5.58E-05	3.49E-06	2.99	0.062
	-180.00	-292.00	5.613E-01	8.140E-05	3.076E-05	1.009E-04	1860.8	116.16	7234	151.08	31.44	2.154	7.57E-05	4.73E-06	3.07	0.064
	-160.00	-256.00	4.518E+01	6.553E-03	2.565E-03	8.416E-03	1796.0	112.12	2478	51.76	27.67	1.896	5.02E-03	3.13E-04	4.58	0.096
	-140.00	-220.00	8.600E+02	1.247E-01	5.070E-02	1.683E-01	1729.8	107.99	1172	24.47	24.00	1.645	8.11E-02	5.07E-03	6.06	0.127
	-120.00	-184.00	6.988E+03	1.013E+00	4.288E-01	1.407E+00	1661.5	103.73	673	14.07	20.48	1.402	5.77E-01	3.60E-02	7.45	0.156
	-100.00	-148.00	3.311E+04	4.602E+00	2.122E+00	6.964E+00	1590.6	99.30	440	9.19	17.03	1.167	2.45E+00	1.53E-01	8.73	0.182
n b p	-81.45	-114.61	1.013E+05	1.470E+01	6.791E+00	2.228E+01	1521.3	94.97	321	6.70	13.98	0.958	6.94E+00	4.33E-01	9.60	0.205
	-80.00	-112.00	1.094E+05	1.587E+01	7.362E+00	2.416E+01	1515.7	94.62	314	6.56	13.75	0.942	7.46E+00	4.66E-01	9.90	0.207
	-60.00	-76.00	2.818E+05	4.087E+01	2.002E+01	6.568E+01	1435.4	89.61	239	4.98	10.63	0.728	1.82E+01	1.14E+00	11.01	0.230
	-40.00	-40.00	6.070E+05	8.804E+01	4.595E+01	1.508E+02	1346.9	84.08	190	3.97	7.69	0.527	3.83E+01	2.39E+00	12.03	0.251

Continued on next page

t p = triple point, n b p = normal boiling point, c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension σ		Vapor Density ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Chlorotrifluoromethane (R-13) $CClF_3$ (continued)															
	-20.00	-4.00	1.148E+06	1.665E+02	9.400E+01	3.084E+02	1245.1	77.73	157	3.28	4.97	0.341	7.36E+01	4.59E+00	13.29	0.278
	0.00	32.00	1.973E+06	2.862E+02	1.799E+02	5.902E+02	1118.3	69.81	130	2.72	2.55	0.175	1.35E+02	8.43E+00	15.11	0.316
	15.56	60.00	2.866E+06	4.156E+02	2.987E+02	9.801E+02	978.2	61.07	91	1.90	0.95	0.085	2.23E+02	1.39E+01	17.12	0.358
	20.00	68.00	3.171E+06	4.599E+02	3.511E+02	1.152E+03	920.8	57.48	77	1.61	0.57	0.039	2.84E+02	1.65E+01	17.82	0.372
c p	28.80	83.84	3.866E+06	5.607E+02	6.822E+02	2.238E+03	577.8	36.07	29	0.60	0.00	0.000	5.78E+02	3.61E+01	28.90	0.604
	Cyclohexane C_6H_{12}				M = 84.161.				Zc = 0.273							
t p	6.54	43.77	5.357E+03	7.769E-01	6.919E-01	2.270E+00	789.4	49.28	1212	25.32	26.91	1.844	1.94E-01	1.21E-02	6.97	0.146
	15.56	60.00	8.441E+03	1.224E+00	1.101E+00	3.614E+00	781.5	48.79	1031	21.53	25.80	1.768	2.96E-01	1.85E-02	7.18	0.150
	20.00	68.00	1.043E+04	1.513E+00	1.368E+00	4.488E+00	777.6	48.54	955	19.94	25.26	1.731	3.60E-01	2.25E-02	7.28	0.152
	40.00	104.00	2.474E+04	3.588E+00	3.322E+00	1.090E+01	759.5	47.41	695	14.52	22.85	1.566	8.00E-01	4.99E-02	7.75	0.162
	60.00	140.00	5.188E+04	7.524E+00	7.141E+00	2.343E+01	740.8	46.25	525	10.96	20.49	1.404	1.58E+00	9.84E-02	8.22	0.172
	60.00	176.00	9.856E+04	1.430E+01	1.393E+01	4.571E+01	721.3	45.03	409	8.54	18.18	1.248	2.83E+00	1.76E-01	8.69	0.181
n b p	60.72	177.30	1.013E+05	1.470E+01	1.434E+01	4.704E+01	720.6	44.99	406	8.47	18.10	1.240	2.90E+00	1.81E-01	8.71	0.182
	100.00	212.00	1.729E+05	2.508E+01	2.516E+01	8.254E+01	701.0	43.76	327	6.83	15.93	1.092	4.90E+00	3.06E-01	9.17	0.192
	120.00	248.00	2.844E+05	4.124E+01	4.267E+01	1.400E+02	679.6	42.42	267	5.58	13.75	0.942	7.96E+00	4.98E-01	9.66	0.202
	140.00	284.00	4.434E+05	6.431E+01	6.883E+01	2.258E+02	656.9	41.01	223	4.65	11.63	0.797	1.24E+01	7.71E-01	10.18	0.213
	160.00	320.00	6.617E+05	9.598E+01	1.066E+02	3.499E+02	632.7	39.50	189	3.94	9.59	0.657	1.64E+01	1.15E+00	10.73	0.224
	160.00	356.00	9.523E+05	1.381E+02	1.601E+02	5.252E+02	606.5	37.87	162	3.39	7.63	0.523	2.66E+01	1.66E+00	11.33	0.237
	200.00	392.00	1.329E+06	1.928E+02	2.347E+02	7.700E+02	577.6	36.06	141	2.95	5.78	0.396	3.78E+01	2.36E+00	12.02	0.251
	220.00	428.00	1.810E+06	2.625E+02	3.387E+02	1.111E+03	544.8	34.01	124	2.60	4.03	0.276	5.33E+01	3.33E+00	12.87	0.269
	240.00	464.00	2.411E+06	3.498E+02	4.863E+02	1.595E+03	505.7	31.57	111	2.31	2.43	0.167	7.55E+01	4.71E+00	14.03	0.293
	260.00	500.00	3.157E+06	4.578E+02	7.096E+02	2.328E+03	453.6	28.32	99	2.08	1.03	0.071	1.09E+02	6.82E+00	15.89	0.332
c p	260.39	536.70	4.075E+06	5.910E+02	1.520E+03	4.987E+03	273.4	17.07	29	0.61	0.00	0.000	2.73E+02	1.71E+01	29.35	0.613
	Dichlorodifluoromethane (R 12) CCl_2F_2				M = 120.914				Zc = 0.280							
t p	-158.00	-252.40	9.302E-02	1.349E-05	5.226E-06	1.715E-05	1814.9	113.30	8190	171.06	36.97	2.533	1.17E-05	7.33E-07	5.65	0.118
	-140.00	-220.00	4.966E+00	7.202E-04	2.856E-04	9.389E-04	1773.1	110.69	3679	76.83	33.87	2.321	5.42E-04	3.39E-05	6.29	0.131
	-120.00	-184.00	1.137E+02	1.650E-02	6.722E-03	2.205E-02	1725.4	107.71	1885	39.37	30.50	2.090	1.06E-02	6.74E-04	6.96	0.146
	-100.00	-148.00	1.174E+03	1.703E-01	7.134E-02	2.341E-01	1678.0	104.75	1127	23.54	27.21	1.864	9.68E-02	6.17E-03	7.67	0.160
	-80.00	-112.00	6.160E+03	8.934E-01	3.864E-01	1.268E+00	1625.5	101.48	750	15.66	23.99	1.644	4.66E-01	2.91E-02	8.38	0.175
	-60.00	-76.00	2.259E+04	3.277E+00	1.466E+00	4.809E+00	1571.5	96.11	538	11.25	20.87	1.430	1.58E+00	9.76E-02	9.11	0.190
	-40.00	-40.00	6.415E+04	9.304E+00	4.316E+00	1.416E+01	1515.7	94.62	409	8.55	17.84	1.222	4.12E+00	2.57E-01	9.88	0.206
n b p	-29.79	-21.62	1.013E+05	1.470E+01	6.951E+00	2.281E+01	1486.3	92.79	362	7.58	16.33	1.119	6.30E+00	3.94E-01	10.16	0.212
	-20.00	-4.00	1.510E+05	2.190E+01	1.057E+01	3.487E+01	1457.4	90.98	325	6.78	14.92	1.022	9.15E+00	5.71E-01	10.84	0.226
	0.00	32.00	3.089E+05	4.479E+01	2.257E+01	7.404E+01	1395.6	87.12	267	5.57	12.11	0.830	1.80E+01	1.12E+00	11.83	0.247
	15.56	60.00	4.996E+05	7.246E+01	3.790E+01	1.244E+02	1344.1	83.91	233	4.87	10.02	0.686	2.85E+01	1.78E+00	12.44	0.260
	20.00	68.00	5.674E+05	8.229E+01	4.354E+01	1.429E+02	1328.7	82.95	225	4.70	9.43	0.646	3.22E+01	2.01E+00	12.61	0.263
	40.00	104.00	9.594E+05	1.392E+02	7.802E+01	2.560E+02	1253.9	78.28	194	4.05	6.91	0.473	5.45E+01	3.40E+00	13.49	0.282
	60.00	140.00	1.521E+06	2.206E+02	1.330E+02	4.362E+02	1166.6	72.83	170	3.55	4.57	0.313	8.88E+01	5.55E+00	14.82	0.310
	80.00	176.00	2.295E+06	3.329E+02	2.214E+02	7.264E+02	1057.2	66.00	141	2.93	2.46	0.169	1.44E+02	9.01E+00	16.93	0.354
	100.00	212.00	3.340E+06	4.844E+02	3.790E+02	1.244E+03	896.6	56.09	96	2.00	0.70	0.048	2.51E+02	1.57E+01	20.16	0.421
c p	111.80	233.24	4.125E+06	5.983E+02	7.536E+02	2.473E+03	558.0	34.83	31	0.65	0.00	0.000	5.58E+02	3.48E+01	31.02	0.648

t p = triple point, n b p = normal boiling point, c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Ethane (R 170) C_2H_6				**M = 30.069**				**Zc = 0.284**							
t p	-182.80	-297.04	1.131E+00	1.640E-04	1.769E-04	5.803E-04	651.9	40.70	1303	27.22	27.03	1.852	4.55E-05	2.84E-06	0.83	0.013
	-180.00	-292.00	2.315E+00	3.357E-04	3.637E-04	1.193E-03	648.8	40.51	1160	24.23	26.69	1.829	9.03E-05	5.64E-06	0.89	0.019
	-160.00	-256.00	1.260E+02	1.828E-02	2.050E-02	6.726E-02	626.9	39.14	592	12.36	24.21	1.659	4.03E-03	2.52E-04	2.50	0.052
	-140.00	-220.00	1.855E+03	2.691E-01	3.129E-01	1.027E+00	604.6	37.74	365	7.61	21.72	1.489	5.04E-02	3.15E-03	3.77	0.079
	-120.00	-184.00	1.259E+04	1.826E+00	2.207E+00	7.240E+00	581.8	36.32	252	5.27	19.24	1.318	2.99E-01	1.87E-02	4.77	0.100
	-100.00	-148.00	5.238E+04	7.598E+00	9.571E+00	3.140E+01	558.1	34.84	188	3.93	16.74	1.147	1.12E+00	6.98E-02	5.56	0.117
n b p	-88.60	-127.48	1.013E+05	1.470E+01	1.899E+01	6.230E+01	544.1	33.97	163	3.41	15.32	1.050	2.08E+00	1.28E-01	5.99	0.125
	-80.00	-112.00	1.575E+05	2.285E+01	3.013E+01	9.886E+01	533.2	33.28	148	3.09	14.25	0.976	3.09E+00	1.93E-01	6.30	0.132
	-60.00	-76.00	3.789E+05	5.496E+01	7.831E+01	2.504E+02	506.3	31.61	120	2.51	11.74	0.805	7.03E+00	4.39E-01	7.00	0.146
	-40.00	-40.00	7.777E+05	1.128E+02	1.664E+02	5.458E+02	476.6	29.76	99	2.08	9.23	0.632	1.40E+01	8.77E-01	7.78	0.163
	-20.00	-4.00	1.422E+06	2.062E+02	3.320E+02	1.089E+03	436.7	27.26	81	1.68	6.71	0.460	2.59E+01	1.62E+00	8.36	0.175
	0.00	32.00	2.387E+06	3.462E+02	6.078E+02	1.994E+03	400.4	25.00	63	1.31	4.17	0.286	4.82E+01	2.88E+00	9.91	0.207
	15.56	60.00	3.416E+06	4.954E+02	9.788E+02	3.211E+03	355.9	22.22	48	1.01	2.18	0.149	7.41E+01	4.63E+00	11.53	0.241
	20.00	68.00	3.763E+06	5.458E+02	1.132E+03	3.714E+03	339.0	21.16	44	0.93	1.61	0.110	8.63E+01	5.39E+00	12.41	0.259
c p	32.27	90.09	4.880E+06	7.078E+02	2.448E+03	8.032E+03	203.3	12.69	22	0.45	0.00	0.000	2.03E+02	1.27E+01	21.66	0.452
	Ethanol C_2H_6O				**M = 46.069**				**Zc = 0.248**							
t p	-114.10	-173.38	7.178E-04	1.041E-07	8.126E-08	2.666E-07	900.7	56.23	102677	2148.63	32.91	2.256	2.50E-08	1.56E-09	5.12	0.107
	-100.00	-148.00	1.372E-02	1.991E-06	1.572E-06	5.157E-06	890.3	55.58	45586	972.96	31.84	2.162	4.39E-07	2.74E-08	5.56	0.116
	-80.00	-112.00	4.099E-01	5.946E-05	4.776E-05	1.567E-04	875.2	54.64	18481	385.98	30.32	2.077	1.18E-05	7.34E-07	6.18	0.129
	-60.00	-76.00	6.171E+00	8.950E-04	7.319E-04	2.401E-03	859.7	53.67	8708	181.88	28.78	1.972	1.60E-04	1.00E-05	6.79	0.142
	-40.00	-40.00	5.809E+01	8.135E-03	6.779E-03	2.224E-02	843.7	52.67	4652	97.15	27.21	1.865	1.33E-03	8.33E-05	7.39	0.154
	-20.00	-4.00	3.481E+02	5.048E-02	4.291E-02	1.408E-01	827.1	51.63	2729	56.99	25.63	1.755	7.84E-03	4.77E-04	7.99	0.167
	0.00	32.00	1.608E+03	2.332E-01	2.025E-01	6.843E-01	809.9	50.56	1718	35.89	24.03	1.647	3.26E-02	2.06E-03	8.57	0.179
	15.56	60.00	4.490E+03	6.512E-01	5.752E-01	1.887E+00	796.0	49.69	1246	26.03	22.77	1.560	8.69E-02	5.42E-03	9.02	0.188
	20.00	68.00	5.887E+03	8.538E-01	7.580E-01	2.487E+00	791.9	49.44	1143	23.87	22.40	1.535	1.12E-01	7.01E-03	9.15	0.191
	40.00	104.00	1.789E+04	2.594E+00	2.359E+00	7.740E+00	773.1	48.26	793	16.57	20.75	1.422	3.21E-01	2.01E-02	9.73	0.203
	60.00	140.00	4.671E+04	6.775E+00	6.322E+00	2.074E+01	753.4	47.03	569	11.88	19.06	1.306	7.95E-01	4.97E-02	10.30	0.215
n p b	78.29	172.92	1.013E+05	1.470E+01	1.407E+01	4.618E+01	734.3	45.84	430	8.97	17.49	1.199	1.65E+00	1.03E-01	10.82	0.226
	80.00	176.00	1.077E+05	1.562E+01	1.500E+01	4.920E+01	732.5	45.73	419	8.75	17.34	1.188	1.74E+00	1.09E-01	10.87	0.227
	100.00	212.00	2.240E+05	3.248E+01	3.216E+01	1.055E+02	710.2	44.33	315	6.57	15.59	1.068	3.51E+00	2.19E-01	11.45	0.239
	120.00	248.00	4.270E+05	6.193E+01	6.345E+01	2.082E+02	686.1	42.83	240	5.02	13.78	0.944	6.58E+00	4.11E-01	12.05	0.252
	140.00	284.00	7.563E+05	1.097E+02	1.169E+02	3.834E+02	659.9	41.20	186	3.88	11.92	0.817	1.15E+01	7.19E-01	12.67	0.265
	160.00	320.00	1.258E+06	1.825E+02	2.035E+02	6.675E+02	630.7	39.37	145	3.03	10.00	0.685	1.92E+01	1.20E+00	13.34	0.279
	180.00	356.00	1.984E+06	2.876E+02	3.388E+02	1.111E+03	597.2	37.28	114	2.38	7.96	0.547	3.12E+01	1.94E+00	14.13	0.295
	200.00	392.00	2.986E+06	4.331E+02	5.468E+02	1.794E+03	556.9	34.77	90	1.88	5.85	0.401	5.08E+01	3.17E+00	15.16	0.317
	220.00	428.00	4.317E+06	6.261E+02	8.749E+02	2.870E+03	503.1	31.41	71	1.48	3.51	0.241	8.54E+01	5.33E+00	16.87	0.352
	240.00	464.00	6.024E+06	8.737E+02	1.554E+03	5.100E+03	395.1	24.67	56	1.18	0.68	0.047	1.72E+02	1.07E+01	22.16	0.463
c p	243.10	469.58	6.384E+06	9.258E+02	2.358E+03	7.738E+03	276.0	17.23	54	1.14	0.00	0.000	2.76E+02	1.72E+01	54.39	1.136

t p = triple point n b p = normal boiling point c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - h_v		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Ethyl Chloride (R 160) C_2H_5Cl					**M = 64.514**				**Zc = 0.275**						
t p	-138.35	-217.03	1.166E-01	1.691E-05	1.118E-05	3.668E-05	1063.3	66.38	4847	101.22	36.46	2.498	6.71E-06	4.19E-07	4.82	0.101
	-120.00	-184.00	3.296E+00	4.780E-04	3.211E-04	1.054E-03	1046.5	65.33	2542	53.09	34.42	2.358	1.67E-04	1.04E-05	5.42	0.113
	-100.00	-148.00	5.134E+01	7.446E-03	5.094E-03	1.671E-02	1027.6	64.15	1470	30.69	32.20	2.206	2.30E-03	1.44E-04	6.07	0.127
	-80.00	-112.00	4.221E+02	6.123E-02	4.270E-02	1.401E-01	1008.1	62.93	951	19.87	29.97	2.054	1.70E-02	1.06E-03	6.72	0.140
	-60.00	-76.00	2.217E+03	3.215E-01	2.288E-01	7.508E-01	987.8	61.67	668	13.95	27.74	1.901	8.09E-02	5.05E-03	7.37	0.154
	-40.00	-40.00	8.412E+03	1.220E+00	8.873E-01	2.911E+00	969.7	60.35	498	10.41	25.52	1.748	2.82E-01	1.76E-02	8.02	0.167
	-20.00	-4.00	2.508E+04	3.637E+00	2.707E+00	8.883E+00	944.5	58.96	389	8.13	23.29	1.596	7.80E-01	4.87E-02	8.67	0.181
	0.00	32.00	6.227E+04	9.032E+00	6.893E+00	2.261E+01	921.3	57.51	315	6.58	21.06	1.443	1.82E+00	1.13E-01	9.33	0.195
n b p	12.27	54.09	1.013E+05	1.470E+01	1.140E+01	3.740E+01	906.3	56.58	281	5.87	19.69	1.349	2.86E+00	1.79E-01	9.73	0.203
	15.56	60.00	1.144E+05	1.659E+01	1.293E+01	4.242E+01	902.2	56.33	273	5.70	19.32	1.324	3.20E+00	2.00E-01	9.84	0.206
	20.00	68.00	1.343E+05	1.948E+01	1.527E+01	5.011E+01	896.6	55.96	263	5.48	18.82	1.290	3.72E+00	2.32E-01	9.99	0.209
	40.00	104.00	2.595E+05	3.764E+01	3.040E+01	9.974E+01	870.4	54.34	224	4.96	16.59	1.137	6.99E+00	4.30E-01	10.68	0.223
	60.00	140.00	4.596E+05	6.666E+01	5.565E+01	1.826E+02	842.2	52.57	195	4.06	14.35	0.983	1.18E+01	7.39E-01	11.39	0.238
	80.00	176.00	7.595E+05	1.102E+02	9.545E+01	3.132E+02	811.4	50.65	172	3.59	12.11	0.830	1.91E+01	1.19E+00	12.15	0.254
	100.00	212.00	1.187E+06	1.722E+02	1.558E+02	5.111E+02	777.2	48.52	154	3.21	9.87	0.676	2.96E+01	1.84E+00	12.97	0.271
	120.00	248.00	1.775E+06	2.574E+02	2.452E+02	8.044E+02	738.2	46.08	139	2.90	7.82	0.522	4.40E+01	2.75E+00	13.91	0.290
	140.00	284.00	2.559E+06	3.712E+02	3.772E+02	1.238E+03	691.8	43.18	127	2.65	5.37	0.368	6.37E+01	3.98E+00	15.03	0.314
	160.00	320.00	3.583E+06	5.197E+02	5.787E+02	1.899E+03	631.4	39.41	117	2.44	3.11	0.213	9.90E+01	6.18E+00	16.76	0.350
	180.00	356.00	4.900E+06	7.107E+02	9.454E+02	3.102E+03	528.5	32.99	109	2.27	0.83	0.057	1.78E+01	1.11E+00	15.31	0.320
c p	187.20	368.96	5.269E+06	7.642E+02	1.666E+03	5.465E+03	322.6	20.14	33	0.69	0.00	0.000	3.23E+02	2.01E+01	32.80	0.685
	Ethylene (R 1150) C_2H_4					**M = 28.054**				**Zc = 0.277**						
t p	-169.18	-272.52	1.397E+02	2.026E-02	2.185E-02	7.169E-02	651.8	40.69	693	14.47	29.43	2.017	4.53E-03	2.83E-04	2.21	0.046
	-160.00	-256.00	5.895E+02	8.560E-02	9.377E-02	3.076E-01	641.0	40.02	462	9.65	27.51	1.885	1.76E-02	1.10E-03	2.70	0.056
	-140.00	-220.00	6.131E+03	8.892E-01	1.014E+00	3.327E+00	616.6	38.49	258	5.38	23.42	1.605	1.56E-01	9.74E-03	3.90	0.079
	-120.00	-184.00	3.466E+04	5.027E+00	5.985E+00	1.964E+01	590.6	36.87	182	3.79	19.49	1.335	7.76E-01	4.84E-02	4.90	0.102
n b p	-103.69	-154.62	1.013E+05	1.470E+01	1.819E+01	5.969E+01	568.0	35.46	148	3.09	16.40	1.124	2.08E+00	1.30E-01	5.76	0.120
	-100.00	-148.00	1.256E+05	1.822E+01	2.277E+01	7.472E+01	582.5	35.12	142	2.96	15.72	1.077	2.55E+00	1.59E-01	5.96	0.124
	-80.00	-112.00	3.401E+05	4.933E+01	6.517E+01	2.138E+02	532.1	33.22	114	2.38	12.14	0.832	6.44E+00	4.02E-01	6.96	0.145
	-60.00	-76.00	7.541E+05	1.094E+02	1.541E+02	5.057E+02	499.8	31.14	90	1.98	8.77	0.601	1.39E+01	8.61E-01	7.93	0.166
	-40.00	-40.00	1.453E+06	2.107E+02	3.214E+02	1.054E+03	460.8	28.77	67	1.40	5.67	0.389	2.67E+01	1.67E+00	8.56	0.179
	-20.00	-4.00	2.529E+06	3.668E+02	6.238E+02	2.046E+03	413.5	25.81	47	0.98	2.91	0.200	4.96E+01	3.10E+00	10.09	0.211
	0.00	32.00	4.099E+06	5.945E+02	1.225E+03	4.019E+03	341.2	21.30	30	0.62	0.67	0.046	9.82E+01	6.13E+00	13.77	0.288
c p	9.21	48.58	5.040E+06	7.310E+02	2.400E+03	7.873E+03	214.2	13.37	22	0.46	0.00	0.000	2.14E+02	1.34E+01	21.80	0.457
	Helium (R 704) He					**M = 4.003**				**Zc = 0.302**						
t p	-270.97	-455.75	5.039E+03	7.309E-01	3.514E+00	1.153E+01	148.2	9.13	4	0.08	0.30	0.020	1.18E+00	7.35E-02	0.59	0.012
n b p	-268.93	-452.07	1.013E+05	1.470E+01	8.267E+01	2.712E+02	125.0	7.80	3	0.07	0.10	0.007	1.59E+01	1.06E+00	1.23	0.026
c p	-267.96	-450.31	2.275E+05	3.299E+01	3.331E+02	1.093E+03	69.6	4.35	2	0.05	0.00	0.000	6.94E+01	4.35E+00	2.43	0.051

t p = triple point, n b p = normal boiling point, c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_f		Liquid Viscosity - μ_f		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m^3	lbm/ft^3	Pa sec ×10^6	lbf sec/ft^2 × 10^8	N/m ×10^3	lbf/ft × 10^3	kg/m^3	lbm/ft^3	Pa sec ×10^6	lbf sec/ft^2 × 10^8
		Heptane-n C_7H_{16}			M = 100.203				Zc = 0.263							
t.p.	-90 59	-131.06	1 152E-01	1.671E-05	1.519E-05	4 984E-05	773 1	48 26	3850	80.41	32 13	2 201	7 60E-06	4 75E-07	3 58	0.075
	-80 00	-112.00	6.264E-01	9.085E-05	8.346E-05	2 738E-04	765 3	47.78	2570	53.68	30 95	2.121	3 91E-05	2.44E-06	3.81	0.080
	-80 00	-76 00	8 796E+00	1.276E-03	1.197E-03	3 926E-03	749.5	46 79	1460	30 49	28 75	1 970	4 97E-04	3.11E-05	4 24	0.089
	-40.00	-40.00	7.107E+01	1.031E-02	9.879E-03	3.241E-02	733 5	45.79	965	20.15	26 58	1 821	3 67E-03	2.29E-04	4.67	0.098
	-20 00	-4.00	3.823E+02	5.544E-02	5.435E-02	1.783E-01	717.2	44.77	689	14.39	24 45	1 675	1 82E-02	1.14E-03	5.10	0.107
	0.00	32.00	1.515E+03	2.197E-01	2.205E-01	7 235E-01	700 5	43.73	526	10 99	22 35	1 531	6.70E-02	4.18E-03	5.53	0.115
	15 56	60 00	3 751E+03	5.440E-01	5.564E-01	1.826E+00	687 4	42 91	433	9.05	20 74	1 421	1 57E-01	9.83E-03	5.86	0.122
	20 00	68.00	4.753E+03	6.894E-01	7.090E-01	2 326E+00	683 6	42 68	414	8.65	20.29	1 390	1.97E-01	1 23E-02	5.95	0.124
	40.00	104.00	1.244E+04	1.804E+00	1.903E+00	6.245E+00	666.5	41 61	338	7 08	18.27	1.252	4.85E-01	3.03E-02	6.38	0.133
	60 00	140 00	2.824E+04	4 096E+00	4 436E+00	1 456E+01	649 1	40 52	281	5 87	16.29	1 116	1.04E+00	6 52E-02	6 81	0.142
	80 00	176.00	5.726E+04	8 305E+00	9 251E+00	3 035E+01	631.1	39.40	239	4 99	14 38	0.984	2.02E+00	1 26E-01	7.23	0 151
n.b.p.	98.43	209 17	1.013E+05	1.470E+01	1 684E+01	5.527E+01	613 4	38.29	207	4.33	12 62	0 865	3.46E+00	2.16E-01	7.63	0.159
	100.00	212.00	1.060E+05	1 538E+01	1.766E+01	5.793E+01	612.4	38.23	198	4 14	12 47	0 855	3 80E+00	2.25E-01	7 36	0.154
	120.00	248.00	1.826E+05	2.648E+01	3.141E+01	1.031E+02	592.6	36.99	167	3.49	10.65	0.729	6.08E+00	3.79E-01	7.82	0.163
	140.00	284.00	2.962E+05	4 297E+01	5.289E+01	1.735E+02	571 1	35 65	143	2 98	8 68	0 606	9.79E+00	6.11E-01	8 34	0.174
	160.00	320.00	4.580E+05	6.642E+01	8.520E+01	2.795E+02	548 1	34.22	122	2 54	7 18	0.492	1.51E+01	9.43E-01	8.97	0.187
	180 00	356.00	6 805E+05	9 870E+01	1 326E+02	4 351E+02	523 2	32 66	103	2.15	5 55	0 390	2.24E+01	1 40E+00	9.70	0.203
	200.00	392.00	9 788E+05	1.420E+02	2.016E+02	6 613E+02	495 2	30 91	87	1 81	4 01	0 275	3.30E+01	2 06E+00	10.60	0.221
	220.00	428 00	1.371E+06	1.988E+02	3 028E+02	9.934E+02	461 6	28.82	72	1.51	2.58	0.177	4.89E+01	3 05E+00	11.90	0.249
	240.00	464.00	1.878E+06	2.723E+02	4.584E+02	1.504E+03	417.7	26.08	58	1.21	1 30	0 089	7 45E+01	4 65E+00	13.60	0 288
	260 00	500 00	2 526E+06	3 663E+02	7 450E+02	2 444E+03	345 7	21 58	45	0 93	0 25	0 017	1 29E+02	8 03E+00	17.90	0.374
	267 11	512 80	2.736E+06	3 968E+02	1 203E+03	3.948E+03	231 8	14.47	27	0.57	0 00	0 000	2.32E+02	1 45E+01	27.10	0.566
		Hexane-n C_6H_{14}			M = 86.177				Zc = 0.264							
t p	-95 31	-139.58	8 039E-01	1.166E-04	1.084E-04	3 558E-04	756.0	47 19	2084	43 52	32 22	2.208	4 69E-05	2.92E-06	3 69	0 077
	-80 00	-112 00	7 246E+00	1 051E-03	9 924E-04	3 256E-03	744 5	45.48	1380	28 82	30 31	2 077	3 89E-04	2.43E-05	4.03	0 084
	-60 00	-76 00	7 158E+01	1 038E-02	1 002E-02	3 269E-02	726 2	45 46	892	18 63	27 86	1 909	3 48E-03	2 17E-04	4 47	0 093
	-40 00	-40.00	4.355E+02	6.317E-02	6 242E-02	2.048E-01	711.5	44.42	633	13 22	25.45	1.744	1.94E-02	1.21E-03	4 91	0 103
	-20 00	-4 00	1.859E+03	2.697E-01	2.729E-01	8.955E-01	694.6	43 36	479	10 00	23 09	1.582	7 63E-02	4.77E-03	5 35	0 112
	0 00	32.00	6 091E+03	8.634E-01	9.171E-01	3.009E+00	677 2	42.28	381	7 96	20 79	1.424	2 33E-01	1.45E-02	5.79	0 121
	15 56	60 00	1 330E+04	1 930E+00	2 044E+00	6.707E+00	663 6	41.43	320	6 69	19 03	1.304	4.64E-01	3.02E-02	6 13	0 128
	20.00	68.00	1.632E+04	2 367E+00	2 523E+00	8 279E+00	659 4	41 17	307	6 41	18 53	1 270	5 65E-01	[illegible]	6 23	0 130
	40 00	104 00	3.745E+04	5 431E+00	5 958E+00	1.955E+01	640.9	40.01	253	5 28	16 34	1.120	1 27E+00	7 94E-02	6 67	0 139
	60.00	140.00	7.617E+04	1 105E+01	1 249E+01	4 098E+01	621 6	38 82	216	4 51	14 21	0 974	2 47E+00	1 54E-01	7 11	0 146
n b p	68 73	155 71	1 013E+05	1 470E+01	1 684E+01	5.525E+01	613.5	38 30	204	4 26	13 31	0 912	3 24E+00	2 02E-01	7 30	0 152
	80 00	176 00	1.409E+05	2 043E+01	2 385E+01	7 827E+01	602 2	37 59	184	3 83	12 15	0 833	4 42E+00	2 76E-01	7 54	0 157
	100.00	212 00	2 417E+05	3 505E+01	4 238E+01	1.391E+02	581 4	36 30	154	3 21	10 17	0 697	7 41E+00	4 63E-01	8 02	0.168
	120 00	248 00	3 903E+05	5 661E+01	7.123E+01	2.337E+02	558 8	34 88	136	2 84	8 27	0 566	1.18E+01	7 39E-01	8 61	0.180
	140 00	284.00	6 008E+05	8 714E+01	1 147E+02	3 762E+02	534 3	33 36	115	2 41	6 46	0 442	1.82E+01	1 14E+00	9 32	0 195
	160 00	320 00	8 894E+05	1 290E+02	1 791E+02	5 877E+02	506 3	31 61	97	2 03	4 75	0 326	2 75E+01	1.72E+00	10.22	0 213
	180.00	356 00	1 276E+06	1 851E+02	2 739E+02	8 987E+02	475 1	29 66	81	1 69	3 18	0 218	4 10E+01	2 56E+00	11 38	0 238
	200 00	392 00	1.785E+06	2 589E+02	4 171E+02	1.368E+03	436 5	27 25	66	1 37	1 76	0 121	6.18E+01	3.86E+00	13.06	0 273
	220.00	428 00	2 447E+06	3 549E+02	6 549E+02	2 149E+03	381 0	23 79	51	1 07	0 57	0.039	9.94E+01	6 17E+00	15.86	0.331
c p	234 28	453 70	3 012E+06	4 369E+02	1 319E+03	4 328E+03	232 8	14 54	27	0 57	0 00	0 000	2 33E+02	1 45E+01	27.20	0 568

t p = triple point, n b p = normal boiling point, c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁵	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Hydrogen-n (R 702) H_2				**M = 2.016**				**Zc = 0.306**							
tp	-259.20	-434.56	7.042E+03	1.021E+00	9.337E+00	3.063E+01	76.9	4.80	27	0.55	2.99	0.205	1.24E-01	7.76E-03	0.65	0.014
nbp	-252.77	-422.99	1.013E+05	1.470E+01	1.462E+02	4.797E+02	70.7	4.41	13	0.28	1.93	0.132	1.33E+00	8.30E-02	1.12	0.023
cp	-239.96	-399.93	1.316E+06	1.908E+02	4.454E+03	1.461E+04	30.1	1.88	4	0.08	0.00	0.000	3.01E+01	1.88E+00	3.64	0.076
	Hydrogen Chloride HCl				**M = 36.461**				**Zc = 0.249**							
tp	-114.18	-173.52	1.350E+04	1.958E+00	1.082E+00	3.551E+00	1272.0	79.41	706	14.75	35.61	2.440	3.75E-01	2.34E-02	7.64	0.164
	-100.00	-148.00	3.980E+04	5.772E+00	3.286E+00	1.078E+01	1234.9	77.09	534	11.15	31.71	2.172	1.02E+00	6.39E-02	8.58	0.179
nbp	-85.00	-121.00	1.013E+05	1.470E+01	8.652E+00	2.839E+01	1194.2	74.55	407	8.49	27.69	1.898	2.50E+00	1.56E-01	9.39	0.196
	-80.00	-112.00	1.339E+05	1.942E+01	1.157E+01	3.796E+01	1180.3	73.68	373	7.79	26.39	1.808	3.20E+00	2.00E-01	9.66	0.202
	-60.00	-76.00	3.486E+05	5.056E+01	3.167E+01	1.039E+02	1122.4	70.07	267	5.58	21.30	1.460	8.30E+00	5.18E-01	10.75	0.225
	-40.00	-40.00	7.595E+05	1.102E+02	7.305E+01	2.397E+02	1060.2	66.18	194	4.04	16.49	1.130	1.70E+01	1.06E+00	11.87	0.248
	-20.00	-4.00	1.459E+06	2.116E+02	1.500E+02	4.920E+02	992.0	61.93	141	2.94	11.97	0.820	3.20E+01	2.00E+00	13.06	0.273
	0.00	32.00	2.559E+06	3.712E+02	2.853E+02	9.359E+02	914.8	57.11	102	2.13	7.82	0.536	5.40E+01	3.37E+00	14.42	0.301
	15.56	60.00	3.782E+06	5.485E+02	4.566E+02	1.498E+03	844.8	52.72	79	1.65	4.91	0.336	8.61E+01	5.37E+00	15.78	0.330
	20.00	68.00	4.202E+06	6.095E+02	5.213E+02	1.711E+03	821.9	51.31	73	1.53	4.13	0.283	9.70E+01	6.06E+00	16.24	0.339
	40.00	104.00	6.573E+06	9.533E+02	9.714E+02	3.187E+03	689.9	43.07	52	1.09	1.12	0.077	1.80E+02	1.12E+01	19.55	0.408
cp	51.50	124.70	8.309E+06	1.205E+03	1.883E+03	6.177E+03	450.0	28.09	43	0.89	0.00	0.000	4.50E+02	2.81E+01	42.64	0.691
	Mercury Hg				**M = 200.61**				**Zc = 0.378**							
cp	-38.93	-37.89	2.996E-04	4.345E-08	2.231E-08	7.321E-08	13891.8	854.75	2055	42.92	489.69	33.564	3.08E-08	1.93E-09	28.84	0.602
	-20.00	-4.00	3.196E-03	4.636E-07	2.390E-08	7.841E-08	13645.0	851.93	1847	38.58	485.10	33.240	3.06E-07	1.90E-08	30.55	0.638
	0.00	32.00	2.728E-02	3.957E-06	2.046E-07	6.713E-07	13595.5	848.74	1687	35.23	480.20	32.904	2.41E-06	1.50E-07	32.39	0.676
	15.56	60.00	1.175E-01	1.704E-05	8.838E-07	2.900E-06	13557.2	846.35	1664	34.75	476.39	32.643	9.82E-06	6.13E-07	33.82	0.706
	20.00	68.00	1.729E-01	2.508E-05	1.301E-06	4.270E-06	13546.3	845.67	1556	32.50	475.30	32.568	1.42E-05	8.89E-07	34.23	0.715
	40.00	104.00	8.626E-01	1.251E-04	6.517E-06	2.138E-05	13497.3	842.61	1457	30.43	470.37	32.231	6.84E-05	4.16E-06	36.09	0.754
	60.00	140.00	3.536E+00	5.129E-04	2.681E-05	8.796E-05	13448.6	839.57	1372	28.65	465.43	31.892	2.56E-04	1.60E-05	37.96	0.793
	80.00	176.00	1.232E+01	1.787E-03	9.375E-05	3.076E-04	13400.1	836.54	1301	27.17	460.48	31.553	8.42E-04	5.25E-05	39.85	0.832
	100.00	212.00	3.745E+01	5.432E-03	2.860E-04	9.384E-04	13351.8	833.53	1241	25.92	455.51	31.212	2.42E-03	1.51E-04	41.75	0.872
	120.00	248.00	1.015E+02	1.472E-02	7.780E-04	2.552E-03	13303.7	830.52	1189	24.83	450.52	30.871	6.18E-03	3.86E-04	43.66	0.912
	140.00	284.00	2.491E+02	3.613E-02	1.916E-03	6.287E-03	13255.8	827.53	1144	23.89	445.52	30.528	1.45E-02	9.08E-04	45.58	0.952
	160.00	320.00	5.616E+02	8.146E-02	4.337E-03	1.423E-02	13208.0	824.55	1104	23.06	440.50	30.184	3.13E-02	1.95E-03	47.51	0.992
	180.00	356.00	1.178E+03	1.709E-01	9.127E-03	2.995E-02	13160.3	821.57	1070	22.35	435.46	29.839	6.27E-02	3.91E-03	49.45	1.033
	200.00	392.00	2.315E+03	3.358E-01	1.800E-02	5.906E-02	13112.7	818.60	1039	21.70	430.41	29.492	1.18E-01	7.37E-03	51.40	1.074
	220.00	428.00	4.304E+03	6.242E-01	3.359E-02	1.102E-01	13065.1	815.63	1012	21.13	425.33	29.145	2.11E-01	1.31E-02	53.38	1.114
	240.00	464.00	7.614E+03	1.104E+00	5.964E-02	1.957E-01	13017.8	812.96	987	20.61	420.24	28.796	3.58E-01	2.24E-02	55.33	1.156
	260.00	500.00	1.289E+04	1.870E+00	1.014E-01	3.325E-01	12970.0	809.69	964	20.14	415.13	28.445	5.64E-01	3.64E-02	57.30	1.197
	280.00	536.00	2.099E+04	3.045E+00	1.657E-01	5.435E-01	12922.4	806.72	944	19.72	410.00	28.094	9.16E-01	5.72E-02	59.29	1.238
	300.00	572.00	3.302E+04	4.788E+00	2.615E-01	8.579E-01	12874.7	803.74	926	19.34	404.85	27.741	1.39E+00	8.69E-02	61.28	1.280
nbp	356.95	674.51	1.013E+05	1.470E+01	8.111E-01	2.661E+00	12737.8	795.19	881	18.41	390.07	26.729	3.89E+00	2.43E-01	66.98	1.399
	400.00	752.00	2.102E+05	3.049E+01	1.687E+00	5.558E+00	12632.8	788.84	853	17.82	378.79	25.956	7.57E+00	4.73E-01	71.32	1.490
	600.00	1112.00	2.346E+06	3.403E+02	1.972E+01	6.470E+01	12129.9	757.25	767	16.02	324.93	22.265	6.67E+01	4.16E+00	91.92	1.920
	800.00	1472.00	1.069E+07	1.550E+03	9.405E+01	3.086E+02	11584.0	723.16	717	14.97	268.15	18.374	2.59E+02	1.62E+01	113.61	2.373
cp	1491.85	2717.33	1.510E+08	2.190E+04	2.799E+03	9.185E+03	5500.0	343.35	396	8.28	0.00	0.000	5.50E+03	3.43E+02	396.46	8.280

t.p. = triple point, n.b.p. = normal boiling point, c.p. = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - h_v = P		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	C	F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁵	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10⁴	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Methane (R 50) CH_4				M = 16.043				Zc = 0.288							
t p	-182.48	-296.46	1.172E+04	1.700E+00	2.648E+00	8.689E+00	451.2	28.17	204	4.27	17.63	1.208	2.51E-01	1.57E-02	3.74	0.078
	-180.00	-292.00	1.596E+04	2.315E+00	3.633E+00	1.192E+01	448.0	27.97	186	3.89	17.15	1.175	3.34E-01	2.08E-02	3.84	0.080
n b p	-161.49	-258.68	1.013E+05	1.470E+01	2.446E+01	8.023E+01	422.5	26.38	114	2.39	13.63	0.934	1.82E+00	1.13E-01	4.48	0.094
	-160.00	-256.00	1.146E+05	1.662E+01	2.781E+01	9.123E+01	420.3	26.24	111	2.32	13.34	0.914	2.04E+00	1.27E-01	4.53	0.095
	-140.00	-220.00	4.424E+05	6.417E+01	1.160E+02	3.806E+02	389.0	24.29	81	1.69	9.63	0.660	7.13E+00	4.45E-01	5.24	0.110
	-120.00	-184.00	1.196E+06	1.735E+02	3.467E+02	1.138E+03	351.8	21.96	60	1.25	6.03	0.413	1.88E+01	1.17E+00	6.29	0.131
	-100.00	-148.00	2.602E+06	3.774E+02	8.819E+02	2.893E+03	300.8	18.78	41	0.86	2.62	0.179	4.48E+01	2.80E+00	8.08	0.168
c p	-82.57	-116.63	4.595E+06	6.664E+02	2.890E+03	9.482E+03	182.1	10.12	16	0.34	0.00	0.000	1.62E+02	1.01E+01	16.45	0.344
	Methanol CH_4O				M = 32.042				Zc = 0.224							
t p	-97.56	-143.61	1.077E-01	1.562E-05	1.245E-05	4.085E-05	881.8	55.05	14310	298.87	31.36	2.149	2.38E-06	1.48E-07	5.67	0.118
	-80.00	-112.00	1.777E+00	2.577E-04	2.083E-04	6.835E-04	869.7	54.29	5770	120.50	30.10	2.062	3.55E-05	2.21E-06	6.29	0.131
	-60.00	-76.00	2.291E+01	3.323E-03	2.731E-03	8.962E-03	855.4	53.40	3253	67.64	28.64	1.962	4.15E-04	2.59E-05	7.00	0.146
	-40.00	-40.00	1.794E+02	2.602E-02	2.176E-02	7.141E-02	840.5	52.47	1893	39.53	27.16	1.861	2.87E-03	1.88E-04	7.70	0.161
	-20.00	-4.00	9.685E+02	1.405E-01	1.197E-01	3.927E-01	825.0	51.50	1118	23.34	25.66	1.758	1.48E-02	9.25E-04	8.41	0.176
	0.00	32.00	3.939E+03	5.713E-01	4.966E-01	1.629E+00	808.8	50.49	748	15.85	24.12	1.653	5.60E-02	3.50E-03	9.11	0.190
	15.56	60.00	1.005E+04	1.458E+00	1.288E+00	4.226E+00	796.7	49.67	590	12.31	22.91	1.570	1.36E-01	8.47E-03	9.68	0.202
	20.00	68.00	1.286E+04	1.865E+00	1.656E+00	5.434E+00	791.8	49.43	555	11.60	22.56	1.546	1.71E-01	1.07E-02	9.81	0.205
	40.00	104.00	3.532E+04	5.122E+00	4.653E+00	1.527E+01	773.9	48.31	439	9.16	20.98	1.437	4.43E-01	2.77E-02	10.51	0.220
	60.00	140.00	8.446E+04	1.225E+01	1.141E+01	3.743E+01	754.8	47.12	359	7.50	19.33	1.325	1.01E+00	6.29E-02	11.21	0.234
n b p	64.70	148.46	1.013E+05	1.470E+01	1.377E+01	4.518E+01	750.2	46.83	344	7.18	18.94	1.298	1.22E+00	7.63E-02	11.38	0.238
	80.00	176.00	1.806E+05	2.620E+01	2.508E+01	8.227E+01	734.5	45.68	299	6.24	17.65	1.210	2.08E+00	1.30E-01	11.92	0.249
	100.00	212.00	3.526E+05	5.115E+01	5.046E+01	1.656E+02	712.6	44.49	248	5.17	15.83	1.081	3.85E+00	2.40E-01	12.64	0.264
	120.00	248.00	6.386E+05	9.262E+01	9.455E+01	3.102E+02	688.7	42.99	202	4.22	14.14	0.969	7.14E+00	4.46E-01	13.37	0.279
	140.00	284.00	1.087E+06	1.576E+02	1.673E+02	5.490E+02	662.2	41.34	161	3.36	12.29	0.842	1.22E+01	7.59E-01	14.13	0.295
	160.00	320.00	1.755E+06	2.545E+02	2.831E+02	9.287E+02	632.1	39.46	123	2.56	10.34	0.709	1.99E+01	1.24E+00	14.95	0.312
	180.00	356.00	2.712E+06	3.934E+02	4.634E+02	1.520E+03	596.9	37.26	91	1.90	8.28	0.567	3.19E+01	1.99E+00	15.88	0.332
	200.00	392.00	4.040E+06	5.860E+02	7.449E+02	2.444E+03	553.0	34.52	64	1.33	6.04	0.414	5.08E+01	3.17E+00	17.04	0.356
	220.00	428.00	5.831E+06	8.457E+02	1.212E+03	3.975E+03	490.7	30.63	42	0.88	3.51	0.240	8.63E+01	5.39E+00	18.93	0.395
c p	239.43	462.97	8.096E+06	1.174E+03	3.033E+03	9.950E+03	272.2	16.99	27	0.56	0.00	0.000	2.72E+02	1.70E+01	26.88	0.561
	Methyl Chloride (R 40) CH_3Cl				M = 50.488				Zc = [illegible]							
t p	-97.72	-143.90	8.739E+02	1.268E-01	7.895E-02	2.590E-01	1128.8	70.47	571	11.92	35.64	2.442	3.03E-02	1.89E-03	6.70	0.140
	-80.00	-112.00	4.029E+03	5.842E-01	3.731E-01	1.224E+00	1101.0	68.73	476	9.94	32.52	2.228	1.27E-01	7.94E-03	7.36	0.154
	-60.00	-76.00	1.600E+04	2.320E+00	1.526E+00	5.008E+00	1068.8	66.71	402	8.40	29.07	1.992	4.60E-01	2.87E-02	8.04	0.168
	-40.00	-40.00	4.858E+04	7.046E+00	4.786E+00	1.570E+01	1034.9	64.61	350	7.30	25.67	1.759	1.26E+00	7.89E-02	8.72	0.182
n b p	-24.22	-11.80	1.013E+05	1.470E+01	1.026E+01	3.365E+01	1007.3	62.88	318	6.64	23.05	1.579	2.55E+00	1.59E-01	9.25	0.193
	-20.00	-4.00	1.210E+05	1.754E+01	1.234E+01	4.048E+01	999.7	62.41	311	6.49	22.36	1.532	2.96E+00	1.85E-01	9.40	0.199
	0.00	32.00	2.593E+05	3.761E+01	2.747E+01	9.012E+01	962.6	60.09	281	5.87	19.12	1.310	5.99E+00	3.74E-01	10.10	0.211
	15.56	60.00	4.328E+05	6.277E+01	4.734E+01	1.553E+02	932.2	58.20	262	5.48	16.66	1.141	9.86E+00	6.16E-01	10.67	0.223
	20.00	68.00	4.956E+05	7.188E+01	5.474E+01	1.796E+02	923.2	57.64	258	5.38	15.97	1.094	1.10E+01	6.87E-01	10.84	0.228
	40.00	104.00	8.664E+05	1.257E+02	1.003E+02	3.291E+02	880.9	54.99	239	4.99	12.91	0.885	1.89E+01	1.18E+00	11.66	0.243

Continued on next page

t p = triple point n b p = normal boiling point c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10^6	lbf sec/ft² × 10^6	N/m × 10^3	lbf/ft × 10^3	kg/m³	lbm/ft³	Pa sec × 10^6	lbf sec/ft² × 10^6
	Methyl Chloride (R 40) CH_3Cl (continued)															
	60.00	140.00	1.413E+06	2.049E+02	1.726E+02	5.663E+02	834.6	52.10	224	4.68	9.99	0.684	3.20E+01	2.00E+00	12.56	0.262
	80.00	176.00	2.180E+06	3.161E+02	2.840E+02	9.317E+02	782.7	48.86	197	4.11	7.18	0.492	4.90E+01	3.06E+00	13.56	0.284
	100.00	212.00	3.220E+06	4.670E+02	4.548E+02	1.492E+03	721.8	45.06	175	3.65	4.55	0.312	7.50E+01	4.68E+00	14.82	0.310
	120.00	248.00	4.583E+06	6.682E+02	7.278E+02	2.388E+03	643.5	40.17	152	3.17	2.16	0.149	1.20E+02	7.49E+00	16.82	0.347
	140.00	284.00	6.374E+06	9.244E+02	1.306E+03	4.283E+03	497.9	31.08	67	1.40	0.19	0.013	2.38E+02	1.49E+01	19.82	0.410
c.p.	143.10	289.58	6.679E+06	9.688E+02	1.875E+03	6.152E+03	363.2	22.68	28	0.58	0.00	0.000	3.63E+02	2.27E+01	27.80	0.581
	Neon (R 720) Ne				**M = 20.179**				**Zc = 0.300**							
t.p.	-248.59	-415.46	4.338E+04	6.292E+00	3.540E+00	1.162E+01	1249.3	77.99	151	3.16	5.67	0.389	4.41E+00	2.75E-01	4.60	0.096
n.b.p.	-246.06	-410.89	1.013E+05	1.470E+01	8.558E+00	2.808E+01	1207.3	75.37	125	2.60	4.77	0.327	9.56E+00	5.97E-01	4.65	0.097
c.p.	-228.70	-379.66	2.664E+06	3.864E+02	5.625E+02	1.846E+03	483.0	30.15	17	0.35	0.00	0.000	4.83E+02	3.02E+01	16.70	0.349
	Nitrogen (R 728) N_2				**M = 28.013**				**Zc = 0.292**							
t.p.	-210.00	-346.00	1.253E+04	1.817E+00	1.472E+00	4.831E+00	867.8	54.17	303	6.33	12.21	0.837	6.75E-01	4.21E-02	4.29	0.089
	-200.00	-328.00	5.894E+04	8.683E+00	7.387E+00	2.424E+01	827.4	51.65	183	3.82	9.65	0.675	2.87E+00	1.79E-01	5.08	0.106
n.b.p.	-195.80	-320.44	1.013E+05	1.470E+01	1.278E+01	4.192E+01	808.6	50.48	157	3.27	8.88	0.608	4.61E+00	2.88E-01	5.38	0.112
	-180.00	-292.00	4.670E+05	6.773E+01	6.534E+01	2.144E+02	728.8	45.50	104	2.18	5.46	0.374	1.99E+01	1.24E+00	6.61	0.138
	-160.00	-256.00	1.754E+06	2.544E+02	3.009E+02	9.872E+02	594.5	37.11	83	1.32	1.72	0.118	8.15E+01	5.09E+00	8.94	0.187
c.p.	-146.95	-232.51	3.400E+06	4.931E+02	1.104E+03	3.623E+03	314.0	19.60	19	0.40	0.00	0.000	3.14E+02	1.96E+01	19.10	0.399
	Nitrous Oxide (R 744A) N_2O				**M = 44.013**				**Zc = 0.274**							
t.p.	-90.85	-131.53	8.785E+04	1.274E+01	7.034E+00	2.308E+01	1273.5	79.50	357	7.47	24.43	1.674	2.73E+00	1.70E-01	9.31	0.194
n.b.p.	-88.48	-127.26	1.013E+05	1.470E+01	8.161E+00	2.677E+01	1266.1	79.04	344	7.18	23.68	1.636	3.11E+00	1.94E-01	9.44	0.197
	-80.00	-112.00	1.669E+05	2.421E+01	1.374E+01	4.508E+01	1238.9	77.34	299	6.25	21.94	1.503	4.83E+00	3.08E-01	9.91	0.207
	-60.00	-76.00	4.490E+05	6.512E+01	3.909E+01	1.283E+02	1171.1	73.11	215	4.48	17.48	1.198	1.23E+01	7.67E-01	11.03	0.230
	-40.00	-40.00	9.832E+05	1.426E+02	9.140E+01	2.999E+02	1098.9	68.47	154	3.21	13.20	0.905	2.56E+01	1.60E+00	12.22	0.255
	-20.00	-4.00	1.866E+06	2.707E+02	1.879E+02	6.164E+02	1013.0	63.24	109	2.29	9.18	0.627	4.75E+01	2.97E+00	13.54	0.283
	0.00	32.00	3.205E+06	4.649E+02	3.579E+02	1.174E+03	913.1	57.00	77	1.62	5.40	0.370	8.42E+01	5.26E+00	15.19	0.317
	15.56	60.00	4.642E+06	6.733E+02	5.819E+02	1.909E+03	813.4	50.78	59	1.23	2.78	0.199	1.38E+02	8.59E+00	17.23	0.360
	20.00	68.00	5.127E+06	7.436E+02	6.720E+02	2.205E+03	778.0	48.57	55	1.14	2.07	0.142	1.63E+02	1.02E+01	18.20	0.380
c.p.	36.42	97.56	7.245E+06	1.051E+03	1.634E+03	5.362E+03	452.0	28.22	41	0.85	0.00	0.000	4.52E+02	2.82E+01	40.73	0.851
	Octane-n C_8H_{18}				**M = 114.230**				**Zc = 0.259**							
t.p.	-56.77	-70.19	2.544E+00	3.690E-04	3.426E-04	1.124E-03	757.1	47.27	2118	44.19	28.06	1.991	1.68E-04	1.06E-05	4.28	0.089
	-40.00	-40.00	1.520E+01	2.205E-03	2.079E-03	6.823E-03	745.5	46.54	1430	29.87	27.39	1.877	9.40E-04	5.87E-05	4.55	0.095
	-20.00	-4.00	9.067E+01	1.314E-02	1.263E-02	4.143E-02	731.4	45.66	967	20.20	25.42	1.742	5.15E-03	3.22E-04	4.87	0.102
	0.00	32.00	4.013E+02	5.821E-02	5.709E-02	1.873E-01	716.8	44.75	714	14.91	23.47	1.608	2.12E-02	1.32E-03	5.20	0.109
	15.56	60.00	1.085E+03	1.574E-01	1.569E-01	5.149E-01	706.2	44.02	576	12.03	21.98	1.506	5.42E-02	3.38E-03	5.46	0.114
	20.00	68.00	1.410E+03	2.045E-01	2.049E-01	6.723E-01	701.8	43.81	546	11.40	21.55	1.477	6.83E-02	4.30E-03	5.54	0.116
	40.00	104.00	4.120E+03	5.976E-01	6.121E-01	2.008E+00	686.3	42.85	435	9.09	19.67	1.348	1.90E-01	1.16E-02	5.88	0.123
	60.00	140.00	1.037E+04	1.504E+00	1.578E+00	5.177E+00	670.3	41.84	356	7.44	17.81	1.220	4.50E-01	2.81E-02	6.22	0.130
	80.00	176.00	2.312E+04	3.353E+00	3.807E+00	1.183E+01	653.6	40.80	298	6.22	15.99	1.096	9.49E-01	5.93E-02	6.56	0.137
	100.00	212.00	4.661E+04	6.761E+00	7.471E+00	2.451E+01	638.2	39.72	256	5.35	14.20	0.973	1.82E+00	1.14E-01	6.92	0.144

Continued on next page

t.p. = triple point, n.b.p. = normal boiling point, c.p. = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁵	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Octane-n C_8H_{18} (continued)															
	120.00	248.00	8.646E+04	1.254E+01	1.427E+01	4.681E+01	617.9	38.58	214	4.47	12.48	0.854	3.24E+00	2.02E-01	7.28	0.152
n b p	125.68	258.22	1.013E+05	1.470E+01	1.687E+01	5.534E+01	612.6	38.24	205	4.29	11.97	0.820	3.75E+00	2.34E-01	7.38	0.154
	140.00	284.00	1.495E+05	2.169E+01	2.547E+01	8.356E+01	595.7	37.37	177	3.69	10.75	0.737	5.41E+00	3.38E-01	7.68	0.160
	160.00	320.00	2.437E+05	3.535E+01	4.299E+01	1.410E+02	578.1	36.09	151	3.15	9.10	0.623	8.60E+00	5.37E-01	8.09	0.169
	180.00	356.00	3.779E+05	5.481E+01	6.930E+01	2.274E+02	556.1	34.71	128	2.68	7.49	0.513	1.32E+01	8.22E-01	8.80	0.180
	200.00	392.00	5.613E+05	8.141E+01	1.076E+02	3.530E+02	532.0	33.21	109	2.28	5.95	0.408	1.96E+01	1.22E+00	9.26	0.193
	220.00	428.00	8.036E+05	1.166E+02	1.622E+02	5.323E+02	505.1	31.53	92	1.92	4.47	0.306	2.88E+01	1.80E+00	10.10	0.211
	240.00	464.00	1.115E+06	1.617E+02	2.397E+02	7.865E+02	474.1	29.60	77	1.80	3.08	0.211	4.24E+01	2.65E+00	11.20	0.234
	260.00	500.00	1.504E+06	2.182E+02	3.517E+02	1.154E+03	436.2	27.23	93	1.31	1.79	0.123	6.22E+01	3.88E+00	12.70	0.265
	280.00	536.00	1.983E+06	2.876E+02	5.283E+02	1.733E+03	382.7	23.89	49	1.03	0.66	0.045	9.83E+01	6.14E+00	15.20	0.317
c p	295.68	564.22	2.486E+06	3.606E+02	1.092E+03	3.583E+03	232.2	14.49	28	0.55	0.00	0.000	2.32E+02	1.45E+01	28.40	0.551
	Oxygen (R 732) O_2				**M = 31.999**				**Zc = 0.288**							
t p	-218.79	-361.82	1.460E+02	2.118E-02	1.139E-02	3.738E-02	1306.8	81.58	839	17.53	22.50	1.542	1.04E-02	6.47E-04	4.92	0.103
	-200.00	-328.00	1.061E+04	1.568E+00	9.018E-01	2.958E+00	1222.2	76.30	325	6.80	17.49	1.198	5.73E-01	3.57E-02	5.85	0.122
n b p	-182.96	-297.33	1.013E+05	1.470E+01	9.055E+00	2.971E+01	1141.0	71.23	194	4.05	13.15	0.901	4.48E+00	2.79E-01	6.96	0.146
	-180.00	-292.00	1.367E+05	1.982E+01	1.237E+01	4.060E+01	1126.1	70.30	181	3.77	12.42	0.851	5.99E+00	3.68E-01	7.20	0.150
	-160.00	-256.00	6.711E+05	9.734E+01	6.731E+01	2.208E+02	1016.8	63.48	123	2.58	7.70	0.528	2.61E+01	1.63E+00	8.87	0.185
	-140.00	-220.00	2.039E+06	2.958E+02	2.371E+02	7.781E+02	876.9	54.74	91	1.89	3.46	0.237	8.10E+01	5.06E+00	10.82	0.226
	-120.00	-184.00	4.770E+06	6.919E+02	8.203E+02	2.691E+03	593.0	37.02	39	0.82	0.13	0.009	2.81E+02	1.75E+01	17.74	0.371
c p	-118.57	-181.42	5.043E+06	7.314E+02	1.179E+03	3.869E+03	436.1	27.23	26	0.54	0.00	0.000	4.38E+02	2.72E+01	25.90	0.541
	Pentane-n C_5H_{12}				**M = 72.150**				**Zc = 0.288**							
t p	-129.73	-201.51	5.295E-02	7.680E-06	7.141E-06	2.343E-05	756.2	47.21	3400	71.01	33.57	2.301	3.24E-06	2.02E-07	3.17	0.066
	-120.00	-184.00	3.512E-01	5.093E-05	4.783E-05	1.569E-04	748.6	46.73	2310	48.25	32.37	2.218	2.01E-05	1.25E-06	3.40	0.071
	-100.00	-148.00	8.076E+00	1.171E-03	1.124E-03	3.688E-03	732.7	45.74	1250	26.11	29.92	2.050	4.08E-04	2.55E-05	3.88	0.081
	-80.00	-112.00	8.865E+01	1.286E-02	1.262E-02	4.140E-02	716.4	44.73	786	16.42	27.51	1.885	4.02E-03	2.51E-04	4.35	0.091
	-60.00	-76.00	5.791E+02	8.400E-02	8.440E-02	2.769E-01	699.7	43.68	546	11.40	25.13	1.722	2.39E-02	1.49E-03	4.82	0.101
	-40.00	-40.00	2.509E+03	3.769E-01	3.883E-01	1.274E+00	682.4	42.60	419	8.75	22.78	1.561	9.78E-02	6.10E-03	5.28	0.110
	-20.00	-4.00	8.821E+03	1.279E+00	1.354E+00	4.441E+00	664.4	41.48	341	7.12	20.48	1.403	3.06E-01	1.91E-02	5.73	0.120
	0.00	32.00	2.424E+04	3.515E+00	3.827E+00	1.256E+01	645.8	40.32	283	5.91	18.22	1.249	7.64E-01	4.89E-02	6.18	0.129
	15.56	60.00	4.743E+04	6.880E+00	7.668E+00	2.516E+01	630.7	39.38	249	5.19	16.50	1.131	1.46E+00	9.12E-02	6.53	0.136
	20.00	68.00	5.658E+04	8.206E+00	9.211E+00	3.022E+01	626.3	39.10	240	5.01	16.01	1.097	1.72E+00	1.07E-01	6.63	0.139
n b p	36.07	96.93	1.013E+05	1.470E+01	1.694E+01	5.557E+01	610.0	38.08	217	4.54	14.27	0.978	2.96E+00	1.85E-01	7.00	0.146
	40.00	104.00	1.163E+05	1.687E+01	1.957E+01	6.422E+01	605.9	37.82	191	3.98	13.85	0.949	3.38E+00	2.10E-01	7.05	0.147
	60.00	140.00	2.161E+05	3.136E+01	3.773E+01	1.238E+02	584.2	36.47	162	3.38	11.75	0.805	6.02E+00	3.76E-01	7.58	0.158
	80.00	176.00	3.708E+05	5.377E+01	6.739E+01	2.211E+02	561.0	35.02	136	2.83	9.70	0.665	1.01E+01	6.31E-01	8.18	0.171
	100.00	212.00	5.963E+05	8.649E+01	1.135E+02	3.723E+02	535.9	33.45	114	2.39	7.73	0.530	1.62E+01	1.01E+00	8.80	0.184
	120.00	248.00	9.105E+05	1.321E+02	1.827E+02	5.995E+02	508.1	31.72	96	2.01	5.84	0.400	2.52E+01	1.57E+00	9.44	0.197
	140.00	284.00	1.333E+06	1.933E+02	2.853E+02	9.360E+02	476.4	29.74	81	1.68	4.06	0.278	3.85E+01	2.40E+00	10.36	0.216
	160.00	320.00	1.895E+06	2.734E+02	4.386E+02	1.440E+03	436.1	27.35	66	1.37	2.40	0.164	5.91E+01	3.69E+00	11.63	0.243
	180.00	356.00	2.594E+06	3.762E+02	6.862E+02	2.251E+03	385.4	24.06	50	1.05	0.92	0.063	9.35E+01	5.84E+00	14.21	0.297
c p	196.50	385.70	3.369E+06	4.886E+02	1.487E+03	4.879E+03	231.0	14.42	27	0.55	0.00	0.000	2.31E+02	1.44E+01	28.53	0.554

t p = triple point n b p = normal boiling point c p = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature, T		Vapor Pressure, P_v		Vapor Head, $h_v = P_v/\rho g$		Liquid Density, ρ_l		Liquid Viscosity, μ_l		Surface Tension, σ		Vapor Density, ρ_g		Vapor Viscosity, μ_g	
	C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10^6	lbf sec/ft² × 10^6	N/m × 10^3	lbf/ft × 10^3	kg/m³	lbm/ft³	Pa sec × 10^5	lbf sec/ft² × 10^6
	Potassium K				**M = 39.10**				**Zc = 0.178**							
tp	63.25	145.85	1.657E-04	2.404E-08	2.042E-08	6.700E-08	827.6	51.66	518	10.82	109.03	7.471	2.32E-09	1.45E-10	6.42	0.134
	80.00	176.00	7.004E-04	1.016E-07	8.669E-08	2.844E-07	823.9	51.43	489	10.21	107.93	7.395	9.33E-09	5.82E-10	6.67	0.139
	100.00	212.00	3.300E-03	4.786E-07	4.106E-07	1.347E-06	819.4	51.16	457	9.55	106.61	7.305	4.16E-08	2.60E-09	6.96	0.145
	120.00	248.00	1.326E-02	1.923E-06	1.659E-06	5.443E-06	815.0	50.88	429	8.95	105.28	7.214	1.59E-07	9.90E-09	7.24	0.151
	140.00	284.00	4.650E-02	6.744E-06	5.850E-06	1.919E-05	810.5	50.60	403	8.41	103.96	7.124	5.29E-07	3.30E-08	7.54	0.157
	160.00	320.00	1.451E-01	2.104E-05	1.835E-05	6.022E-05	806.0	50.32	379	7.91	102.64	7.033	1.58E-06	9.83E-08	7.83	0.163
	180.00	356.00	4.089E-01	5.930E-05	5.202E-05	1.707E-04	801.5	50.03	357	7.46	101.32	6.943	4.24E-06	2.65E-07	8.12	0.170
	200.00	392.00	1.055E+00	1.530E-04	1.350E-04	4.428E-04	796.9	49.75	337	7.05	100.00	6.852	1.05E-05	6.55E-07	8.41	0.176
	250.00	482.00	8.179E+00	1.186E-03	1.062E-03	3.484E-03	785.5	49.04	295	6.16	96.70	6.626	7.37E-05	4.60E-06	9.14	0.191
	300.00	572.00	4.417E+01	6.407E-03	5.820E-03	1.909E-02	774.0	48.32	261	5.45	93.40	6.400	3.64E-04	2.27E-05	9.88	0.206
	350.00	662.00	1.813E+02	2.630E-02	2.425E-02	7.957E-02	762.3	47.59	233	4.87	90.10	6.174	1.38E-03	8.61E-05	10.62	0.222
	400.00	752.00	6.014E+02	8.723E-02	8.171E-02	2.681E-01	750.5	46.85	210	4.40	86.80	5.948	4.25E-03	2.65E-04	11.37	0.237
	450.00	842.00	1.686E+03	2.445E-01	2.327E-01	7.634E-01	738.6	46.11	192	4.01	83.50	5.722	1.12E-02	6.97E-04	12.12	0.253
	500.00	932.00	4.125E+03	5.982E-01	5.788E-01	1.899E+00	726.7	45.36	176	3.68	80.20	5.496	2.57E-02	1.61E-03	12.87	0.269
	550.00	1022.00	9.034E+03	1.310E+00	1.289E+00	4.230E+00	714.6	44.61	163	3.41	76.90	5.270	5.34E-02	3.33E-03	13.63	0.285
	600.00	1112.00	1.805E+04	2.619E+00	2.621E+00	8.599E+00	702.4	43.85	153	3.19	73.60	5.043	1.02E-01	6.34E-03	14.39	0.300
	700.00	1292.00	5.798E+04	8.409E+00	8.723E+00	2.862E+01	677.8	42.31	136	2.84	67.00	4.591	3.00E-01	1.87E-02	15.92	0.332
nbp	756.68	1394.02	1.013E+05	1.470E+01	1.557E+01	5.108E+01	663.6	41.43	128	2.68	63.26	4.335	5.02E-01	3.14E-02	16.80	0.351
	800.00	1472.00	1.490E+05	2.162E+01	2.328E+01	7.638E+01	652.8	40.75	124	2.58	60.40	4.139	7.18E-01	4.48E-02	17.47	0.365
	1000.00	1832.00	6.239E+05	9.049E+01	1.057E+02	3.468E+02	601.9	37.58	107	2.24	47.20	3.234	2.68E+00	1.67E-01	20.61	0.430
	1200.00	2192.00	1.750E+06	2.539E+02	3.245E+02	1.065E+03	550.0	34.33	96	2.00	34.00	2.330	6.85E+00	4.28E-01	23.82	0.497
cp	1899.85	3451.73	1.670E+07	2.422E+03	6.430E+03	2.766E+04	202.0	12.61	47	0.97	0.00	0.000	2.02E+02	1.26E+01	46.57	0.973
	Propane (R 290) C_3H_8				**M = 44.098**				**Zc = 0.280**							
tp	-187.71	-305.88	3.034E-04	4.400E-08	4.221E-08	1.385E-07	732.9	45.75	10789	225.34	36.26	2.486	1.88E-08	1.18E-09	1.30	0.027
	-180.00	-292.00	4.275E-03	6.200E-07	6.010E-07	1.972E-06	725.2	45.27	6041	126.17	35.11	2.406	2.43E-07	1.52E-08	1.48	0.031
	-160.00	-256.00	7.584E-01	1.100E-04	1.097E-04	3.598E-04	705.1	44.02	2039	42.59	32.11	2.200	3.55E-05	2.22E-06	1.99	0.042
	-140.00	-220.00	2.861E+01	4.150E-03	4.260E-03	1.398E-02	684.9	42.76	1001	20.90	29.15	1.997	1.14E-03	7.11E-05	2.55	0.053
	-120.00	-184.00	3.940E+02	5.715E-02	6.046E-02	1.984E-01	664.6	41.49	608	12.69	26.24	1.798	1.36E-02	8.50E-04	3.15	0.066
	-100.00	-148.00	2.817E+03	4.086E-01	4.461E-01	1.464E+00	643.9	40.20	418	8.74	23.38	1.602	8.64E-02	5.40E-03	3.80	0.079
	-80.00	-112.00	1.283E+04	1.862E+00	2.102E+00	6.895E+00	622.7	38.88	311	6.49	20.57	1.410	3.56E-01	2.22E-02	4.49	0.094
	-60.00	-76.00	4.240E+04	6.150E+00	7.195E+00	2.361E+01	600.9	37.52	241	5.04	17.83	1.222	1.08E+00	6.73E-02	5.21	0.109
nbp	-42.04	-43.67	1.013E+05	1.470E+01	1.779E+01	5.838E+01	580.7	36.25	196	4.09	15.42	1.056	2.42E+00	1.51E-01	5.90	0.123
	-40.00	-40.00	1.110E+05	1.610E+01	1.957E+01	6.422E+01	578.2	36.10	192	4.00	15.15	1.038	2.63E+00	1.64E-01	5.97	0.125
	-20.00	-4.00	2.448E+05	3.551E+01	4.504E+01	1.478E+02	554.2	34.60	153	3.20	12.54	0.860	5.52E+00	3.44E-01	6.77	0.141
	0.00	32.00	4.753E+05	6.894E+01	9.173E+01	3.010E+02	528.4	32.99	123	2.56	10.03	0.687	1.04E+01	6.48E-01	7.60	0.159
	15.56	60.00	7.441E+05	1.079E+02	1.498E+02	4.915E+02	506.5	31.62	102	2.14	8.13	0.557	1.61E+01	1.01E+00	8.52	0.178
	20.00	68.00	8.381E+05	1.216E+02	1.710E+02	5.610E+02	499.9	31.21	97	2.03	7.61	0.521	1.81E+01	1.13E+00	8.74	0.183
	40.00	104.00	1.373E+06	1.991E+02	2.995E+02	9.827E+02	467.3	29.17	76	1.59	5.30	0.363	3.03E+01	1.89E+00	9.37	0.196
	60.00	140.00	2.122E+06	3.077E+02	5.058E+02	1.659E+03	427.8	26.70	58	1.22	3.16	0.216	4.96E+01	3.10E+00	10.40	0.217
	80.00	176.00	3.135E+06	4.547E+02	8.569E+02	2.811E+03	373.1	23.29	44	0.91	1.23	0.085	8.44E+01	5.27E+00	13.17	0.275
cp	96.67	206.01	4.242E+06	6.153E+02	1.976E+03	6.482E+03	219.0	13.67	21	0.45	0.00	0.000	2.19E+02	1.37E+01	21.40	0.447

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_f		Liquid Viscosity - μ_f		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶	N/m ×10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec ×10⁶	lbf sec/ft² × 10⁶
	Propylene (R 1270) C_3H_6				**M = 42.080**				**Zc = 0.289**							
tp	-185.26	-301.47	9.572E-04	1.388E-07	1.269E-07	4.165E-07	768.9	48.00	13761	287.40	38.38	2.630	5.49E-08	3.43E-09	3.77	0.079
	-180.00	-292.00	5.994E-03	8.693E-07	8.012E-07	2.629E-06	762.8	47.62	8310	173.56	37.50	2.570	3.27E-07	2.04E-08	3.83	0.080
	-160.00	-256.00	1.227E+00	1.780E-04	1.693E-04	5.554E-04	739.5	46.16	2083	43.51	34.21	2.344	5.48E-05	3.42E-06	4.11	0.086
	-140.00	-220.00	4.521E+01	6.557E-03	6.435E-03	2.111E-02	716.3	44.72	892	18.62	30.97	2.122	1.72E-03	1.07E-04	4.42	0.092
	-120.00	-184.00	5.945E+02	8.622E-02	8.740E-02	2.868E-01	693.6	43.30	515	10.75	27.78	1.904	1.97E-02	1.23E-03	4.78	0.100
	-100.00	-148.00	4.035E+03	5.853E-01	6.134E-01	2.012E+00	670.8	41.88	352	7.36	24.66	1.690	1.18E-01	7.39E-03	5.19	0.108
	-80.00	-112.00	1.754E+04	2.544E+00	2.761E+00	9.059E+00	647.8	40.44	266	5.55	21.60	1.480	4.64E-01	2.90E-02	5.65	0.118
	-60.00	-76.00	5.583E+04	8.098E+00	9.122E+00	2.993E+01	624.1	38.96	211	4.41	18.62	1.276	1.36E+00	8.46E-02	6.08	0.127
nbp	-47.69	-53.84	1.013E+05	1.470E+01	1.696E+01	5.566E+01	609.1	38.02	185	3.87	16.82	1.153	2.36E+00	1.47E-01	6.65	0.139
	-40.00	-40.00	1.421E+05	2.060E+01	2.417E+01	7.929E+01	599.4	37.42	171	3.57	15.71	1.077	3.23E+00	2.01E-01	6.94	0.145
	-20.00	-4.00	3.067E+05	4.448E+01	5.457E+01	1.790E+02	573.1	35.78	138	2.88	12.90	0.884	6.64E+00	4.15E-01	7.57	0.158
	0.00	32.00	5.856E+05	8.494E+01	1.096E+02	3.597E+02	544.6	34.00	110	2.29	10.19	0.698	1.24E+01	7.71E-01	8.17	0.171
	15.56	60.00	9.072E+05	1.316E+02	1.778E+02	5.832E+02	520.4	32.49	90	1.89	8.16	0.559	1.91E+01	1.19E+00	8.73	0.182
	20.00	68.00	1.019E+06	1.478E+02	2.025E+02	6.645E+02	513.0	32.03	85	1.78	7.59	0.520	2.14E+01	1.34E+00	8.93	0.186
	40.00	104.00	1.652E+06	2.395E+02	3.533E+02	1.159E+03	476.7	29.76	64	1.33	5.14	0.352	3.57E+01	2.23E+00	10.05	0.210
	60.00	140.00	2.534E+06	3.675E+02	5.979E+02	1.962E+03	432.1	26.98	46	0.96	2.88	0.197	5.90E+01	3.68E+00	11.75	0.245
	80.00	176.00	3.727E+06	5.406E+02	1.034E+03	3.393E+03	367.5	22.94	31	0.66	0.91	0.062	1.03E+02	6.45E+00	14.23	0.297
cp	92.42	198.36	4.665E+06	6.765E+02	2.129E+03	6.984E+03	223.5	13.95	24	0.50	0.00	0.000	2.23E+02	1.40E+01	23.94	0.500
	Sodium Na				**M = 22.99**				**Zc = 0.230**							
tp	106.85	224.33	2.612E-05	3.789E-09	2.879E-09	9.447E-09	925.1	57.75	806	16.82	197.12	13.507	1.90E-10	1.18E-11	8.12	0.170
	500.00	932.00	5.536E+02	8.029E-02	6.782E-02	2.225E-01	832.3	51.96	285	5.95	157.80	10.813	2.03E-03	1.27E-04	17.56	0.367
	550.00	1022.00	1.452E+03	2.107E-01	1.806E-01	5.924E-01	820.2	51.21	264	5.52	152.80	10.470	5.04E-03	3.15E-04	18.76	0.392
	600.00	1112.00	3.405E+03	4.939E-01	4.296E-01	1.410E+00	808.2	50.45	247	5.15	147.80	10.128	1.12E-02	7.02E-04	19.96	0.417
	650.00	1202.00	7.265E+03	1.054E+00	9.306E-01	3.053E+00	796.1	49.70	232	4.84	142.80	9.785	2.29E-02	1.43E-03	21.16	0.442
	700.00	1292.00	1.432E+04	2.076E+00	1.862E+00	6.109E+00	784.0	48.94	219	4.58	137.80	9.442	4.33E-02	2.70E-03	22.36	0.467
	750.00	1382.00	2.636E+04	3.823E+00	3.482E+00	1.142E+01	771.9	48.19	208	4.35	132.80	9.100	7.67E-02	4.79E-03	23.56	0.492
	800.00	1472.00	4.578E+04	6.640E+00	6.144E+00	2.016E+01	759.8	47.43	198	4.14	127.80	8.757	1.29E-01	8.03E-03	24.76	0.517
	850.00	1562.00	7.561E+04	1.097E+01	1.031E+01	3.383E+01	747.7	46.68	190	3.97	122.80	8.414	2.05E-01	1.28E-02	25.96	0.542
nbp	881.45	1618.61	1.013E+05	1.470E+01	1.396E+01	4.580E+01	740.2	46.21	185	3.87	119.66	8.199	2.70E-01	1.68E-02	26.71	0.558
	900.00	1652.00	1.195E+05	1.733E+01	1.657E+01	5.436E+01	735.6	45.92	182	3.81	117.80	8.072	3.14E-01	1.96E-02	27.16	0.567
	950.00	1742.00	1.818E+05	2.636E+01	2.562E+01	8.405E+01	723.5	45.16	176	3.67	112.80	7.729	4.64E-01	2.90E-02	28.36	0.592
	1000.00	1832.00	2.672E+05	3.876E+01	3.830E+01	1.257E+02	711.4	44.41	170	3.55	107.80	7.387	6.63E-01	4.14E-02	29.56	0.617
	1050.00	1922.00	3.812E+05	5.529E+01	5.559E+01	1.824E+02	699.3	43.65	164	3.43	102.80	7.044	9.20E-01	5.74E-02	30.76	0.642
	1100.00	2012.00	5.295E+05	7.680E+01	7.858E+01	2.578E+02	687.2	42.90	159	3.33	97.80	6.701	1.24E+00	7.77E-02	31.96	0.667
	1150.00	2102.00	7.182E+05	1.042E+02	1.085E+02	3.559E+02	675.1	42.15	155	3.24	92.80	6.359	1.65E+00	1.03E-01	33.16	0.692
	1200.00	2192.00	9.534E+05	1.383E+02	1.466E+02	4.810E+02	663.1	41.39	151	3.15	87.80	6.016	2.13E+00	1.33E-01	34.36	0.718
	1250.00	2282.00	1.242E+06	1.801E+02	1.944E+02	6.380E+02	651.1	40.64	147	3.08	82.80	5.674	2.72E+00	1.70E-01	35.56	0.743
	1300.00	2372.00	1.589E+06	2.304E+02	2.535E+02	8.317E+02	639.1	39.90	144	3.00	77.80	5.331	3.41E+00	2.13E-01	36.76	0.768
	1350.00	2462.00	2.001E+06	2.903E+02	3.254E+02	1.068E+03	627.2	39.15	141	2.94	72.80	4.988	4.23E+00	2.64E-01	37.96	0.793
		32.00														
	1400.00	2552.00	2.485E+06	3.604E+02	4.118E+02	1.351E+03	615.3	38.41	138	2.88	67.80	4.646	5.20E+00	3.24E-01	39.16	0.818
cp	2459.85	4459.73	4.136E+07	5.999E+03	2.320E+04	7.612E+04	181.8	11.35	75	1.57	0.00	0.000	1.82E+02	1.13E+01	75.00	1.566

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P		Vapor Head - h_v		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶
	Sulfur Dioxide (R 764) SO_2				**M = 64.060**				**Zc = 0.269**							
t p	-75 48	-103 864	1 816E+03	2 344E-01	1 024E-01	3 359E-01	1609 2	100 48	1293	27 00	42 22	2 883	6 43E-02	4 01E-03	8 79	0 184
	-60 00	-76 00	5 549E+03	8 049E-01	3 590E-01	1 178E+00	1576 1	98 39	949	19 93	38 93	2 667	2 07E-01	1 29E-02	9 52	0 199
	-40 00	-40 00	2 090E+04	3 031E+00	1 391E+00	4 565E+00	1531 6	95 61	668	13 95	34 74	2 380	7 18E-01	4 47E-02	10 46	0 219
	-20 00	-4 00	6 231E+04	9 038E+00	4 279E+00	1 404E+01	1484 8	92 69	487	10 17	30 63	2 099	1 97E+00	1 23E-01	11 40	0 238
n b p	-10 02	13 96	1 013E+05	1 470E+01	7 075E+00	2 321E+01	1460 4	91 17	420	8 77	28 60	1 960	3 08E+00	1 93E-01	11 68	0 248
	0 00	32 00	1 553E+05	2 252E+01	1 103E+01	3 619E+01	1435 2	89 60	364	7 60	26 60	1 822	4 55E+00	2 84E-01	12 35	0 258
	15 56	60 00	2 860E+05	4 149E+01	2 092E+01	6 863E+01	1394 4	87 05	293	6 12	23 52	1 612	8 32E+00	5 19E-01	13 09	0 273
	20 00	68 00	3 360E+05	4 873E+01	2 478E+01	8 132E+01	1382 4	86 30	275	5 75	22 68	1 553	9 23E+00	5 76E-01	13 30	0 278
	40 00	104 00	6 499E+05	9 426E+01	5 000E+01	1.640E+02	1325 4	82 74	210	4 39	18 82	1 290	1 70E+01	1 06E+00	14 29	0 298
	60 00	140 00	1 148E+06	1 665E+02	9 268E+01	3 041E+02	1263 0	78 85	160	3 35	15 10	1 035	3 03E+01	1 89E+00	15 33	0 320
	80 00	176 00	1 883E+06	2 731E+02	1 609E+02	5 279E+02	1193 4	74 50	122	2 55	11 52	0 789	4 87E+01	3 04E+00	16 47	0 344
	100 00	212 00	2 905E+06	4 214E+02	2 662E+02	8 734E+02	1113 0	69 48	92	1 93	8 10	0 555	7 66E+01	4 78E+00	17 60	0 372
	120 00	248 00	4 260E+06	6 179E+02	4 281E+02	1 405E+03	1014 7	63 35	69	1 45	4 90	0 335	1 22E+01	7 62E-01	17 68	0 369
	140 00	284 00	5 985E+06	8 680E+02	6 952E+02	2 281E+03	877 8	54 80	51	1 07	2 00	0 137	2 11E+02	1 32E+01	23 10	0 483
c p	157 60	315 68	7 884E+06	1 143E+03	1 531E+03	5 023E+03	525 1	32 78	39	0 82	0 00	0 000	5 25E+02	3 28E+01	39 17	0 818
	Sulfur Trioxide SO_3				**M = 80.058**				**Zc = 0.256**							
t p	16 80	62 24	2 113E+04	3 065E+00	1 122E+00	3 681E+00	1920 5	119 89	1890	39 48	34 80	2 384	7 02E-01	4 38E-02	14 08	0 294
	20 00	68 00	2 571E+04	3 729E+00	1 373E+00	4 506E+00	1908 8	119 16	1772	37 02	34 12	2 338	8 44E-01	5 27E-02	14 22	0 297
	40 00	104 00	8 094E+04	1 174E+01	4 498E+00	1 476E+01	1834 7	114 53	1183	24 70	29 97	2 053	2 49E+00	1 55E-01	15 10	0 315
n b p	44 75	112 55	1 013E+05	1 470E+01	5 687E+00	1 866E+01	1816 7	113 41	1075	22 44	28 99	1 987	3 07E+00	1 92E-01	15 31	0 320
	60 00	140 00	2 051E+05	2 975E+01	1 189E+01	3 903E+01	1758 2	109 76	793	16 56	25 91	1 775	5 83E+00	3 70E-01	15 99	0 334
	80 00	176 00	4 392E+05	6 370E+01	2 668E+01	8 752E+01	1678 9	104 81	540	11 28	21 98	1 505	1 30E+01	8 12E-01	16 91	0 353
	100 00	212 00	8 251E+05	1 197E+02	5 271E+01	1 730E+02	1596 1	99 64	376	7 85	18 14	1 243	2 50E+01	1 56E+00	17 88	0 374
	120 00	248 00	1 400E+06	2 030E+02	9 461E+01	3 104E+02	1508 6	94 18	270	5 63	14 46	0 991	3 70E+01	2 31E+00	18 87	0 394
	140 00	284 00	2 194E+06	3 182E+02	1 581E+02	5 189E+02	1414 6	88 31	200	4 18	10 94	0 749	5 60E+01	3 50E+00	19 98	0 417
	160 00	320 00	3 236E+06	4 693E+02	2 516E+02	8 255E+02	1311 3	81 86	155	3 23	7 61	0 522	8 60E+01	5 37E+00	21 25	0 444
	180 00	356 00	4 556E+06	6 608E+02	3 896E+02	1 278E+03	1192 3	74 43	124	2 80	4 53	0 310	1 37E+02	8 55E+00	23 06	0 482
	200 00	392 00	6 197E+06	8 988E+02	6 075E+02	1 993E+03	1040 3	64 94	105	2 19	1 80	0 124	2 33E+02	1 45E+01	26 42	0 552
c p	217 70	423 86	8 207E+06	1 190E+03	1 328E+03	4 358E+03	630 0	39 33	94	1 95	0 00	0 000	6 30E+02	3 93E+01	93 55	1 954
	Trichlorofluoromethane (R 11) CCl_3F				**M = 137.368**				**Zc = 0.279**							
t p	-111 11	-168 00	4 738E+00	6 872E-04	2 748E-04	9 015E-04	1758 3	109 77	4872	101 75	37 39	2 562	4 63E-04	3 02E-05	5 70	0 119
	-100 00	-148 00	2 210E+01	3 206E-03	1 297E-03	4 255E-03	1737 6	108 46	3350	69 97	35 74	2 449	2 11E-03	1 32E-04	6 10	0 127
	-80 00	-112 00	2 105E+02	3 052E-02	1 263E-02	4 143E-02	1699 4	106 09	1933	40 38	32 82	2 249	1 80E-02	1 12E-03	6 81	0 142
	-60 00	-76 00	1 223E+03	1 774E-01	7 514E-02	2 465E-01	1660 1	103 64	1258	26 27	29 95	2 052	9 91E-02	6 19E-03	7 52	0 157
	-40 00	-40 00	4 968E+03	7 234E-01	3 140E-01	1 030E+00	1619 5	101 11	893	18 85	27 12	1 858	3 82E-01	2 26E-02	8 22	0 172
	-20 00	-4 00	1 566E+04	2 271E+00	1 012E+00	3 320E+00	1577 6	98 49	676	14 13	24 35	1 569	1 04E+00	6 47E-02	8 92	0 186
	0 00	32 00	4 035E+04	5 853E+00	2 662E+00	8 801E+00	1533 9	95 76	538	11 24	21 64	1 483	2 48E+00	1 55E-01	9 63	0 201
	15 56	60 00	7 577E+04	1 099E+01	5 155E+00	1 691E+01	1496 7	93 56	462	9 66	19 57	1 341	4 43E+00	2 77E-01	10 18	0 213
	20 00	68 00	8 942E+04	1 297E+01	6 126E+00	2 010E+01	1488 4	92 92	444	9 28	18 99	1 301	5 17E+00	3 23E-01	10 30	0 215
n b p	23 52	74 34	1 013E+05	1 470E+01	6 980E+00	2 290E+01	1480 2	92 41	431	9 00	18 53	1 270	5 91E+00	3 69E-01	10 70	0 223

Continued on next page

t p = triple point; n b p = normal boiling point; c p = critical point

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P_v/\gamma$		Liquid Density - ρ		Liquid Viscosity - μ		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	°C	°F	Pa	psia	m	ft	kg/m^3	lbm/ft^3	Pa sec × 10^6	lbf sec/ft^2 × 10^8	N/m × 10^3	lbf/ft × 10^3	kg/m^3	lbm/ft^3	Pa sec × 10^6	lbf sec/ft^2 × 10^6
	Trichlorofluoromethane (R 11) CCl_3F (continued)															
	40.00	104.00	1.762E+05	2.556E+01	1.247E+01	4.091E+01	1440.6	89.93	377	7.87	16.40	1.124	9.71E+00	6.06E-01	11.90	0.249
	60.00	140.00	3.164E+05	4.589E+01	2.321E+01	7.615E+01	1389.9	86.77	325	6.80	13.89	0.952	1.64E+01	1.02E+00	12.70	0.265
	80.00	176.00	5.277E+05	7.653E+01	4.028E+01	1.321E+02	1335.9	83.40	283	5.92	11.46	0.786	2.75E+01	1.72E+00	13.10	0.274
	100.00	212.00	8.293E+05	1.203E+02	6.621E+01	2.172E+02	1277.3	79.74	246	5.14	9.13	0.625	4.30E+01	2.68E+00	13.60	0.284
	120.00	248.00	1.242E+06	1.802E+02	1.045E+02	3.427E+02	1212.6	75.70	210	4.39	6.90	0.473	6.52E+01	4.07E+00	14.30	0.299
	140.00	284.00	1.789E+06	2.595E+02	1.601E+02	5.254E+02	1139.1	71.12	174	3.63	4.80	0.329	9.73E+01	6.08E+00	15.60	0.326
	160.00	320.00	2.495E+06	3.618E+02	2.421E+02	7.942E+02	1051.0	65.61	135	2.83	2.86	0.196	1.46E+02	9.11E+00	17.90	0.374
	180.00	356.00	3.369E+06	4.914E+02	3.708E+02	1.217E+03	931.6	58.16	96	2.01	1.14	0.078	2.29E+02	1.43E+01	21.50	0.449
c.p.	198.05	388.49	4.408E+06	6.393E+02	8.114E+02	2.662E+03	553.9	34.58	33	0.70	0.00	0.000	5.54E+02	3.46E+01	33.30	0.695
	Water (R 718) H_2O				M = 18.015				Zc = 0.233							
t.p.	0.01	32.02	6.117E+02	8.872E-02	6.239E-02	2.047E-01	999.8	62.41	1791	37.41	75.65	5.184	4.86E-03	3.03E-04	9.22	0.193
	4.44	40.00	8.386E+02	1.216E-01	8.551E-02	2.806E-01	1000.0	62.43	1549	32.38	75.02	5.141	6.55E-03	4.09E-04	9.32	0.195
	10.00	50.00	1.228E+03	1.781E-01	1.253E-01	4.110E-01	999.7	62.41	1308	27.32	74.22	5.086	9.41E-03	5.87E-04	9.46	0.198
	15.56	60.00	1.766E+03	2.561E-01	1.802E-01	5.913E-01	999.1	62.37	1124	23.48	73.41	5.030	1.33E-02	8.28E-04	9.62	0.201
	20.00	68.00	2.339E+03	3.392E-01	2.389E-01	7.839E-01	998.2	62.31	1003	20.95	72.74	4.984	1.73E-02	1.08E-03	9.73	0.203
	21.11	70.00	2.502E+03	3.629E-01	2.557E-01	8.388E-01	998.0	62.31	978	20.39	72.57	4.973	1.84E-02	1.15E-03	9.73	0.203
	26.67	80.00	3.491E+03	5.064E-01	3.572E-01	1.172E+00	996.7	62.22	859	17.94	71.72	4.914	2.53E-02	1.58E-03	9.92	0.207
	30.00	86.00	4.246E+03	6.158E-01	4.348E-01	1.427E+00	995.6	62.15	798	16.67	71.20	4.879	3.04E-02	1.90E-03	10.01	0.209
	32.22	90.00	4.813E+03	6.981E-01	4.933E-01	1.618E+00	995.0	62.12	782	15.91	70.85	4.855	3.42E-02	2.14E-03	10.06	0.210
	37.78	100.00	6.545E+03	9.492E-01	6.720E-01	2.205E+00	993.1	62.00	682	14.24	69.96	4.794	4.57E-02	2.85E-03	10.26	0.214
	40.00	104.00	7.381E+03	1.071E+00	7.586E-01	2.489E+00	992.2	61.94	653	13.64	69.60	4.769	5.12E-02	3.20E-03	10.31	0.215
	43.33	110.00	8.791E+03	1.275E+00	9.046E-01	2.968E+00	990.9	61.86	615	12.63	69.05	4.732	6.04E-02	3.77E-03	10.41	0.218
	48.89	120.00	1.167E+04	1.693E+00	1.204E+00	3.950E+00	988.5	61.71	558	11.64	68.13	4.669	7.68E-02	4.82E-03	10.61	0.222
	50.00	122.00	1.234E+04	1.790E+00	1.274E+00	4.180E+00	988.0	61.68	547	11.43	67.95	4.656	8.31E-02	5.19E-03	10.62	0.222
	54.44	130.00	1.533E+04	2.223E+00	1.586E+00	5.201E+00	985.9	61.55	509	10.63	67.19	4.604	1.02E-01	6.38E-03	10.76	0.225
	60.00	140.00	1.993E+04	2.891E+00	2.067E+00	6.783E+00	983.2	61.38	467	9.75	66.24	4.539	1.30E-01	8.13E-03	10.94	0.228
	65.56	150.00	2.564E+04	3.718E+00	2.667E+00	8.751E+00	980.1	61.19	430	8.99	65.27	4.472	1.65E-01	1.03E-02	11.12	0.232
	70.00	158.00	3.119E+04	4.522E+00	3.251E+00	1.067E+01	977.8	61.04	405	8.45	64.48	4.419	1.98E-01	1.24E-02	11.26	0.235
	71.11	160.00	3.269E+04	4.741E+00	3.412E+00	1.119E+01	977.0	60.99	399	8.33	64.29	4.405	2.07E-01	1.29E-02	11.28	0.235
	76.67	170.00	4.132E+04	5.993E+00	4.327E+00	1.420E+01	973.7	60.79	370	7.74	63.28	4.335	2.58E-01	1.61E-02	11.46	0.239
	80.00	176.00	4.737E+04	6.871E+00	4.971E+00	1.631E+01	971.8	60.67	356	7.41	62.66	4.295	2.93E-01	1.83E-02	11.60	0.242
	82.22	180.00	5.178E+04	7.511E+00	5.443E+00	1.786E+01	970.2	60.57	345	7.21	62.27	4.267	3.19E-01	1.99E-02	11.67	0.244
	87.78	190.00	6.440E+04	9.340E+00	6.793E+00	2.229E+01	966.6	60.34	323	6.75	61.24	4.195	3.91E-01	2.44E-02	11.87	0.248
	90.00	194.00	7.012E+04	1.017E+01	7.406E+00	2.430E+01	965.3	60.26	315	6.58	60.82	4.167	4.22E-01	2.64E-02	11.93	0.249
	93.33	200.00	7.947E+04	1.153E+01	8.416E+00	2.761E+01	962.8	60.11	303	6.34	60.19	4.124	4.76E-01	2.97E-02	12.05	0.252
	98.89	210.00	9.737E+04	1.412E+01	1.036E+01	3.400E+01	956.3	59.70	285	5.97	59.13	4.052	5.76E-01	3.60E-02	12.25	0.256
n.b.p.	100.00	212.00	1.013E+05	1.470E+01	1.078E+01	3.537E+01	958.4	59.83	282	5.90	58.92	4.037	5.98E-01	3.73E-02	12.28	0.256
	110.00	230.00	1.432E+05	2.078E+01	1.536E+01	5.039E+01	951.0	59.37	255	5.33	56.97	3.903	8.26E-01	5.16E-02	12.62	0.264
	120.00	248.00	1.985E+05	2.879E+01	2.146E+01	7.040E+01	943.2	58.88	232	4.85	54.97	3.767	1.12E+00	7.00E-02	12.97	0.271
	130.00	266.00	2.700E+05	3.916E+01	2.945E+01	9.663E+01	934.9	58.36	213	4.44	52.94	3.627	1.50E+00	9.34E-02	13.32	0.276

Continued on next page

t.p. = triple point, n.b.p. = normal boiling point, c.p. = critical point

Table D.1 Critical and Saturated Properties of Selected Fluids (*Continued*)

	Temperature - T		Vapor Pressure - P_v		Vapor Head - $h_v = P/\gamma$		Liquid Density - ρ_l		Liquid Viscosity - μ_l		Surface Tension - σ		Vapor Density - ρ_g		Vapor Viscosity - μ_g	
	C	F	Pa	psia	m	ft	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁶	N/m × 10³	lbf/ft × 10³	kg/m³	lbm/ft³	Pa sec × 10⁶	lbf sec/ft² × 10⁵
	Water (R 718) H_2O (continued)															
	140 00	284 00	3 612E+05	5 239E+01	3 977E+01	1 305E+02	926 2	57 82	196	4 10	50 86	3 485	1 96E+00	1 23E-01	13 67	0 286
	150 00	302 00	4 757E+05	6 900E+01	5 290E+01	1 735E+02	917 1	57 25	182	3 80	48 75	3 340	2 55E+00	1 59E-01	14 02	0 293
	160 00	320 00	6 177E+05	8 958E+01	6 940E+01	2 277E+02	907 5	56 65	170	3 54	46 60	3 193	3 28E+00	2 03E-01	14 37	0 300
	170 00	338 00	7 916E+05	1 148E+02	8 992E+01	2 950E+02	897 5	56 03	159	3 32	44 41	3 043	4 12E+00	2 57E-01	14 72	0 307
	180 00	356 00	1 002E+06	1 453E+02	1 152E+02	3 779E+02	887 1	55 38	149	3 12	42 19	2 891	5 15E+00	3 22E-01	15 07	0 315
	190 00	374 00	1 254E+06	1 819E+02	1 460E+02	4 789E+02	876 2	54 70	141	2 94	39 95	2 737	6 39E+00	3 99E-01	15 42	0 322
	200 00	392 00	1 564E+06	2 253E+02	1 832E+02	6 011E+02	864 7	53 98	134	2 79	37 68	2 582	7 85E+00	4 90E-01	15 78	0 330
	210 00	410 00	1 906E+06	2 765E+02	2 279E+02	7 478E+02	852 8	53 24	127	2 65	35 39	2 425	9 58E+00	5 99E-01	16 13	0 337
	220 00	428 00	2 318E+06	3 362E+02	2 812E+02	9 228E+02	840 3	52 46	121	2 53	33 07	2 266	1 16E+01	7 25E-01	16 49	0 344
	230 00	446 00	2 795E+06	4 054E+02	3 445E+02	1 130E+03	827 3	51 64	116	2 41	30 74	2 106	1 40E+01	8 72E-01	16 85	0 352
	240 00	464 00	3 345E+06	4 851E+02	4 192E+02	1 375E+03	813 5	50 79	111	2 31	28 40	1 946	1 67E+01	1 04E+00	17 22	0 360
	250 00	482 00	3 974E+06	5 763E+02	5 071E+02	1 664E+03	799 1	49 88	106	2 21	26 05	1 785	2 00E+01	1 25E+00	17 59	0 367
	260 00	500 00	4 690E+06	6 802E+02	6 101E+02	2 002E+03	783 8	48 93	102	2 12	23 69	1 624	2 37E+01	1 48E+00	17 98	0 376
	270 00	518 00	5 500E+06	7 977E+02	7 305E+02	2 397E+03	767 7	47 92	97	2 03	21 34	1 462	2 81E+01	1 75E+00	18 38	0 384
	280 00	536 00	6 413E+06	9 302E+02	8 713E+02	2 858E+03	750 5	46 85	93	1 95	19 00	1 302	3 32E+01	2 07E+00	18 80	0 393
	290 00	554 00	7 438E+06	1 079E+03	1 036E+03	3 399E+03	732 2	45 71	90	1 87	16 67	1 142	3 91E+01	2 44E+00	19 25	0 402
	300 00	572 00	8 584E+06	1 245E+03	1 229E+03	4 031E+03	712 4	44 47	86	1 79	14 37	0 984	4 62E+01	2 88E+00	19 74	0 412
	310 00	590 00	9 861E+06	1 430E+03	1 455E+03	4 774E+03	691 0	43 13	82	1 71	12 09	0 829	5 45E+01	3 40E+00	20 28	0 424
	320 00	608 00	1 128E+07	1 636E+03	1 723E+03	5 654E+03	667 4	41 66	78	1 64	9 87	0 676	6 46E+01	4 03E+00	20 89	0 436
	330 00	626 00	1 285E+07	1 864E+03	2 044E+03	6 706E+03	641 0	40 02	74	1 55	7 71	0 528	7 70E+01	4 81E+00	21 62	0 452
	340 00	644 00	1 459E+07	2 117E+03	2 436E+03	7 994E+03	610 8	38 13	70	1 47	5 63	0 386	9 27E+01	5 79E+00	22 52	0 470
	350 00	662 00	1 652E+07	2 396E+03	2 931E+03	9 617E+03	574 7	35 88	66	1 37	3 67	0 252	1 13E+02	7 08E+00	23 72	0 495
	360 00	680 00	1 866E+07	2 706E+03	3 602E+03	1 182E+04	528 1	32 97	60	1 26	1 88	0 129	1 44E+02	8 97E+00	25 54	0 533
	370 00	698 00	2 103E+07	3 050E+03	4 733E+03	1 553E+04	453 1	28 29	52	1 09	0 39	0 027	2 00E+02	1 25E+01	29 26	0 611
c p	373 98	705 16	2 206E+07	3 199E+03	6 984E+03	2 291E+04	322 0	20 10	39	0 81	0 00	0 000	3 22E+02	2 01E+01	39 00	0 815

t p = triple point n b p = normal boiling point, c p = critical point

Table D.2 Properties of Selected Gases

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				
						Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa·s×10⁶	lbf-sec/ft²×10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
					Acetic Acid $C_2H_4O_2$					
117.90	244.22	3.10E+00	1.93E-01	10.6	0.662	1327	0.3169	1189	0.2840	1.116
150.00	302.00	2.86E+00	1.79E-01	11.7	0.731	1402	0.3349	1264	0.3019	1.109
200.00	392.00	2.37E+00	1.48E-01	13.4	0.837	1513	0.3614	1375	0.3284	1.100
250.00	482.00	1.77E+00	1.10E-01	15.1	0.941	1614	0.3855	1476	0.3525	1.093
300.00	572.00	1.21E+00	7.53E-02	16.7	1.043	1707	0.4077	1569	0.3747	1.088
400.00	752.00	7.01E-01	4.37E-02	19.8	1.236	1872	0.4471	1734	0.4142	1.080
					Acetone C_3H_6O					
56.29	133.32	2.15E+00	1.34E-01	8.36	0.175	1369	0.3270	1226	0.2929	1.117
100.00	212.00	1.90E+00	1.18E-01	9.44	0.197	1502	0.3587	1359	0.3245	1.105
150.00	302.00	1.67E+00	1.04E-01	10.7	0.224	1649	0.3938	1506	0.3596	1.095
200.00	392.00	1.50E+00	9.34E-02	12.1	0.252	1788	0.4272	1645	0.3930	1.087
250.00	482.00	1.35E+00	8.44E-02	13.4	0.280	1920	0.4586	1777	0.4244	1.081
300.00	572.00	1.24E+00	7.71E-02	14.7	0.307	2043	0.4880	1900	0.4538	1.075
350.00	662.00	1.14E+00	7.09E-02	16.0	0.334	2158	0.5154	2015	0.4812	1.071
400.00	752.00	1.05E+00	6.56E-02	17.2	0.360	2264	0.5408	2121	0.5067	1.067
450.00	842.00	9.78E-01	6.11E-02	18.4	0.385	2363	0.5645	2220	0.5303	1.064
500.00	932.00	9.15E-01	5.71E-02	19.6	0.409	2455	0.5864	2312	0.5522	1.062

Acetonitrile C_2H_3N

81.60	178.88	1.55E+00	9.65E-02	8.49	0.177	1396	0.3334	1193	0.2850	1.170
100.00	212.00	1.47E+00	9.17E-02	8.93	0.186	1435	0.3428	1233	0.2945	1.164
150.00	302.00	1.30E+00	8.09E-02	10.1	0.211	1540	0.3678	1337	0.3194	1.151
200.00	392.00	1.16E+00	7.23E-02	11.2	0.235	1640	0.3916	1437	0.3433	1.141
250.00	482.00	1.05E+00	6.54E-02	12.4	0.258	1735	0.4143	1532	0.3659	1.132
300.00	572.00	9.57E-01	5.97E-02	13.5	0.282	1824	0.4357	1622	0.3873	1.125
350.00	662.00	8.80E-01	5.49E-02	14.6	0.305	1909	0.4559	1706	0.4075	1.119
400.00	752.00	8.14E-01	5.08E-02	15.7	0.328	1988	0.4748	1785	0.4264	1.113
450.00	842.00	7.58E-01	4.73E-02	16.8	0.351	2062	0.4926	1860	0.4442	1.109
500.00	932.00	7.09E-01	4.43E-02	17.9	0.374	2132	0.5093	1930	0.4609	1.105

Acetylene C_2H_2

-84.15	-119.47	1.73E+00	1.08E-01	6.34	0.132	1434	0.3426	1115	0.2663	1.286
-50.00	-58.00	1.45E+00	9.07E-02	7.59	0.159	1535	0.3666	1216	0.2904	1.263
0.00	32.00	1.17E+00	7.32E-02	9.36	0.195	1663	0.3972	1344	0.3209	1.238
50.00	122.00	9.87E-01	6.16E-02	11.1	0.231	1773	0.4235	1454	0.3473	1.220
100.00	212.00	8.52E-01	5.32E-02	12.7	0.265	1871	0.4468	1551	0.3705	1.206
150.00	302.00	7.51E-01	4.69E-02	14.2	0.297	1958	0.4676	1638	0.3913	1.195
200.00	392.00	6.71E-01	4.19E-02	15.7	0.328	2036	0.4863	1717	0.4101	1.186
250.00	482.00	6.07E-01	3.79E-02	17.1	0.358	2107	0.5033	1788	0.4271	1.179
300.00	572.00	5.54E-01	3.46E-02	18.5	0.387	2172	0.5189	1853	0.4426	1.172
350.00	662.00	5.09E-01	3.18E-02	19.8	0.414	2232	0.5331	1913	0.4569	1.167
400.00	752.00	4.71E-01	2.94E-02	21.1	0.441	2287	0.5463	1968	0.4700	1.162
450.00	842.00	4.39E-01	2.74E-02	22.3	0.466	2338	0.5584	2019	0.4821	1.158
500.00	932.00	4.10E-01	2.56E-02	23.5	0.491	2385	0.5697	2066	0.4934	1.155

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity -μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				
						Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa sx10⁶	lbf-sec/ft²x10⁶	J/kg K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
					Air (R 729)					
-194.35	-317.83	4.37E+00	2.73E-01	5.44	0.114	1001	0.2390	713.5	0.1704	1.402
-150.00	-238.00	2.90E+00	1.81E-01	8.59	0.179	1001	0.2390	713.5	0.1704	1.402
-100.00	-148.00	2.06E+00	1.28E-01	11.8	0.246	1001	0.2390	713.5	0.1704	1.402
-50.00	-58.00	1.59E+00	9.93E-02	14.6	0.306	1001	0.2390	713.6	0.1704	1.402
0.00	32.00	1.30E+00	8.10E-02	17.2	0.360	1001	0.2391	714.2	0.1706	1.402
50.00	122.00	1.10E+00	6.84E-02	19.6	0.410	1003	0.2396	716.1	0.1710	1.401
100.00	212.00	9.49E-01	5.92E-02	21.8	0.458	1007	0.2405	720.1	0.1720	1.399
150.00	302.00	8.36E-01	5.22E-02	23.9	0.499	1013	0.2420	726.4	0.1735	1.395
200.00	392.00	7.48E-01	4.67E-02	25.8	0.539	1022	0.2440	734.6	0.1755	1.391
250.00	482.00	6.78E-01	4.23E-02	27.6	0.577	1031	0.2464	744.5	0.1778	1.385
300.00	572.00	6.19E-01	3.87E-02	29.4	0.614	1042	0.2490	755.5	0.1804	1.380
350.00	662.00	5.71E-01	3.56E-02	31.0	0.648	1054	0.2518	767.1	0.1832	1.374
400.00	752.00	5.30E-01	3.31E-02	32.6	0.681	1066	0.2546	779.0	0.1861	1.366
450.00	842.00	4.95E-01	3.09E-02	34.1	0.713	1078	0.2574	790.9	0.1889	1.363
500.00	932.00	4.84E-01	2.90E-02	35.6	0.744	1090	0.2602	802.6	0.1917	1.358
600.00	1112.00	4.15E-01	2.59E-02	38.4	0.802	1112	0.2656	824.9	0.1970	1.348
700.00	1292.00	3.76E-01	2.35E-02	41.0	0.856	1132	0.2705	845.4	0.2019	1.339
800.00	1472.00	3.46E-01	2.16E-02	43.5	0.908	1151	0.2749	863.8	0.2063	1.332
1000.00	1832.00	3.02E-01	1.88E-02	48.1	1.004	1182	0.2823	895.0	0.2138	1.321

Ammonia (R 717) NH_3

-33.43	-28.17	8.89E-01	5.55E-02	8.0	0.169	1996	0.4767	1508	0.3601	1.324
0.00	32.00	7.71E-01	4.82E-02	9.0	0.194	2049	0.4894	1561	0.3728	1.313
50.00	122.00	6.47E-01	4.04E-02	11.1	0.232	2135	0.5098	1646	0.3932	1.297
100.00	212.00	5.59E-01	3.49E-02	12.9	0.270	2226	0.5316	1737	0.4149	1.281
150.00	302.00	4.92E-01	3.07E-02	14.7	0.308	2319	0.5540	1831	0.4374	1.267
200.00	392.00	4.39E-01	2.74E-02	16.5	0.346	2415	0.5767	1926	0.4601	1.253
250.00	482.00	3.97E-01	2.48E-02	18.4	0.383	2510	0.5995	2022	0.4828	1.242
300.00	572.00	3.62E-01	2.26E-02	20.2	0.421	2604	0.6220	2116	0.5053	1.231
350.00	662.00	3.36E-01	2.10E-02	21.9	0.458	2697	0.6441	2208	0.5275	1.221
400.00	752.00	3.09E-01	1.93E-02	23.7	0.495	2787	0.6657	2299	0.5491	1.212
450.00	842.00	2.88E-01	1.80E-02	25.5	0.532	2875	0.6868	2387	0.5701	1.205
500.00	932.00	2.66E-01	1.68E-02	27.2	0.569	2961	0.7072	2473	0.5906	1.197

Argon (R 740) Ar

-185.86	-302.55	5.72E+00	3.57E-01	7.23	0.151	520.1	0.1242	312	0.0745	1.667
-150.00	-238.00	4.01E+00	2.50E-01	10.2	0.213	520.1	0.1242	312	0.0745	1.667
-100.00	-148.00	2.83E+00	1.76E-01	14.1	0.295	520.1	0.1242	312	0.0745	1.667
-50.00	-58.00	2.18E+00	1.36E-01	17.7	0.370	520.1	0.1242	312	0.0745	1.667
0.00	32.00	1.78E+00	1.11E-01	21.1	0.441	520.1	0.1242	312	0.0745	1.667
50.00	122.00	1.51E+00	9.41E-02	24.2	0.506	520.1	0.1242	312	0.0745	1.667
100.00	212.00	1.31E+00	8.15E-02	27.2	0.568	520.1	0.1242	312	0.0745	1.667
150.00	302.00	1.15E+00	7.18E-02	30.0	0.626	520.1	0.1242	312	0.0745	1.667
200.00	392.00	1.03E+00	6.42E-02	32.6	0.681	520.1	0.1242	312	0.0745	1.667
250.00	482.00	9.30E-01	5.81E-02	35.0	0.731	520.1	0.1242	312	0.0745	1.667
300.00	572.00	8.49E-01	5.30E-02	37.4	0.780	520.1	0.1242	312	0.0745	1.667
350.00	662.00	7.81E-01	4.88E-02	39.6	0.828	520.1	0.1242	312	0.0745	1.667
400.00	752.00	7.23E-01	4.51E-02	41.8	0.873	520.1	0.1242	312	0.0745	1.667
450.00	842.00	6.73E-01	4.20E-02	43.9	0.917	520.1	0.1242	312	0.0745	1.667
500.00	932.00	6.29E-01	3.93E-02	46.0	0.960	520.1	0.1242	312	0.0745	1.667

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m^3	lbm/ft^3	Pa·s x10^6	lbf-sec/ft^{2}x10^6	J/kg K	Btu/lbm-°R	J/kg K	Btu/lbm-°R	$k = c_p/c_v$
					Benzene C_6H_6					
80.09	176.16	2.76E+00	1.72E-01	8.97	0.187	1265	0.3021	1158	0.2766	1.092
100.00	212.00	2.61E+00	1.63E-01	9.49	0.198	1338	0.3196	1232	0.2942	1.086
150.00	302.00	2.28E+00	1.43E-01	10.8	0.225	1513	0.3613	1406	0.3358	1.076
200.00	392.00	2.03E+00	1.27E-01	12.1	0.252	1673	0.3995	1566	0.3741	1.068
250.00	482.00	1.83E+00	1.15E-01	13.3	0.278	1819	0.4343	1712	0.4089	1.062
300.00	572.00	1.67E+00	1.04E-01	14.5	0.304	1951	0.4661	1845	0.4406	1.058
350.00	662.00	1.53E+00	9.58E-02	15.7	0.329	2072	0.4949	1965	0.4694	1.054
400.00	752.00	1.42E+00	8.85E-02	16.9	0.354	2181	0.5210	2075	0.4956	1.051
450.00	842.00	1.32E+00	8.21E-02	18.1	0.378	2281	0.5447	2174	0.5193	1.049
500.00	932.00	1.22E+00	7.64E-02	19.3	0.402	2371	0.5662	2264	0.5408	1.047
					Bromine Br_2					
58.75	137.75	5.95E+00	3.72E-01	17.0	0.356	228	0.0544	176	0.0419	1.296
100.00	212.00	5.29E+00	3.30E-01	19.0	0.397	229	0.0547	177	0.0423	1.294
150.00	302.00	4.64E+00	2.90E-01	21.4	0.447	230	0.0550	178	0.0426	1.292
200.00	392.00	4.14E+00	2.59E-01	23.8	0.497	231	0.0553	179	0.0428	1.290
250.00	482.00	3.74E+00	2.34E-01	26.2	0.547	232	0.0555	180	0.0431	1.289
300.00	572.00	3.41E+00	2.13E-01	28.6	0.597	233	0.0557	181	0.0432	1.287
350.00	662.00	3.13E+00	1.96E-01	31.0	0.647	234	0.0558	182	0.0434	1.286
400.00	752.00	2.90E+00	1.81E-01	33.3	0.696	234	0.0559	182	0.0435	1.286
450.00	842.00	2.70E+00	1.68E-01	35.7	0.745	235	0.0560	182	0.0436	1.285
500.00	932.00	2.52E+00	1.58E-01	38.0	0.793	235	0.0561	183	0.0437	1.285

Butane-iso (R 600A) C_4H_{10}

-11.72	10.90	2.81E+00	1.76E-01	6.66	0.139	1490	0.3560	1347	0.3218	1.106
0.00	32.00	2.68E+00	1.67E-01	6.97	0.146	1549	0.3699	1406	0.3357	1.102
50.00	122.00	2.23E+00	1.40E-01	8.23	0.172	1796	0.4289	1652	0.3947	1.087
100.00	212.00	1.92E+00	1.20E-01	9.41	0.196	2033	0.4856	1890	0.4514	1.076
150.00	302.00	1.69E+00	1.05E-01	10.5	0.220	2257	0.5390	2114	0.5048	1.068
200.00	392.00	1.50E+00	9.39E-02	11.6	0.242	2465	0.5888	2322	0.5546	1.062
250.00	482.00	1.36E+00	8.49E-02	12.6	0.263	2658	0.6349	2515	0.6007	1.057
300.00	572.00	1.24E+00	7.74E-02	13.5	0.283	2837	0.6775	2693	0.6433	1.053
350.00	662.00	1.14E+00	7.10E-02	14.5	0.302	3001	0.7169	2858	0.6827	1.050
400.00	752.00	1.05E+00	6.57E-02	15.4	0.321	3154	0.7533	3011	0.7192	1.048
450.00	842.00	9.80E-01	6.11E-02	16.2	0.339	3295	0.7871	3152	0.7529	1.045
500.00	932.00	9.16E-01	5.72E-02	17.1	0.357	3426	0.8184	3283	0.7842	1.044

Butane-n (R 600) C_4H_{10}

-0.50	31.10	2.71E+00	1.69E-01	6.89	0.144	1584	0.3784	1441	0.3442	1.099
0.00	32.00	2.70E+00	1.69E-01	6.90	0.144	1586	0.3789	1443	0.3448	1.099
50.00	122.00	2.25E+00	1.40E-01	8.17	0.171	1815	0.4335	1672	0.3994	1.086
100.00	212.00	1.93E+00	1.20E-01	9.40	0.196	2040	0.4872	1897	0.4530	1.075
150.00	302.00	1.69E+00	1.06E-01	10.6	0.221	2254	0.5385	2111	0.5043	1.068
200.00	392.00	1.51E+00	9.41E-02	11.7	0.245	2457	0.5868	2314	0.5526	1.062
250.00	482.00	1.36E+00	8.50E-02	12.8	0.268	2646	0.6319	2502	0.5977	1.057
300.00	572.00	1.24E+00	7.75E-02	13.9	0.290	2821	0.6738	2678	0.6397	1.053
350.00	662.00	1.14E+00	7.11E-02	14.9	0.311	2984	0.7128	2841	0.6787	1.050
400.00	752.00	1.05E+00	6.57E-02	15.9	0.332	3136	0.7490	2993	0.7148	1.048
450.00	842.00	9.80E-01	6.11E-02	16.8	0.351	3277	0.7826	3134	0.7485	1.046
500.00	932.00	9.16E-01	5.72E-02	17.7	0.370	3408	0.8139	3265	0.7797	1.044

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa s x10⁶	lbf-sec/ft² x10⁶	J/kg K	Btu/lbm-°R	J/kg K	Btu/lbm-°R	$k = c_p/c_v$
				Carbon Dioxide (R 744) CO_2						
-78.48	-109.26	2.80E+00	1.75E-01	9.75	0.204	733.4	0.1752	544.4	0.1300	1.347
-50.00	-58.00	2.43E+00	1.52E-01	11.2	0.235	762.8	0.1822	573.9	0.1371	1.329
0.00	32.00	1.98E+00	1.24E-01	13.8	0.287	815.8	0.1949	626.9	0.1497	1.301
50.00	122.00	1.67E+00	1.04E-01	16.2	0.337	866.9	0.2071	678.0	0.1619	1.279
150.00	302.00	1.27E+00	7.92E-02	20.6	0.430	957.0	0.2266	768.1	0.1835	1.246
200.00	392.00	1.13E+00	7.08E-02	22.6	0.473	995.6	0.2378	806.7	0.1927	1.234
250.00	482.00	1.03E+00	6.40E-02	24.6	0.514	1030	0.2461	841.3	0.2010	1.225
300.00	572.00	9.36E-01	5.85E-02	26.5	0.553	1061	0.2535	872.5	0.2084	1.217
350.00	662.00	8.61E-01	5.38E-02	28.3	0.591	1089	0.2602	900.5	0.2151	1.210
400.00	752.00	7.97E-01	4.98E-02	30.0	0.627	1115	0.2663	925.8	0.2211	1.204
450.00	842.00	7.42E-01	4.63E-02	31.7	0.662	1138	0.2717	948.8	0.2266	1.199
500.00	932.00	6.94E-01	4.33E-02	33.3	0.695	1159	0.2767	970.0	0.2316	1.195
600.00	1112.00	6.14E-01	3.83E-02	36.3	0.759	1195	0.2854	1006	0.2403	1.188
700.00	1292.00	5.51E-01	3.44E-02	39.2	0.819	1225	0.2927	1037	0.2476	1.182
800.00	1472.00	5.00E-01	3.12E-02	41.9	0.875	1252	0.2989	1063	0.2538	1.178
1000.00	1832.00	4.21E-01	2.63E-02	46.9	0.979	1293	0.3089	1105	0.2638	1.171

				Carbon Disulfide CS_2						
46 22	115.20	2.95E+00	1.84E-01	10.7	0.223	610	0.1458	501.3	0.1197	1.218
100 00	212.00	2.53E+00	1.58E-01	12.5	0.262	639	0.1526	529 8	0.1265	1.206
150 00	302.00	2.22E+00	1.38E-01	14.3	0.298	661	0 1578	551.5	0.1317	1.198
200 00	392.00	1.98E+00	1.23E-01	16.0	0.334	679	0 1621	569.6	0.1361	1.192
250 00	482.00	1.78E+00	1.11E-01	17.7	0.369	694	0.1658	584.9	0.1397	1.187
300.00	572 00	1.63E+00	1.02E-01	19.4	0.405	707	0 1689	598.0	0.1428	1.183
350.00	662.00	1.49E+00	9.33E-02	21.1	0.440	718	0.1716	609.2	0.1455	1.179
400 00	752 00	1.38E+00	8.63E-02	22.7	0.475	728	0.1739	619.0	0.1479	1.176
450.00	842.00	1 29E+00	8.03E-02	24.4	0.509	737	0.1760	627.6	0.1499	1.174
500.00	932.00	1.20E+00	7.50E-02	26.0	0 544	744	0 1778	635.2	0.1517	1 172
				Carbon Monoxide CO						
-191.45	-312 61	4.36E+00	2.72E-01	5.41	0.113	1040	0 2483	742.9	0.1774	1.400
-150 00	-238.00	2 81E+00	1.75E-01	8 21	0.172	1040	0.2483	742.9	0.1774	1.400
-100 00	-148 00	1.98E+00	1.24E-01	11.3	0.235	1040	0.2483	742 9	0.1774	1.400
-50.00	-58.00	1.53E+00	9.56E-02	14.0	0.293	1040	0 2483	742.9	0 1774	1.400
0.00	32.00	1.25E+00	7.80E-02	16.5	0.345	1040	0 2484	743.1	0.1775	1.399
50 00	122 00	1.06E+00	6.59E-02	18.8	0.393	1041	0.2487	744.3	0.1778	1.399
100 00	212 00	9.15E-01	5.71E-02	20.9	0.437	1044	0 2494	747.4	0.1785	1.397
150.00	302 00	8 07E-01	5.04E-02	22.9	0.479	1050	0.2508	753.1	0.1799	1.394
200.00	392.00	7.21E-01	4.50E-02	24.8	0.519	1058	0.2527	761 3	0.1818	1.390
250.00	482.00	6.52E-01	4.07E-02	26.6	0.556	1068	0.2552	771.6	0.1843	1.385
300.00	572 00	5 96E-01	3.72E-02	28.3	0.592	1080	0 2580	783.5	0 1871	1.379
350.00	662.00	5.48E-01	3.42E-02	30.0	0.626	1093	0 2611	796.2	0 1902	1.373
400 00	752.00	5 07E-01	3.17E-02	31.5	0.659	1106	0 2642	809.3	0.1933	1.367
450.00	842 00	4.72E-01	2.95E-02	33.1	0.690	1119	0.2674	822.5	0.1965	1.361
500.00	932 00	4 42E-01	2.76E-02	34.5	0.721	1132	0 2704	835.5	0.1995	1.355
600.00	1112.00	3.91E-01	2.44E-02	37.3	0.779	1157	0 2763	860.0	0.2054	1.345
700.00	1292.00	3 51E-01	2.19E-02	39.9	0 834	1179	0.2816	882.0	0.2107	1.337
800.00	1472.00	3.18E-01	1.99E-02	42.4	0.886	1198	0.2862	901.6	0.2153	1.329
1000.00	1832 00	2.68E-01	1.67E-02	47.1	0.983	1230	0.2939	933.7	0.2230	1.318

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity -μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				
						Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa·sx10⁶	lbf-sec/ft²x10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Carbon Tetrachloride CCl_4						
76.64	169.95	5.58E+00	3.48E-01	11.6	0.242	576.5	0.1377	522.5	0.1248	1.103
100.00	212.00	5.19E+00	3.24E-01	12.3	0.257	588.1	0.1405	534.0	0.1275	1.101
150.00	302.00	4.53E+00	2.83E-01	13.8	0.289	608.2	0.1453	554.2	0.1324	1.098
200.00	392.00	4.03E+00	2.51E-01	15.3	0.319	623.7	0.1490	569.7	0.1361	1.095
250.00	482.00	3.63E+00	2.26E-01	16.7	0.349	636.0	0.1519	581.9	0.1390	1.093
300.00	572.00	3.30E+00	2.06E-01	18.1	0.378	645.7	0.1542	591.7	0.1413	1.091
350.00	662.00	3.03E+00	1.89E-01	19.4	0.406	653.6	0.1561	599.6	0.1432	1.090
400.00	752.00	2.80E+00	1.75E-01	20.8	0.434	660.2	0.1577	606.1	0.1448	1.089
450.00	842.00	2.60E+00	1.63E-01	22.0	0.460	665.6	0.1590	611.6	0.1461	1.088
500.00	932.00	2.43E+00	1.52E-01	23.3	0.486	670.2	0.1601	616.1	0.1472	1.088
				Chlorine Cl_2						
-34.03	-29.25	3.68E+00	2.30E-01	10.7	0.224	462.2	0.1104	344.9	0.0824	1.340
0.00	32.00	3.21E+00	2.00E-01	12.3	0.257	472.3	0.1128	355.0	0.0848	1.330
50.00	122.00	2.70E+00	1.69E-01	14.5	0.304	484.3	0.1157	367.0	0.0877	1.319
100.00	212.00	2.33E+00	1.46E-01	16.7	0.349	493.5	0.1179	376.3	0.0899	1.312
150.00	302.00	2.05E+00	1.28E-01	18.8	0.392	500.7	0.1196	383.4	0.0916	1.308
200.00	392.00	1.83E+00	1.14E-01	20.8	0.435	506.3	0.1209	389.0	0.0929	1.301
250.00	482.00	1.66E+00	1.03E-01	22.8	0.477	510.8	0.1220	393.5	0.0940	1.298
300.00	572.00	1.51E+00	9.43E-02	24.8	0.517	514.4	0.1229	397.1	0.0949	1.295
350.00	662.00	1.39E+00	8.67E-02	26.7	0.557	517.4	0.1236	400.1	0.0956	1.293
400.00	752.00	1.29E+00	8.02E-02	28.5	0.596	519.9	0.1242	402.6	0.0962	1.291
450.00	842.00	1.20E+00	7.46E-02	30.3	0.634	521.9	0.1247	404.7	0.0967	1.290
500.00	932.00	1.12E+00	6.98E-02	32.1	0.671	523.7	0.1251	406.5	0.0971	1.288

Chlorodifluoromethane (R 22) $CHClF_2$

-40 82	-41.48	4.71E+00	2.94E-01	10.2	0 212	563	0.1345	467	0.1115	1.206
0.00	32.00	3 94E+00	2 46E-01	11.9	0 249	614	0.1467	518	0.1238	1 186
50 00	122.00	3.30E+00	2.06E-01	14 0	0.293	672	0.1606	576	0.1376	1.167
100.00	212 00	2 84E+00	1 78E-01	16 1	0 337	725	0.1732	629	0.1502	1 153
150.00	302.00	2 50E+00	1.56E-01	18 2	0.379	773	0.1847	677	0.1617	1.142
200.00	392.00	2.23E+00	1 40E-01	20.2	0 421	817	0.1951	721	0.1721	1.133
250.00	482.00	2 01E+00	1 26E-01	22.1	0.462	856	0.2045	760	0.1815	1.127
300.00	572.00	1 84E+00	1 15E-01	24.1	0.503	892	0.2129	795	0.1900	1.121
350 00	662.00	1.69E+00	1 06E-01	26.0	0.543	923	0.2205	827	0.1975	1.116
400.00	752.00	1.57E+00	9.77E-02	27.9	0.582	951	0.2272	855	0.2042	1.112
450.00	842 00	1.46E+00	9 10E-02	29.7	0.621	976	0 2332	880	0.2102	1.109
500 00	932.00	1.36E+00	8 51E-02	31.6	0.659	998	0.2384	902	0 2155	1.107

Chloroform $CHCl_3$

61.18	142.12	4.50E+00	2.81E-01	11.3	0.708	579	0.1384	510	0.1217	1.137
100.00	212 00	3.99E+00	2 49E-01	12.6	0.790	607	0 1449	537	0 1283	1.130
150.00	302 00	3 49E+00	2.18E-01	14.3	0.893	637	0.1522	568	0.1356	1.123
200.00	392 00	3.11E+00	1 94E-01	15 9	0 995	663	0.1584	594	0 1418	1.117
250.00	482 00	2.80E+00	1.75E-01	17.5	1.095	686	0.1637	616	0.1471	1.113
300.00	572.00	2.55E+00	1.60E-01	19.1	1.194	705	0 1683	635	0.1517	1 110
350.00	662.00	2.35E+00	1 47E-01	20.7	1 291	722	0.1724	652	0.1557	1.107
400.00	752.00	2 17E+00	1.36E-01	22.2	1.387	736	0.1759	667	0.1592	1.104
450.00	842.00	2 02E+00	1.26E-01	23.7	1.482	749	0.1790	680	0.1623	1.102
500.00	932 00	1 89E+00	1.18E-01	25.2	1.575	761	0.1817	691	0.1651	1 101

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity -μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft² x10⁶	J/kg K	Btu/lbm-°R	J/kg K	Btu/lbm-°R	$k = c_p/c_v$
0.00	32.00	7.11E+00	4.44E-01	11.7	0.245	680	0.1624	626	0.1496	1.086
50.00	122.00	5.92E+00	3.69E-01	13.6	0.285	748	0.1786	694	0.1657	1.078
100.00	212.00	5.09E+00	3.18E-01	15.5	0.324	807	0.1928	753	0.1800	1.071
150.00	302.00	4.47E+00	2.79E-01	17.3	0.362	860	0.2053	606	0.1925	1.067
200.00	392.00	3.98E+00	2.48E-01	19.1	0.399	906	0.2163	852	0.2035	1.063
250.00	482.00	3.60E+00	2.25E-01	20.8	0.435	946	0.2259	892	0.2130	1.060
300.00	572.00	3.28E+00	2.05E-01	22.5	0.469	981	0.2342	927	0.2214	1.058
350.00	662.00	3.02E+00	1.89E-01	24.1	0.503	1011	0.2415	957	0.2286	1.056
400.00	752.00	2.60E+00	1.75E-01	25.6	0.536	1038	0.2478	984	0.2350	1.055
460.00	842.00	2.60E+00	1.63E-01	27.2	0.567	1061	0.2534	1007	0.2406	1.053
500.00	932.00	2.44E+00	1.52E-01	28.6	0.598	1082	0.2584	1028	0.2456	1.052
Chlorotrifluoromethane (R 13) $CClF_3$										
-81.45	-114.61	6.94E+00	4.33E-01	10.1	0.211	505	0.1207	426	0.1017	1.187
-50.00	-56.00	5.85E+00	3.65E-01	11.3	0.236	550	0.1313	470	0.1122	1.169
0.00	32.00	4.72E+00	2.95E-01	13.3	0.279	613	0.1484	533	0.1274	1.149
50.00	122.00	3.97E+00	2.48E-01	15.5	0.324	669	0.1597	589	0.1407	1.135
100.00	212.00	3.43E+00	2.14E-01	17.7	0.370	717	0.1713	638	0.1523	1.125
150.00	302.00	3.02E+00	1.86E-01	19.9	0.417	759	0.1814	680	0.1624	1.117
200.00	392.00	2.69E+00	1.68E-01	22.2	0.463	796	0.1901	716	0.1711	1.111
250.00	482.00	2.43E+00	1.52E-01	24.4	0.510	827	0.1976	748	0.1786	1.106
300.00	572.00	2.22E+00	1.39E-01	26.6	0.555	854	0.2040	775	0.1850	1.103
350.00	662.00	2.04E+00	1.28E-01	28.7	0.600	877	0.2095	798	0.1905	1.100
400.00	752.00	1.89E+00	1.18E-01	30.8	0.643	897	0.2143	818	0.1953	1.097
450.00	842.00	1.76E+00	1.10E-01	32.7	0.684	914	0.2184	835	0.1994	1.095
500.00	932.00	1.65E+00	1.03E-01	34.6	0.723	930	0.2221	850	0.2031	1.094

Cyclohexane C_6H_{12}

80 72	177 30	2.90E+00	1.81E-01	8 33	0 174	1545	0 3690	1446	0.3454	1.068
100.00	212.00	2.75E+00	1.72E-01	8 76	0.183	1645	0.3929	1546	0.3693	1.064
150.00	302 00	2 42E+00	1.51E-01	9 86	0 206	1899	0 4536	1800	0.4300	1.055
200.00	392 00	2.17E+00	1.35E-01	10 9	0.228	2140	0 5111	2041	0 4875	1.048
250 00	482 00	1.96E+00	1 22E-01	12 0	0.251	2365	0.5648	2266	0.5412	1.044
300 00	572 00	1.79E+00	1.12E-01	13 0	0 272	2572	0.6143	2473	0 5907	1 040
350 00	662 00	1.65E+00	1.03E-01	14.1	0 294	2762	0.6597	2663	0.6361	1.037
400 00	752 00	1.52E+00	9 51E-02	15.1	0.315	2936	0.7013	2837	0.6777	1.035
450 00	842.00	1.42E+00	8 85E-02	16.1	0.336	3095	0 7393	2997	0 7157	1.033
500 00	932.00	1.33E+00	8 28E-02	17 1	0 356	3241	0 7742	3142	0 7506	1 031

Dichlorodifluoromethane (R 12) CCl_2F_2

-29.79	-21 62	6.30E+00	3.94E-01	10.5	0 219	542	0.1294	473	0.1130	1.145
0.00	32.00	5 54E+00	3 46E-01	11.6	0 242	578	0 1380	509	0.1216	1 135
50 00	122.00	4.63E+00	2.89E-01	13.4	0 280	628	0 1501	560	0 1337	1.123
100 00	212 00	3.99E+00	2 49E-01	15 3	0 319	669	0 1597	600	0 1433	1 115
150 00	302.00	3 50E+00	2.19E-01	17 1	0.357	701	0 1675	632	0.1510	1 109
200.00	392.00	3.13E+00	1.95E-01	18 9	0.395	727	0 1737	659	0 1573	1 104
250 00	482 00	2.82E+00	1.76E-01	20.7	0.433	749	0.1789	680	0.1625	1 101
300 00	572 00	2.57E+00	1 60E-01	22.5	0 471	767	0.1832	698	0.1668	1 098
350 00	662 00	2 36E+00	1.48E-01	24 3	0 508	782	0.1868	713	0.1704	1.096
400.00	752 00	2.19E+00	1 37E-01	26.1	0 546	795	0.1899	726	0.1734	1 095
450 00	842 00	2 04E+00	1 27E-01	27 9	0.583	806	0 1925	737	0 1761	1.093
500.00	932 00	1 91E+00	1 19E-01	29 7	0 621	815	0 1948	747	0 1783	1.092

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft² x10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Ethane (R 170) C_2H_6						
-88.60	-127.48	2.06E+00	1.28E-01	6.01	0.126	1372	0.3277	1095	0.2616	1.252
-50.00	-58.00	1.68E+00	1.05E-01	7.14	0.149	1474	0.3519	1197	0.2859	1.231
0.00	32.00	1.36E+00	8.47E-02	8.58	0.179	1647	0.3934	1370	0.3273	1.202
50.00	122.00	1.14E+00	7.12E-02	10.0	0.209	1848	0.4415	1572	0.3754	1.176
100.00	212.00	9.85E-01	6.15E-02	11.4	0.238	2061	0.4924	1785	0.4263	1.155
150.00	302.00	8.68E-01	5.42E-02	12.8	0.266	2276	0.5435	1999	0.4775	1.138
200.00	392.00	7.77E-01	4.85E-02	14.1	0.294	2485	0.5934	2208	0.5274	1.125
250.00	482.00	7.03E-01	4.39E-02	15.4	0.321	2684	0.8412	2408	0.5751	1.115
300.00	572.00	6.41E-01	4.00E-02	16.6	0.347	2874	0.6864	2597	0.6204	1.106
350.00	662.00	5.87E-01	3.67E-02	17.9	0.373	3052	0.7290	2776	0.6629	1.100
400.00	752.00	5.40E-01	3.37E-02	19.1	0.398	3219	0.7689	2943	0.7028	1.094
450.00	842.00	5.03E-01	3.14E-02	20.2	0.422	3375	0.8062	3099	0.7402	1.089
500.00	932.00	4.70E-01	2.94E-02	21.4	0.446	3521	0.8411	3245	0.7750	1.085
600.00	1112.00	4.16E-01	2.60E-02	23.5	0.492	3785	0.9041	3509	0.8381	1.079
700.00	1292.00	3.74E-01	2.33E-02	25.6	0.535	4017	0.9593	3740	0.8933	1.074
800.00	1472.00	3.39E-01	2.12E-02	27.6	0.576	4220	1.0079	3943	0.9418	1.070
1000.00	1832.00	2.86E-01	1.78E-02	31.2	0.651	4559	1.0889	4283	1.0229	1.065

Ethanol C_2H_6O

78.29	172.92	1.65E+00	1.03E-01	10.4	0.218	1600	0.3822	1420	0.3390	1.127
100.00	212.00	1.54E+00	9.61E-02	11.0	0.231	1672	0.3994	1492	0.3563	1.121
150.00	302.00	1.34E+00	8.38E-02	12.4	0.259	1834	0.4380	1653	0.3949	1.109
200.00	392.00	1.19E+00	7.46E-02	13.7	0.287	1987	0.4747	1807	0.4316	1.100
250.00	482.00	1.08E+00	6.73E-02	15.0	0.314	2131	0.5090	1951	0.4659	1.093
300.00	572.00	9.82E-01	6.13E-02	16.3	0.341	2265	0.5409	2084	0.4978	1.087
350.00	662.00	9.03E-01	5.63E-02	17.6	0.367	2389	0.5705	2208	0.5274	1.082
400.00	752.00	8.35E-01	5.21E-02	18.8	0.393	2503	0.5979	2323	0.5548	1.078
450.00	842.00	7.77E-01	4.85E-02	20.0	0.418	2609	0.6232	2429	0.5801	1.074
500.00	932.00	7.27E-01	4.54E-02	21.2	0.443	2707	0.6466	2527	0.6035	1.071

Ethyl Chloride (R 160) C_2H_5Cl

12.27	54.09	2.86E+00	1.79E-01	9.3	0.195	943	0.2253	814	0.1945	1.156
50.00	122.00	2.49E+00	1.56E-01	10.5	0.220	1030	0.2460	901	0.2152	1.143
100.00	212.00	2.14E+00	1.33E-01	12.1	0.253	1143	0.2730	1014	0.2423	1.127
150.00	302.00	1.87E+00	1.17E-01	13.7	0.286	1251	0.2988	1122	0.2680	1.115
200.00	392.00	1.67E+00	1.04E-01	15.3	0.320	1352	0.3229	1223	0.2921	1.105
250.00	482.00	1.51E+00	9.42E-02	16.9	0.353	1445	0.3452	1317	0.3145	1.098
300.00	572.00	1.38E+00	8.59E-02	18.5	0.385	1532	0.3658	1403	0.3350	1.092
350.00	662.00	1.26E+00	7.89E-02	20.0	0.418	1611	0.3847	1462	0.3540	1.087
400.00	752.00	1.17E+00	7.30E-02	21.6	0.451	1684	0.4021	1555	0.3714	1.083
450.00	842.00	1.09E+00	6.79E-02	23.2	0.484	1751	0.4181	1622	0.3874	1.079
500.00	932.00	1.02E+00	6.35E-02	24.7	0.516	1812	0.4329	1684	0.4021	1.077

Ethylene (R 1150) C_2H_4

-103.68	-154.62	2.09E+00	1.30E-01	5.7	0.119	1225	0.2925	928	0.2217	1.319
-100.00	-148.00	2.05E+00	1.28E-01	5.9	0.122	1230	0.2938	934	0.2230	1.317
-50.00	-58.00	1.56E+00	9.71E-02	7.7	0.160	1333	0.3183	1036	0.2475	1.286
0.00	32.00	1.26E+00	7.87E-02	9.4	0.196	1477	0.3528	1181	0.2820	1.251
50.00	122.00	1.06E+00	6.63E-02	11.0	0.230	1644	0.3927	1348	0.3219	1.220

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity -μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure: Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa·s×10⁶	lbf-sec/ft²×10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Ethylene (R 1150) C_2H_4 (continued)						
100.00	212.00	9.19E-01	5.73E-02	12.6	0.263	1819	0.4346	1523	0.3638	1.195
150.00	302.00	8.09E-01	5.05E-02	14.1	0.294	1994	0.4762	1697	0.4054	1.175
200.00	392.00	7.23E-01	4.52E-02	15.5	0.323	2161	0.5162	1865	0.4455	1.159
250.00	482.00	6.54E-01	4.08E-02	16.8	0.351	2320	0.5542	2024	0.4834	1.146
300.00	572.00	5.97E-01	3.73E-02	18.1	0.378	2469	0.5897	2172	0.5189	1.136
350.00	662.00	5.49E-01	3.43E-02	19.3	0.404	2607	0.6227	2311	0.5519	1.128
400.00	752.00	5.08E-01	3.17E-02	20.5	0.429	2735	0.6534	2439	0.5826	1.122
450.00	842.00	4.73E-01	2.95E-02	21.7	0.452	2854	0.6817	2558	0.6110	1.116
500.00	932.00	4.42E-01	2.76E-02	22.8	0.475	2964	0.7080	2668	0.6372	1.111
				Helium (R 704) He						
-268.93	-452.07	1.69E+01	1.06E+00	1.2	0.026	5193	1.2404	3116	0.7443	1.667
-250.00	-418.00	2.11E+00	1.32E-01	3.8	0.079	5193	1.2404	3116	0.7443	1.667
-200.00	-328.00	6.67E-01	4.16E-02	7.9	0.165	5193	1.2404	3116	0.7443	1.667
-150.00	-238.00	3.96E-01	2.47E-02	11.2	0.235	5193	1.2404	3116	0.7443	1.667
-100.00	-148.00	2.82E-01	1.76E-02	13.8	0.289	5193	1.2404	3116	0.7443	1.667
-50.00	-58.00	2.19E-01	1.36E-02	16.3	0.339	5193	1.2404	3116	0.7443	1.667
0.00	32.00	1.79E-01	1.11E-02	18.6	0.388	5193	1.2404	3116	0.7443	1.667
50.00	122.00	1.51E-01	9.42E-03	20.8	0.435	5193	1.2404	3116	0.7443	1.667
100.00	212.00	1.31E-01	8.16E-03	23.0	0.480	5193	1.2404	3116	0.7443	1.667
150.00	302.00	1.15E-01	7.20E-03	25.1	0.524	5193	1.2404	3116	0.7443	1.667

200.00	392.00	1.03E-01	6.44E-03	27.2	0.567	5193	1.2404	3116	0.7443	1.667
250.00	482.00	9.32E-02	5 82E-03	29 2	0.609	5193	1.2404	3116	0.7443	1.867
300.00	572.00	8.51E-02	5.31E-03	31.1	0.649	5193	1.2404	3116	0.7443	1.667
350.00	662.00	7.83E-02	4.89E-03	33.0	0.689	5193	1.2404	3116	0.7443	1.667
400.00	752.00	7.25E-02	4.52E-03	34.8	0.728	5193	1.2404	3116	0.7443	1.667
450.00	842.00	6.75E-02	4.21E-03	36.7	0.765	5193	1.2404	3116	0.7443	1.667
500.00	932.00	6.31E-02	3.94E-03	38.4	0.802	5193	1.2404	3116	0.7443	1.667
600.00	1112.00	5.59E-02	3.49E-03	41.8	0.874	5193	1.2404	3116	0.7443	1.867
700.00	1292.00	5.01E-02	3.13E-03	45.1	0.942	5193	1.2404	3116	0.7443	1.667
800.00	1472.00	4.55E-02	2 84E-03	48.3	1 008	5193	1.2404	3116	0.7443	1.667
1000.00	1832.00	3.83E-02	2.39E-03	54.1	1.131	5193	1.2404	3116	0.7443	1.667
				Heptane-n C_7H_{16}						
98 43	209.17	3.48E+00	2.16E-01	7.6	0.159	1983	0.4736	1900	0.4537	1.044
100.00	212.00	3.44E+00	2.15E-01	7.3	0.153	1990	0.4752	1907	0.4554	1.044
150.00	302.00	2.98E+00	1.86E-01	8.3	0.174	2203	0.5262	2120	0.5064	1.039
200 00	392.00	2.63E+00	1.64E-01	9.3	0.194	2405	0.5743	2322	0.5545	1.036
250.00	482.00	2.37E+00	1 48E-01	10.2	0.214	2592	0.6192	2509	0.5994	1.033
300.00	572.00	2.15E+00	1.34E-01	11.2	0.233	2766	0.6607	2683	0.6409	1.031
350.00	662.00	1.97E+00	1.23E-01	12.1	0.252	2927	0.6991	2844	0.6793	1.029
400.00	752.00	1 82E+00	1.14E-01	12.9	0.270	3076	0.7346	2993	0.7148	1.028
450 00	842.00	1.70E+00	1.06E-01	13.8	0 288	3213	0.7673	3130	0 7475	1.027
500.00	932.00	1.58E+00	9.89E-02	14.6	0.305	3339	0.7976	3256	0.7778	1.025
				Hexane-n C_6H_{14}						
68.73	155.71	3 24E+00	2.02E-01	7.3	0 152	1856	0.4432	1759	0.0001	1 055
100.00	212.00	2.92E+00	1.83E-01	8.2	0.171	1994	0.4763	1898	0.0001	1.051
150.00	302.00	2.54E+00	1.59E-01	9.3	0.193	2209	0.5276	2112	0.0001	1.046
200.00	392.00	2.25E+00	1.41E-01	10.3	0.215	2411	0.5759	2315	0.0001	1.042
250.00	482.00	2.03E+00	1.27E-01	11.3	0 237	2601	0.6211	2504	0.0001	1.039

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity -μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft² x10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Hexane-n C_6H_{14} (continued)						
300.00	572.00	1.85E+00	1.15E-01	12.3	0.258	2776	0.6630	2680	0.0002	1.036
350.00	662.00	1.69E+00	1.06E-01	13.3	0.278	2938	0.7018	2842	0.0002	1.034
400.00	752.00	1.57E+00	9.78E-02	14.2	0.297	3088	0.7376	2992	0.0002	1.032
450.00	842.00	1.45E+00	9.07E-02	15.1	0.316	3227	0.7707	3130	0.0002	1.031
500.00	932.00	1.35E+00	6.43E-02	16.0	0.335	3355	0.8013	3258	0.0002	1.030
				Hydrogen-n (R 702) H_2						
-252.77	-422.99	1.33E+00	8.30E-02	1.1	0.022	14463	3.4545	10339	2.4694	1.399
-250.00	-418.00	1.12E+00	6.96E-02	1.3	0.026	14463	3.4545	10339	2.4694	1.399
-200.00	-328.00	3.37E-01	2.10E-02	3.4	0.070	14463	3.4545	10339	2.4694	1.399
-150.00	-238.00	1.99E-01	1.25E-02	4.8	0.101	14463	3.4545	10339	2.4694	1.399
-100.00	-148.00	1.42E-01	8.66E-03	6.1	0.127	14463	3.4546	10339	2.4695	1.399
-50.00	-56.00	1.10E-01	6.67E-03	7.3	0.152	14464	3.4546	10340	2.4696	1.399
0.00	32.00	8.99E-02	5.62E-03	8.3	0.174	14465	3.4549	10341	2.4699	1.399
50.00	122.00	7.60E-02	4.75E-03	9.4	0.196	14468	3.4556	10344	2.4706	1.399
100.00	212.00	6.58E-02	4.11E-03	10.4	0.216	14473	3.4569	10349	2.4718	1.399
150.00	302.00	5.81E-02	3.62E-03	11.3	0.236	14482	3.4589	10357	2.4738	1.398
200.00	392.00	5.19E-02	3.24E-03	12.2	0.255	14494	3.4616	10370	2.4788	1.398
250.00	482.00	4.70E-02	2.93E-03	13.1	0.273	14511	3.4658	10386	2.4807	1.397
300.00	572.00	4.29E-02	2.88E-03	14.0	0.291	14532	3.4710	10408	2.4859	1.396
350.00	662.00	3.94E-02	2.46E-03	14.8	0.309	14560	3.4775	10435	2.4925	1.395
400.00	752.00	3.65E-02	2.28E-03	15.6	0.326	14593	3.4854	10469	2.5004	1.394

450.00	842.00	3.40E-02	2.12E-03	16.4	0.343	14632	3.4948	10508	2.5098	1.392
500.00	932.00	3.18E-02	1.98E-03	17.2	0.359	14678	3.5058	10554	2.5207	1.391
600.00	1112.00	2.81E-02	1.76E-03	18.7	0.391	14789	3.5323	10665	2.5472	1.387
700.00	1292.00	2.52E-02	1.58E-03	20.2	0.422	14926	3.5651	10802	2.5801	1.382
800.00	1472.00	2.29E-02	1.43E-03	21.6	0.452	15090	3.6042	10966	2.6192	1.376
1000.00	1832.00	1.93E-02	1.20E-03	24.4	0.509	15493	3.7005	11369	2.7154	1.363

Hydrogen Chloride HCl

-85.00	-121.00	2.50E+00	1.56E-01	7.8	0.164	799	0.1908	571	0.1364	1.399
-50.00	-58.00	2.02E+00	1.26E-01	10.8	0.225	799	0.1908	571	0.1364	1.399
0.00	32.00	1.64E+00	1.02E-01	13.3	0.278	799	0.1908	571	0.1364	1.399
50.00	122.00	1.38E+00	8.62E-02	15.8	0.329	799	0.1908	571	0.1364	1.399
100.00	212.00	1.19E+00	7.46E-02	18.2	0.379	799	0.1909	571	0.1365	1.399
150.00	302.00	1.05E+00	6.57E-02	20.5	0.428	800	0.1911	572	0.1367	1.399
200.00	392.00	9.42E-01	5.88E-02	22.8	0.476	802	0.1915	574	0.1371	1.397
250.00	482.00	8.51E-01	5.32E-02	25.0	0.522	805	0.1922	577	0.1377	1.395
300.00	572.00	7.77E-01	4.85E-02	27.2	0.567	809	0.1931	581	0.1387	1.393
350.00	662.00	7.15E-01	4.46E-02	29.3	0.611	813	0.1943	585	0.1398	1.390
400.00	752.00	6.62E-01	4.13E-02	31.3	0.655	819	0.1957	591	0.1412	1.386
450.00	842.00	6.16E-01	3.84E-02	33.4	0.697	826	0.1972	598	0.1427	1.382
500.00	932.00	5.76E-01	3.60E-02	35.3	0.738	833	0.1989	605	0.1444	1.377

Mercury Hg

356.95	674.51	3.89E+00	2.43E-01	61.9	1.292	103.7	0.0248	62.23	0.0149	1.666
400.00	752.00	3.64E+00	2.27E-01	66.2	1.382	103.7	0.0248	62.23	0.0149	1.666
450.00	842.00	3.39E+00	2.11E-01	71.2	1.487	103.7	0.0248	62.23	0.0149	1.666
500.00	932.00	3.17E+00	1.98E-01	76.2	1.592	103.7	0.0248	62.23	0.0149	1.666
600.00	1112.00	2.60E+00	1.75E-01	86.2	1.800	103.7	0.0248	62.23	0.0149	1.666

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft² x10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Methane (R 50) CH_4						
-161.49	-258.68	1.82E+00	1.13E-01	4.5	0.094	2064	0.4929	1545	0.3691	1.335
-150.00	-238.00	1.63E+00	1.02E-01	5.7	0.119	2064	0.4930	1546	0.3692	1.335
-100.00	-148.00	1.14E+00	7.13E-02	7.3	0.153	2071	0.4946	1553	0.3708	1.334
-50.00	-58.00	8.80E-01	5.49E-02	8.9	0.185	2103	0.5024	1585	0.3786	1.327
0.00	32.00	7.17E-01	4.48E-02	10.4	0.217	2175	0.5196	1657	0.3958	1.313
50.00	122.00	6.06E-01	3.78E-02	11.9	0.248	2268	0.5466	1770	0.4228	1.293
100.00	212.00	5.23E-01	3.26E-02	13.3	0.278	2434	0.5814	1916	0.4577	1.270
150.00	302.00	4.62E-01	2.88E-02	14.7	0.307	2603	0.6218	2085	0.4980	1.249
200.00	392.00	4.13E-01	2.58E-02	16.0	0.335	2786	0.6653	2267	0.5415	1.229
250.00	482.00	3.74E-01	2.33E-02	17.4	0.363	2974	0.7104	2456	0.5866	1.211
300.00	572.00	3.41E-01	2.13E-02	18.6	0.389	3164	0.7557	2646	0.6319	1.196
350.00	662.00	3.14E-01	1.96E-02	19.9	0.415	3351	0.8004	2833	0.6766	1.183
400.00	752.00	2.90E-01	1.81E-02	21.1	0.440	3533	0.8439	3015	0.7201	1.172
450.00	842.00	2.70E-01	1.69E-02	22.2	0.464	3709	0.8858	3190	0.7620	1.162
500.00	932.00	2.53E-01	1.58E-02	23.3	0.487	3877	0.9259	3358	0.8021	1.154
600.00	1112.00	2.24E-01	1.40E-02	25.4	0.531	4189	1.0006	3671	0.8769	1.141
700.00	1292.00	2.01E-01	1.25E-02	27.4	0.571	4471	1.0680	3953	0.9442	1.131
800.00	1472.00	1.82E-01	1.14E-02	29.1	0.608	4724	1.1284	4206	1.0046	1.123
1000.00	1832.00	1.54E-01	9.59E-03	32.2	0.672	5155	1.2312	4637	1.1075	1.112

Methanol CH_4O

64.70	148.46	1.22E+00	7.63E-02	11.4	0.710	1456	0.3478	1197	0.2858	1.217
100.00	212.00	1.07E+00	6.65E-02	12.2	0.763	1539	0.3677	1280	0.3057	1.203
150.00	302.00	9.32E-01	5.82E-02	13.9	0.870	1663	0.3972	1403	0.3352	1.185
200.00	392.00	8.30E-01	5.18E-02	15.6	0.975	1788	0.4270	1528	0.3650	1.170
250.00	482.00	7.49E-01	4.67E-02	17.3	1.079	1911	0.4563	1651	0.3943	1.157
300.00	572.00	6.83E-01	4.26E-02	18.9	1.182	2029	0.4845	1769	0.4225	1.147
350.00	662.00	6.27E-01	3.92E-02	20.6	1.283	2141	0.5113	1881	0.4493	1.138
400.00	752.00	5.81E-01	3.63E-02	22.2	1.384	2247	0.5366	1987	0.4746	1.131
450.00	842.00	5.41E-01	3.37E-02	23.8	1.484	2346	0.5603	2086	0.4983	1.124
500.00	932.00	5.06E-01	3.16E-02	25.3	1.583	2439	0.5825	2180	0.5206	1.119

Methyl Chloride (R 40) CH_3Cl

-24.22	-11.60	2.55E+00	1.59E-01	9.3	0.193	744	0.1778	580	0.1384	1.284
0.00	32.00	2.31E+00	1.44E-01	10.0	0.210	774	0.1849	610	0.1456	1.270
50.00	122.00	1.93E+00	1.20E-01	11.8	0.247	842	0.2012	678	0.1618	1.243
100.00	212.00	1.66E+00	1.04E-01	13.6	0.284	914	0.2183	749	0.1789	1.220
150.00	302.00	1.46E+00	9.12E-02	15.3	0.320	985	0.2353	821	0.1960	1.201
200.00	392.00	1.30E+00	8.15E-02	17.0	0.356	1054	0.2519	890	0.2125	1.185
250.00	482.00	1.18E+00	7.36E-02	18.7	0.391	1120	0.2676	956	0.2282	1.172
300.00	572.00	1.08E+00	6.71E-02	20.4	0.426	1182	0.2823	1017	0.2430	1.162
350.00	662.00	9.89E-01	6.17E-02	22.0	0.460	1240	0.2961	1075	0.2567	1.153
400.00	752.00	9.15E-01	5.71E-02	23.7	0.494	1293	0.3089	1129	0.2695	1.146
450.00	842.00	8.51E-01	5.31E-02	25.3	0.528	1343	0.3208	1178	0.2814	1.140
500.00	932.00	7.96E-01	4.97E-02	26.9	0.561	1389	0.3318	1225	0.2925	1.134

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				
						Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft²x10⁸	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Neon (R 720) Ne						
-246.05	-410.89	9.56E+00	5.97E-01	4.7	0.097	1030	0.2460	618	0.1476	1.667
-200.00	-328.00	3.38E+00	2.11E-01	11.5	0.239	1030	0.2460	618	0.1476	1.667
-150.00	-238.00	2.00E+00	1.25E-01	17.0	0.355	1030	0.2460	618	0.1476	1.667
-100.00	-148.00	1.42E+00	8.87E-02	21.7	0.453	1030	0.2460	618	0.1476	1.667
-50.00	-58.00	1.10E+00	6.86E-02	25.9	0.541	1030	0.2460	618	0.1476	1.667
0.00	32.00	9.00E-01	5.62E-02	29.8	0.622	1030	0.2460	618	0.1476	1.667
50.00	122.00	7.61E-01	4.75E-02	33.4	0.698	1030	0.2460	618	0.1476	1.667
100.00	212.00	6.59E-01	4.11E-02	36.8	0.769	1030	0.2460	618	0.1476	1.667
150.00	302.00	5.81E-01	3.63E-02	40.1	0.837	1030	0.2460	618	0.1476	1.667
200.00	392.00	5.20E-01	3.24E-02	43.2	0.902	1030	0.2460	618	0.1476	1.667
250.00	482.00	4.70E-01	2.93E-02	46.2	0.964	1030	0.2460	618	0.1476	1.667
300.00	572.00	4.29E-01	2.68E-02	49.1	1.025	1030	0.2460	618	0.1476	1.667
350.00	662.00	3.95E-01	2.46E-02	51.9	1.083	1030	0.2460	618	0.1476	1.667
400.00	752.00	3.65E-01	2.28E-02	54.6	1.140	1030	0.2460	618	0.1476	1.667
450.00	842.00	3.40E-01	2.12E-02	57.2	1.195	1030	0.2460	618	0.1476	1.667
500.00	932.00	3.18E-01	1.99E-02	59.8	1.249	1030	0.2460	618	0.1476	1.667
600.00	1112.00	2.82E-01	1.76E-02	64.8	1.353	1030	0.2460	618	0.1476	1.667
700.00	1292.00	2.53E-01	1.58E-02	69.6	1.453	1030	0.2460	618	0.1476	1.667
800.00	1472.00	2.29E-01	1.43E-02	74.2	1.550	1030	0.2460	618	0.1476	1.667
1000.00	1632.00	1.93E-01	1.21E-02	63.0	1.733	1030	0.2460	618	0.1476	1.667

Nitrogen (R 728) N_2

-195.80	-320.44	4.61E+00	2.88E-01	5.4	0.112	1039	0.2481	742	0.1772	1.400
-150.00	-238.00	2.80E+00	1.75E-01	8.4	0.174	1039	0.2481	742	0.1772	1.400
-100.00	-148.00	1.98E+00	1.24E-01	11.4	0.238	1039	0.2481	742	0.1772	1.400
-50.00	-58.00	1.53E+00	9.56E-02	14.1	0.294	1039	0.2481	742	0.1772	1.400
0.00	32.00	1.25E+00	7.81E-02	16.6	0.346	1039	0.2481	742	0.1772	1.400
50.00	122.00	1.06E+00	6.60E-02	18.9	0.394	1039	0.2482	743	0.1774	1.400
100.00	212.00	9.15E-01	5.71E-02	21.0	0.439	1041	0.2487	744	0.1778	1.399
150.00	302.00	8.07E-01	5.04E-02	23.1	0.482	1045	0.2495	748	0.1787	1.397
200.00	392.00	7.21E-01	4.50E-02	25.0	0.522	1051	0.2510	754	0.1801	1.394
250.00	482.00	6.52E-01	4.07E-02	26.8	0.561	1059	0.2529	762	0.1820	1.390
300.00	572.00	5.95E-01	3.72E-02	28.6	0.598	1068	0.2552	772	0.1843	1.385
350.00	862.00	5.48E-01	3.42E-02	30.3	0.633	1079	0.2578	782	0.1869	1.379
400.00	752.00	5.07E-01	3.16E-02	32.0	0.667	1091	0.2606	794	0.1897	1.374
450.00	842.00	4.72E-01	2.95E-02	33.5	0.701	1103	0.2635	806	0.1926	1.368
500.00	932.00	4.41E-01	2.76E-02	35.1	0.733	1115	0.2664	818	0.1955	1.363
600.00	1112.00	3.91E-01	2.44E-02	38.0	0.794	1139	0.2720	842	0.2011	1.352
700.00	1292.00	3.50E-01	2.19E-02	40.8	0.853	1161	0.2773	864	0.2064	1.343
800.00	1472.00	3.18E-01	1.99E-02	43.5	0.909	1181	0.2620	884	0.2112	1.336
1000.00	1832.00	2.68E-01	1.67E-02	48.6	1.015	1215	0.2901	918	0.2192	1.323

Nitrous Oxide (R 744A) N_2O

-88.48	-127.26	3.11E+00	1.94E-01	9.44	0.197	748	0.1788	559	0.1336	1.338
-50.00	-58.00	2.44E+00	1.52E-01	11.0	0.230	793	0.1894	604	0.1443	1.313
0.00	32.00	1.98E+00	1.23E-01	13.5	0.282	849	0.2028	660	0.1576	1.286
50.00	122.00	1.67E+00	1.04E-01	15.9	0.332	900	0.2150	711	0.1699	1.266
100.00	212.00	1.44E+00	9.00E-02	18.2	0.380	947	0.2261	758	0.1810	1.249
150.00	302.00	1.27E+00	7.93E-02	20.4	0.425	988	0.2359	799	0.1908	1.237
200.00	392.00	1.13E+00	7.08E-02	22.4	0.469	1024	0.2446	835	0.1995	1.226
250.00	482.00	1.03E+00	6.41E-02	24.4	0.510	1057	0.2524	868	0.2073	1.218
300.00	572.00	9.36E-01	5.85E-02	26.3	0.550	1086	0.2594	897	0.2143	1.211
350.00	662.00	8.61E-01	5.38E-02	28.2	0.588	1112	0.2657	923	0.2205	1.205

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·sx10⁶	lbf-sec/ft²x10⁶	J/kg·K	Btu/lbm·°R	J/kg·K	Btu/lbm·°R	$k = c_p/c_v$
				Nitrous Oxide (R 744A) N_2O (continued)						
400.00	752.00	7.97E-01	4.98E-02	29.9	0.625	1136	0.2713	947	0.2262	1.199
450.00	842.00	7.42E-01	4.83E-02	31.6	0.660	1157	0.2764	968	0.2313	1.195
500.00	932.00	6.94E-01	4.33E-02	33.2	0.694	1177	0.2810	988	0.2359	1.191
				Octane-n C_8H_{18}						
125.88	258 22	3.75E+00	2.34E-01	7.4	0.461	2097	0.5010	2025	0.4836	1.036
150.00	302.00	3.45E+00	2.15E-01	7.7	0.480	2200	0.5254	2127	0.5080	1.034
200.00	392.00	3.03E+00	1.89E-01	8.6	0.536	2401	0.5734	2328	0.5560	1.031
250.00	482.00	2.71E+00	1.69E-01	9.5	0.593	2588	0.6181	2515	0.6007	1.029
300.00	572.00	2.46E+00	1.54E-01	10.4	0.650	2761	0.6594	2688	0.6420	1.027
350.00	662.00	2.26E+00	1.41E-01	11.3	0.707	2920	0.6975	2848	0.6801	1.026
400.00	752.00	2.08E+00	1.30E-01	12.2	0.765	3068	0.7327	2995	0.7153	1.024
450.00	842.00	1.93E+00	1.20E-01	13.2	0.823	3203	0.7651	3131	0.7477	1.023
500.00	932.00	1.79E+00	1.12E-01	14.1	0.881	3329	0.7951	3256	0.7777	1.022

Oxygen (R 732) O_2

-182.96	-297.33	4.48E+00	2.79E-01	7.0	0.146	909	0.2172	649	0.1551	1.400
-150.00	-238.00	3.21E+00	2.00E-01	9.4	0.196	909	0.2172	649	0.1551	1.400
-100.00	-146.00	2.26E+00	1.41E-01	12.9	0.270	909	0.2172	649	0.1551	1.400
-50.00	-58.00	1.75E+00	1.09E-01	16.1	0.336	910	0.2174	650	0.1553	1.400
0.00	32.00	1.43E+00	8.92E-02	19.0	0.397	914	0.2182	654	0.1561	1.397
50.00	122.00	1.21E+00	7.54E-02	21.7	0.454	921	0.2200	661	0.1579	1.393
100.00	212.00	1.05E+00	6.53E-02	24.3	0.508	933	0.2227	673	0.1607	1.366
150.00	302.00	9.22E-01	5.75E-02	26.7	0.558	947	0.2261	687	0.1641	1.378
200.00	392.00	8.24E-01	5.14E-02	29.0	0.606	962	0.2298	702	0.1678	1.370
250.00	482.00	7.45E-01	4.65E-02	31.2	0.652	978	0.2336	718	0.1716	1.362
300.00	572.00	6.80E-01	4.25E-02	33.3	0.696	994	0.2373	734	0.1753	1.354
350.00	662.00	6.26E-01	3.91E-02	35.4	0.739	1009	0.2409	749	0.1788	1.347
400.00	752.00	5.79E-01	3.62E-02	37.4	0.780	1023	0.2442	763	0.1822	1.341
450.00	842.00	5.39E-01	3.37E-02	39.3	0.820	1035	0.2473	776	0.1852	1.335
500.00	932.00	5.04E-01	3.15E-02	41.1	0.859	1047	0.2501	787	0.1881	1.330
600.00	1112.00	4.46E-01	2.79E-02	44.7	0.933	1068	0.2551	808	0.1931	1.321
700.00	1292.00	4.01E-01	2.50E-02	48.1	1.004	1085	0.2593	826	0.1972	1.315
800.00	1472.00	3.63E-01	2.27E-02	51.3	1.072	1100	0.2627	840	0.2007	1.309
1000.00	1632.00	3.06E-01	1.91E-02	57.5	1.201	1122	0.2681	863	0.2060	1.301

Pentane-n C_5H_{12}

36.07	96.93	2.96E+00	1.85E-01	7.0	0.146	1712	0.4090	1597	0.3815	1.072
50.00	122.00	2.83E+00	1.76E-01	7.6	0.159	1774	0.4236	1658	0.3961	1.069
100.00	212.00	2.41E+00	1.51E-01	8.7	0.183	1995	0.4766	1680	0.4491	1.061
150.00	302.00	2.11E+00	1.32E-01	9.8	0.205	2214	0.5287	2098	0.5012	1.055
200.00	392.00	1.88E+00	1.17E-01	10.9	0.227	2422	0.5785	2307	0.5510	1.050
250.00	482.00	1.69E+00	1.06E-01	11.9	0.248	2618	0.6253	2503	0.5978	1.046
300.00	572.00	1.54E+00	9.63E-02	12.8	0.268	2801	0.6690	2686	0.8414	1.043
350.00	662.00	1.41E+00	8.82E-02	13.8	0.288	2970	0.7095	2855	0.6819	1.040
400.00	752.00	1.31E+00	8.15E-02	14.7	0.306	3127	0.7469	3012	0.7194	1.038
450.00	842.00	1.22E+00	7.59E-02	15.5	0.325	3272	0.7815	3157	0.7540	1.037
500.00	932.00	1.14E+00	7.10E-02	16.4	0.342	3406	0.8135	3291	0.7860	1.035

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa·s x10⁶	lbf-sec/ft² x10⁶	J/kg K	Btu/lbm-°R	J/kg K	Btu/lbm-°R	$k = c_p/c_v$
				Potassium K						
766.68	1394.02	5.02E-01	3.14E-02	16.8	0.351	532	0.1270	319	0.0762	1.666
800.00	1472.00	4.73E-01	2.95E-02	17.7	0.370	532	0.1270	319	0.0762	1.666
1000.00	1832.00	3.83E-01	2.39E-02	22.3	0.465	532	0.1270	319	0.0762	1.666
1200.00	2192.00	3.28E-01	2.05E-02	26.3	0.548	532	0.1271	319	0.0763	1.666
				Propane (R 290) C_3H_8						
-42.04	-43.67	2.42E+00	1.51E-01	5.9	0.123	1384	0.3305	1195	0.2855	1.158
0.00	32.00	2.01E+00	1.26E-01	7.6	0.159	1558	0.3722	1370	0.3271	1.138
50.00	122.00	1.68E+00	1.05E-01	8.9	0.185	1782	0.4257	1594	0.3806	1.118
100.00	212.00	1.45E+00	9.05E-02	10.1	0.211	2009	0.4800	1821	0.4349	1.104
150.00	302.00	1.28E+00	7.96E-02	11.3	0.236	2231	0.5329	2042	0.4878	1.092
200.00	392.00	1.14E+00	7.11E-02	12.5	0.260	2442	0.5833	2254	0.5383	1.084
250.00	482.00	1.03E+00	6.42E-02	13.6	0.285	2641	0.6307	2452	0.5657	1.077
300.00	572.00	9.38E-01	5.86E-02	14.8	0.309	2826	0.6750	2637	0.6299	1.071
350.00	662.00	8.63E-01	5.39E-02	15.9	0.332	2998	0.7161	2810	0.6711	1.067
400.00	752.00	7.98E-01	4.98E-02	17.0	0.356	3158	0.7543	2970	0.7093	1.063
450.00	842.00	7.43E-01	4.64E-02	18.1	0.378	3306	0.7897	3118	0.7447	1.060
500.00	932.00	6.95E-01	4.34E-02	19.2	0.401	3444	0.8226	3256	0.7776	1.058
600.00	1112.00	6.05E-01	3.78E-02	21.3	0.444	3691	0.8816	3502	0.8366	1.054
800.00	1472.00	5.00E-01	3.12E-02	25.2	0.527	4092	0.9774	3904	0.9324	1.048
1000.00	1832.00	4.22E-01	2.63E-02	28.8	0.603	4402	1.0514	4213	1.0063	1.045

Propylene (R 1270) C_3H_6

-47.69	-53.84	2.36E+00	1.47E-01	6.7	0.139	1275	0.3045	1077	0.2573	1.183
0.00	32.00	1.92E+00	1.20E-01	7.9	0.165	1430	0.3415	1232	0.2943	1.160
50.00	122.00	1.61E+00	1.00E-01	9.4	0.195	1611	0.3848	1413	0.3376	1.140
100.00	212.00	1.38E+00	8.64E-02	10.8	0.225	1798	0.4293	1600	0.3821	1.123
150.00	302.00	1.22E+00	7.60E-02	12.1	0.252	1981	0.4731	1783	0.4259	1.111
200.00	392.00	1.09E+00	6.79E-02	13.4	0.279	2157	0.5151	1959	0.4679	1.101
250.00	482.00	9.83E-01	6.14E-02	14.6	0.305	2322	0.5547	2125	0.5075	1.093
300.00	572.00	8.97E-01	5.60E-02	15.8	0.329	2477	0.5917	2280	0.5445	1.087
350.00	662.00	8.24E-01	5.15E-02	16.9	0.353	2622	0.6262	2424	0.5790	1.082
400.00	752.00	7.63E-01	4.76E-02	18.0	0.375	2756	0.6582	2558	0.6110	1.077
450.00	842.00	7.10E-01	4.43E-02	19.0	0.397	2880	0.6879	2683	0.6407	1.074
500.00	932.00	6.64E-01	4.14E-02	20.0	0.418	2995	0.7154	2798	0.6683	1.071

Sodium Na

881.45	1618.61	2.70E-01	1.68E-02	26.7	0.558	886	0.2117	525	0.1253	1.689
1000.00	1832.00	2.32E-01	1.45E-02	28.1	0.587	886	0.2117	525	0.1253	1.689
1100.00	2012.00	1.94E-01	1.21E-02	29.6	0.618	886	0.2117	525	0.1253	1.689
1200.00	2192.00	1.94E-01	1.21E-02	31.2	0.651	886	0.2117	525	0.1253	1.689
1300.00	2372.00	1.81E-01	1.13E-02	32.8	0.686	886	0.2117	525	0.1253	1.689
1400.00	2552.00	1.69E-01	1.06E-02	34.6	0.723	886	0.2117	525	0.1254	1.689
1500.00	2732.00	1.59E-01	9.94E-03	36.5	0.762	887	0.2118	525	0.1254	1.689

Sulfur Dioxide (R 764) SO_2

-10.02	13.96	3.08E+00	1.93E-01	11.9	0.248	600	0.1432	470	0.1122	1.276
0.00	32.00	2.97E+00	1.85E-01	12.0	0.252	606	0.1448	476	0.1138	1.272
50.00	122.00	2.51E+00	1.56E-01	14.0	0.293	638	0.1524	508	0.1214	1.255
100.00	212.00	2.17E+00	1.35E-01	16.2	0.338	668	0.1595	538	0.1285	1.241
150.00	302.00	1.91E+00	1.19E-01	18.3	0.382	694	0.1658	565	0.1348	1.230

Continued on next page.

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure				Ratio
						Constant Pressure c_p		Constant Volume c_v		
°C	°F	kg/m³	lbm/ft³	Pa-sx10⁶	lbf-sec/ft²x10⁶	J/kg K	Btu/lbm-°R	J/kg K	Btu/lbm-°R	$k = c_p/c_v$
			Sulfur Dioxide (R 764) SO_2(continued)							
200.00	392 00	1 70E+00	1.06E-01	20.3	0.424	718	0.1715	588	0.1405	1.221
250.00	482 00	1.54E+00	9.58E-02	22.3	0.465	739	0.1764	609	0.1454	1.213
300.00	572 00	1 40E+00	8.73E-02	24 2	0 504	757	0.1808	627	0.1498	1.207
350.00	662 00	1.28E+00	8.01E-02	26.0	0.543	773	0.1846	643	0.1536	1.202
400.00	752.00	1.19E+00	7.40E-02	27.8	0 580	787	0.1860	657	0.1570	1.197
450.00	842 00	1.10E+00	6.88E-02	29.5	0.617	800	0.1910	670	0.1600	1.194
500.00	932.00	1.03E+00	6.42E-02	31.2	0 652	811	0.1937	681	0.1627	1.191
600.00	1112.00	9.07E-01	5.66E-02	34.5	0 720	830	0.1982	700	0.1672	1.185
700 00	1292 00	8 11E-01	5.06E-02	37.6	0 786	845	0.2019	716	0.1709	1.181
800.00	1472.00	7.33E-01	4.57E-02	40.6	0.849	858	0.2050	728	0.1740	1 178
1000 00	1832 00	6.13E-01	3.82E-02	46.3	0.968	878	0.2097	748	0.1787	1.174

Sulfur Trioxide SO_3

44.75	112.55	3.07E+00	1.92E-01	15.3	0 320	655	0.1564	551	0 1315	1 189
50.00	122.00	3.02E+00	1.88E-01	15 4	0.322	660	0.1576	556	0.1328	1.187
100.00	212.00	2.61E+00	1.63E-01	17.1	0.356	705	0.1683	601	0.1435	1.173
150.00	302.00	2.31E+00	1.44E-01	19.1	0.400	743	0.1775	639	0.1527	1.162
200.00	392.00	2.06E+00	1 29E-01	21.2	0.442	776	0.1853	672	0 1605	1.155
250.00	482.00	1.86E+00	1.16E-01	23.2	0.485	804	0.1921	700	0.1672	1.148
300.00	572.00	1 70E+00	1.06E-01	25.2	0.526	828	0.1978	724	0.1730	1.143
350.00	662.00	1 57E+00	9.77E-02	27.1	0.567	849	0.2029	746	0.1781	1.139
400.00	752.00	1 45E+00	9.05E-02	29 0	0.607	868	0.2072	764	0.1824	1.136
450.00	842.00	1.35E+00	8.42E-02	30.9	0.646	884	0.2111	780	0 1863	1.133
500.00	932 00	1.26E+00	7.68E-02	32.8	0 684	898	0.2145	794	0.1897	1.131
600.00	1112.00	1.12E+00	6.98E-02	36.4	0.759	922	0.2202	818	0.1954	1.127
700.00	1292.00	1.00E+00	6.26E-02	39.8	0 831	941	0.2248	637	0.2000	1.124
800.00	1472.00	9.09E-01	5.66E-02	43.1	0 901	957	0.2286	853	0 2037	1.122
1000.00	1832.00	7.66E-01	4.78E-02	49.3	1 030	981	0.2343	877	0.2095	1.118

Trichlorofluoromethane (R 11) CCl_3F

23.82	74.88	5.91E+00	3.69E-01	10.7	0.223	571	0.1365	511	0.1220	1 118
50 00	122.00	5 31E+00	3.32E-01	12.4	0.256	591	0.1413	531	0.1268	1 114
100.00	212.00	4.56E+00	2.85E-01	14.3	0 299	623	0.1488	563	0.1344	1.108
150.00	302.00	4 00E+00	2.50E-01	16.2	0.338	648	0 1548	588	0.1404	1 103
200.00	392.00	3.57E+00	2.23E-01	18 1	0.378	668	0 1596	608	0.1452	1.100
250 00	482.00	3.21E+00	2.01E-01	19.9	0.416	685	0.1635	624	0.1491	1.097
300.00	572.00	2 92E+00	1.82E-01	21 8	0.455	698	0.1668	638	0.1523	1 095
350.00	662 00	2 69E+00	1 68E-01	23 6	0.493	709	0 1695	649	0.1550	1 093
400.00	752 00	2.49E+00	1 55E-01	25 4	0.531	719	0 1717	659	0.1573	1.092
450 00	842.00	2.31E+00	1.45E-01	27.2	0.569	727	0 1737	667	0.1592	1 091
500.00	932.00	2.17E+00	1 35E-01	29.0	0.606	734	0.1754	674	0.1609	1.090

Table D.2 Properties of Selected Gases (*Continued*)

Temperature t		Gas Density - ρ At One Atmosphere		Gas Viscosity - μ At One Atmosphere		Ideal Gas Specific Heats at Zero Pressure: Constant Pressure c_p		Constant Volume c_v		Ratio
°C	°F	kg/m³	lbm/ft³	Pa·sx10⁶	lbf-sec/ft²x10⁶	J/kg·K	Btu/lbm-°R	J/kg·K	Btu/lbm-°R	$k = c_p/c_v$
				Water (R 718) H_2O						
100.00	212.00	5.98E-01	3.73E-02	12.3	0.256	1890	0.4514	1429	0.3412	1.323
150.00	302.00	5.26E-01	3.29E-02	14.2	0.296	1912	0.4566	1450	0.3464	1.318
200.00	392.00	4.66E-01	2.91E-02	16.2	0.338	1937	0.4627	1476	0.3524	1.313
250.00	482.00	4.21E-01	2.63E-02	18.2	0.381	1965	0.4694	1504	0.3592	1.307
300.00	572.00	3.84E-01	2.40E-02	20.3	0.424	1996	0.4767	1534	0.3664	1.301
350.00	662.00	3.53E-01	2.20E-02	22.4	0.467	2028	0.4843	1566	0.3741	1.295
400.00	752.00	3.27E-01	2.04E-02	24.5	0.511	2061	0.4923	1600	0.3821	1.289
450.00	842.00	3.04E-01	1.90E-02	26.5	0.554	2095	0.5005	1634	0.3903	1.282
500.00	932.00	2.84E-01	1.77E-02	28.6	0.597	2130	0.5088	1669	0.3986	1.277
600.00	1112.00	2.52E-01	1.57E-02	32.6	0.681	2201	0.5256	1739	0.4153	1.265
700.00	1292.00	2.26E-01	1.41E-02	36.6	0.763	2270	0.5423	1809	0.4321	1.255
800.00	1472.00	2.05E-01	1.28E-02	40.4	0.843	2339	0.5567	1878	0.4485	1.246
1000.00	1832.00	1.72E-01	1.08E-02	48.7	1.018	2471	0.5901	2009	0.4799	1.230

Table D.3 Density and Viscosity of Steam and Compressed Water

t °F	ρ lbm/ft³	Z = pv/RT	μ lbf-sec/ft²	ν ft²/sec	t °F	ρ lbm/ft³	Z = pv/RT	μ lbf-sec/ft²	ν ft²/sec
	1 psia					10 psia			
32	62.42	5.472E-05	3.737E-05	1.926E-05	32	62.42	5.472E-04	3.737E-05	1.926E-05
100	62.00	4.841E-05	1.423E-05	7.385E-06	100	61.35	4.892E-04	1.432E-05	7.510E-06
(101.74)	61.96	4.828E-05	1.398E-05	7.260E-06	(193.21)	60.28	4.268E-04	6.604E-06	3.525E-06
(101.74)	0.002998	0.9980	2.145E-07	2.302E-03	(193.21)	0.02603	0.9884	2.490E-07	3.078E-04
200	0.002548	0.9993	2.528E-07	3.192E-03	200	0.02575	0.9889	2.518E-07	3.147E-04
300	0.002211	1.0000	2.963E-07	4.312E-03	300	0.02223	0.9945	2.957E-07	4.279E-04
400	0.001954	1.0001	3.422E-07	5.636E-03	400	0.01960	0.9970	3.418E-07	5.612E-04
600	0.001585	1.0003	4.374E-07	8.881E-03	600	0.01587	0.9990	4.373E-07	8.868E-04
800	0.001333	1.0004	5.338E-07	1.289E-02	800	0.01334	0.9997	5.337E-07	1.288E-03
1000	0.001150	1.0004	6.288E-07	1.759E-02	1000	0.01151	1.0000	6.289E-07	1.759E-03
1200	0.001012	1.0004	7.216E-07	2.295E-02	1200	0.01012	1.0002	7.216E-07	2.295E-03
1400	0.000903	1.0005	8.115E-07	2.892E-02	1400	0.00903	1.0003	8.116E-07	2.892E-03
	14.696 psia					50 psia			
32	62.42	8.042E-04	3.737E-05	1.926E-05	32	62.42	0.002736	3.736E-05	1.926E-05
100	62.00	7.114E-04	1.432E-05	7.432E-06	100	62.00	0.002420	1.423E-05	7.385E-06
200	60.10	6.226E-04	6.343E-06	3.396E-06	200	60.13	0.002117	6.337E-06	3.391E-06
(212.00)	59.81	6.144E-04	5.885E-06	3.166E-06	(281.02)	57.90	0.001958	4.144E-06	2.303E-06
(212.00)	0.03733	0.9845	2.565E-07	2.211E-04	(281.02)	0.1175	0.9653	2.844E-07	7.791E-05
300	0.03277	0.9916	2.954E-07	2.901E-04	300	0.1140	0.9694	2.931E-07	8.269E-05
400	0.02884	0.9954	3.417E-07	3.812E-04	400	0.09938	0.9829	3.403E-07	1.102E-04
600	0.02333	0.9983	4.373E-07	6.030E-04	600	0.07981	0.9929	4.368E-07	1.761E-04
800	0.01961	0.9993	5.337E-07	8.757E-04	800	0.06690	0.9965	5.336E-07	2.566E-04
1000	0.01691	0.9999	6.289E-07	1.196E-03	1000	0.05764	0.9982	6.290E-07	3.511E-04
1200	0.01487	1.0001	7.217E-07	1.562E-03	1200	0.05064	0.9991	7.219E-07	4.586E-04
1400	0.01327	1.0003	8.116E-07	1.968E-03	1400	0.04517	0.9996	8.118E-07	5.782E-04
	100 psia					200 psia			
32	62.42	0.005472	3.734E-05	1.925E-05	32	62.46	0.010938	3.731E-05	1.922E-05
100	62.00	0.004841	1.423E-05	7.385E-06	100	62.03	0.009675	1.423E-05	7.380E-06
200	60.13	0.004234	6.339E-06	3.392E-06	200	60.13	0.008468	6.343E-06	3.394E-06
300	57.31	0.003858	3.831E-06	2.151E-06	300	57.34	0.007712	3.834E-06	2.151E-06
(327.82)	56.37	0.003784	3.437E-06	1.962E-06	(381.80)	54.38	0.007341	2.877E-06	1.702E-06
(327.82)	0.2257	0.9450	3.034E-07	4.325E-05	(381.80)	0.4372	0.9131	3.254E-07	2.395E-05
400	0.2026	0.9641	3.385E-07	5.375E-05	400	0.4238	0.9221	3.346E-07	2.540E-05
600	0.1609	0.9852	4.362E-07	8.724E-05	600	0.3270	0.9695	4.350E-07	4.280E-05
800	0.1344	0.9924	6.291E-07	1.507E-04	800	0.2709	0.9844	5.335E-07	6.336E-05
1000	0.1155	0.9959	6.291E-07	1.752E-04	1000	0.2321	0.9913	6.295E-07	8.725E-05
1200	0.1014	0.9978	7.221E-07	2.291E-04	1200	0.2034	0.9951	7.227E-07	1.143E-04
1400	0.09042	0.9989	8.122E-07	2.890E-04	1400	0.1811	0.9972	8.129E+00	1.444E+03
	300 psia					400 psia			
32	62.46	0.01641	3.727E-05	1.920E-05	32	62.50	0.02186	3.724E-05	1.917E-05
100	62.03	0.01451	1.423E-05	7.380E-06	100	62.07	0.01934	1.423E-05	7.376E-06
200	60.17	0.01269	6.346E-06	3.393E-06	200	60.17	0.01693	6.350E-06	3.396E-06
300	57.34	0.01157	3.838E-06	2.154E-06	300	57.37	0.01541	3.841E-06	2.154E-06
400	53.68	0.01092	2.730E-06	1.636E-06	400	53.71	0.01455	2.733E-06	1.637E-06
(417.35)	52.94	0.01085	2.601E-06	1.581E-06	(444.60)	51.71	0.01437	2.424E-06	1.508E-06
(417.35)	0.6482	0.8863	3.399E-07	1.687E-05	(444.60)	0.8613	0.8625	3.513E-07	1.312E-05
600	0.4989	0.9531	4.339E-07	2.798E-05	600	0.6774	0.9360	4.328E-07	2.056E-05
800	0.4097	0.9763	5.333E-07	4.188E-05	800	0.5509	0.9680	5.332E-07	3.114E-05
1000	0.3498	0.9867	6.298E-07	5.792E-05	1000	0.4686	0.9821	6.302E-07	4.327E-05
1200	0.3059	0.9924	7.233E-07	7.607E-05	1200	0.4090	0.9897	7.240E-07	5.695E-05
1400	0.2721	0.9956	8.136E-07	9.619E-05	1400	0.3634	0.9940	8.143E-07	7.209E-05

Table D.3 Density and Viscosity of Steam and Compressed Water (*Continued*)

t °F	ρ lbm/ft³	Z = pv/RT	μ lbf-sec/ft³	ν ft²/sec	t °F	ρ lbm/ft³	Z = pv/RT	μ lbf-sec/ft²	ν ft²/sec
500 psia					1000 psia				
32	62.54	0.02731	3.721E-05	1.914E-05	32	62.62	0.05455	3.705E-05	1.904E-05
100	62.07	0.02417	1.423E-05	7.376E-06	100	62.19	0.04826	1.423E-05	7.362E-06
200	60.20	0.02114	6.354E-06	3.396E-06	200	60.31	0.04221	6.372E-06	3.399E-06
300	57.41	0.01926	3.846E-06	2.155E-06	300	57.54	0.03842	3.862E-06	2.160E-06
400	53.73	0.01818	2.736E-06	1.638E-06	400	53.91	0.03624	2.753E-06	1.643E-06
(467.01)	50.63	0.01790	2.294E-06	1.458E-06	(544.58)	46.32	0.03611	1.912E-06	1.328E-06
(467.01)	1.078051	0.8406	3.608E-07	1.077E-05	(544.58)	2.238	0.7474	3.971E-07	5.710E-06
600	0.86326	0.9180	4.318E-07	1.609E-05	600	1.947	0.8142	4.284E-07	7.081E-06
800	0.694589	0.9598	5.332E-07	2.470E-05	800	1.455	0.9167	5.341E-07	1.181E-05
1000	0.588512	0.9776	6.307E-07	3.448E-05	1000	1.206	0.9544	6.335E-07	1.691E-05
1200	0.512636	0.9870	7.246E-07	4.548E-05	1200	1.039	0.9737	7.284E-07	2.255E-05
1400	0.455021	0.9924	8.151E-07	5.763E-05	1400	0.91735	0.9845	8.192E-07	2.873E-05
1500 psia					2000 psia				
32	62.74	0.08168	3.690E-05	1.892E-05	32	62.85	0.1087	3.675E-05	1.881E-05
100	62.27	0.07229	1.424E-05	7.358E-06	100	62.38	0.09621	1.424E-05	7.344E-06
200	60.42	0.06320	6.390E-06	3.403E-06	200	60.50	0.08417	6.408E-06	3.408E-06
300	57.64	0.05754	3.879E-06	2.165E-06	300	57.77	0.07654	3.897E-06	2.170E-06
400	54.08	0.05419	2.733E-06	1.626E-06	400	54.23	0.07205	2.786E-06	1.653E-06
(596.90)	42.63	0.05594	1.683E-06	1.270E-06	600	42.88	0.07392	1.694E-06	1.271E-06
(596.90)	3.608	0.6610	4.284E-07	3.821E-06	(635.80)	38.99	0.07865	1.507E-05	1.244E-05
600	3.571	0.6657	4.298E-07	3.872E-06	(635.80)	5.311	0.5774	4.615E-07	2.796E-06
800	2.299	0.8700	5.369E-07	7.514E-06	800	3.255	0.8192	5.421E-07	5.358E-06
1000	1.854	0.9310	6.375E-07	1.106E-05	1000	2.537	0.9072	6.427E-07	8.151E-06
1200	1.581	0.9604	7.329E-07	1.492E-05	1200	2.137	0.9472	7.381E-07	1.111E-05
1400	1.387	0.9767	8.238E-07	1.911E-05	1400	1.864	0.9691	8.289E-07	1.431E-05
2500 psia					3000 psia				
32	62.93	0.1357	3.661E-05	1.872E-05	32	63.05	0.1625	3.646E-05	1.860E-05
100	62.46	0.1201	1.424E-05	7.335E-06	100	62.54	0.1440	1.425E-05	7.331E-06
200	60.61	0.1050	6.427E-06	3.412E-06	200	60.68	0.1259	6.446E-06	3.418E-06
300	57.87	0.0955	3.915E-06	2.177E-06	300	58.00	0.1144	3.933E-06	2.182E-06
400	54.41	0.0898	2.804E-06	1.658E-06	400	54.55	0.1074	2.821E-06	1.664E-06
600	43.44	0.0912	1.719E-06	1.273E-06	600	43.49	0.1093	1.746E-06	1.292E-06
(668.10)	34.98	0.1064	1.335E-06	1.228E-06	(695.33)	29.17	0.1496	1.113E-06	1.228E-06
(668.10)	7.651	0.4866	5.068E-07	2.131E-06	(695.33)	11.76	0.3710	5.968E-07	1.633E-06
800	4.361	0.7643	5.507E-07	4.063E-06	800	5.684	0.7037	5.642E-07	3.194E-06
1000	3.259	0.8825	6.492E-07	6.408E-06	1000	4.021	0.8585	6.573E-07	5.259E-06
1200	2.709	0.9340	7.442E-07	8.840E-06	1200	3.297	0.9208	7.510E-07	7.329E-06
1400	2.348	0.9616	8.345E-07	1.144E-05	1400	2.840	0.9540	8.407E-07	9.524E-06
4000 psia					5000 psia				
32	63.29	0.2159	3.619E-05	1.840E-05	32	63.29	0.2699	3.594E-05	1.827E-05
100	62.89	0.1909	1.427E-05	7.300E-06	100	62.89	0.2386	1.428E-05	7.306E-06
200	60.98	0.1670	6.485E-06	3.422E-06	200	60.98	0.2088	6.525E-06	3.443E-06
300	58.22	0.1519	3.970E-05	2.194E-05	300	58.82	0.1879	4.009E-06	2.193E-06
400	54.84	0.1425	2.857E-06	1.676E-06	400	55.25	0.1768	2.894E-06	1.685E-06
600	44.82	0.1415	1.803E-06	1.294E-06	600	45.66	0.1736	1.865E-06	1.314E-06
800	9.502	0.5613	6.205E-07	2.101E-06	800	16.92	0.3940	8.026E-07	1.526E-06
1000	5.708	0.8063	6.789E-07	3.827E-06	1000	7.622	0.7548	7.092E-07	2.994E-06
1200	4.524	0.8948	7.674E-07	5.458E-06	1200	5.828	0.8682	7.875E-07	4.347E-06
1400	3.844	0.9398	8.545E-07	7.152E-06	1400	4.878	0.9257	8.704E-07	5.741E-06

t °C	ρ kg/m³	Z =pv/RT	μ Pa s	ν m²/s	t °C	ρ kg/m³	Z =pv/RT	μ Pa s	ν m²/s
0.01 Bar (1 kPa)					0.1 Bar (10 kPa)				
0	999.80	7 934E-06	1.789E-03	1.789E-06	0	999.78	7.934E-05	1.789E-03	1.789E-06
(6.98)	999.89	7.736E-06	1 428E-03	1.428E-06	25	997.02	7.289E-05	8 905E-04	8 932E-07
(6.98)	0 007740	0.9993	9.385E-06	1.213E-03	(45.82)	989.82	6.863E-05	5.888E-04	5.948E-07
25	0 007271	0.9995	9.871E-06	1.358E-03	(45.82)	0 068150	0.9968	1.049E-05	1 539E-04
50	0.006707	0 9997	1.063E-05	1.585E-03	50	0.067250	0 9970	1 062E-05	1.579E-04
100	0.005808	0.9998	1.235E-05	2.127E-03	100	0.058150	0.9986	1.234E-05	2.122E-04
200	0.004580	0.9999	1.621E-05	3.539E-03	200	0.045818	0.9995	1 620E-05	3.536E-04
300	0.003781	1.0000	2.030E-05	5.370E-03	300	0.037813	0 9998	2.030E-05	5.369E-04
400	0.003219	1.0000	2.445E-05	7.596E-03	400	0.032192	0.9999	2.445E-05	7.595E-04
500	0.002803	1.0000	2.857E-05	1.019E-02	500	0.028027	0 9999	2 857E-05	1.019E-03
600	0.002482	1.0000	3.261E-05	1 314E-02	600	0 024816	1.0000	3.261E-05	1.314E-03
700	0.002227	1.0000	3.655E-05	1.642E-02	700	0.022266	1.0000	3.655E-05	1.642E-03
1.01325 Bar (101.325 kPa)					5 Bar (500 kPa)				
0	999 83	8.039E-04	1.782E-03	1.782E-06	0	1000 03	3 966E-03	1 791E-03	1 791E-06
25	997 06	7.385E-04	8.908E-04	8 934E-07	25	997.24	3.644E-03	8.907E-04	8.932E-07
50	988.03	6.876E-04	5.471E-04	5.537E-07	50	988 20	3.393E-03	5.471E 04	5.536E-07
75	974.86	6.469E-04	3.784E-04	3 882E-07	100	958.58	3.029E-03	2 824E-04	2.946E-07
(100.00)	958.39	6.139E-04	2.823E-04	2 946E-07	(151 87)	915.31	2.785E-03	1.793E-04	1 959E-07
(100.00)	0.59750	0 9847	1.228E-05	2.055E-05	(151 87)	2.6677	0.9555	1 409E-05	5 282E-06
200	0 46645	0.9948	1.618E-05	3.469E-05	200	2.3537	0.9728	1.607E-05	6 828E-06
300	0.38398	0.9976	2.029E-05	5.284E-05	300	1.9137	0.9877	2.025E-05	1.058E-05
400	0.32657	0.9987	2.445E-05	7 487E-05	400	1.6200	0.9935	2.444E-05	1 509E-05
500	0 28418	0 9993	2.857E-05	1 005E-04	500	1.4066	0 9962	2 858E-05	2.032E-05
600	0.25156	0.9995	3.216E-05	1 278E-04	600	1.2437	0.9976	3.263E-05	2.624E-05
700	0 22567	0 9997	3.655E-05	1 620E-04	700	1.1149	0 9985	3 657E-05	3 280E-05
10 Bar (1 MPa)					15 Bar (1.5 MPa)				
0	1000.29	7.930E-03	1.790E-03	1.789E-06	0	1000.54	1.189E-02	1.789E-03	1.788E-06
25	997.47	7 286E-03	8.906E-04	8 929E-07	25	997.69	1.093E-02	8.905E-04	8.926E-07
50	988.42	6 784E-03	5 472E-04	5 536E-07	50	988.64	1.017E-02	5 473E-04	5.536E-07
100	958 81	6 056E-03	2.826E-04	2 947E-07	100	959 05	9.082E-03	2.827E-04	2 948E-07
(179 92)	887.15	5 391E-03	1.494E-04	1.684E-07	(198 33)	866 69	7.954E-03	1.349E-04	1.556E-07
(179.92)	5.1445	0 9296	1.507E-05	2.929E-06	(198.33)	7 5920	0 9080	1 572E-05	2.071E-06
200	4.8566	0.9429	1 593E-05	3 280E-06	200	7.5510	0.9097	1.579E-06	2 091E-07
300	3.8771	0 9751	2 020E-05	5 210E-06	300	5 8950	0 9619	2.019E-05	3.425E-06
400	3 2617	0 9869	2 442E-05	7 487E-06	400	4.9262	0.9801	2.441E 05	4 955E-06
500	2 8241	0 9924	2 858E-05	1 012E-05	500	4 2526	0.9885	2 858E-05	6 721E-06
600	2.4932	0.9953	3.264E-05	1.309E-05	600	3.7486	0.9930	3.265E-05	8.710E-06
700	2.2331	0 9971	3 659E-05	1 639E-05	700	3.3546	0.9956	3.660E-05	1.091E-05
20 Bar (2 MPa)					25 Bar (2.5 MPa)				
0	1000.79	1.585E-02	1.787E-03	1.786E-06	0	1001 05	1 981E-02	1.780E-03	1 778E-06
25	997 92	1.456E-02	8 904E-04	8 923E-07	25	998 14	1.820E-02	8.898E-04	8.915E-07
50	988 86	1 356E-02	5 474E-04	5.536E-07	50	989.08	1.695E-02	5.479E-04	5 539E-07
100	959.28	1 211E-02	2 829E-04	2.949E-07	100	959 52	1 513E-02	2 836E-04	2 956E-07
200	865.08	1 059E-02	1 339E-04	1 548E-07	200	865 47	1.323E-02	1.345E-04	1.554E-07
(212.42)	849 85	1 050E-02	1 256E-04	1.478E-07	(223 99)	835.19	1.305E-02	1.189E-04	1 424E-07
(212.42)	10.041	0 8888	1 621E-05	1 614E-06	(223.99)	12.508	0.8711	1.663E-05	1 330E-06
300	7.971	0.9485	2.009E-05	2.520E-06	300	10.113	0.9345	1 986E-05	1 964E-06
400	6.614	0 9733	2.440E-05	3 689E-06	400	8.327	0 9664	2.439E-05	2.929E-06
500	5.693	0 9845	2 860E-05	5.024E-06	500	7.144	0.9807	2.861E-05	4 005E-06
600	5 010	0 9906	3 268E-05	6.523E-06	600	6.278	0 9882	3.270E-05	5 209E-06
700	4.479	0.9941	3 665E-05	8.182E-06	700	5.608	0.9926	3.666E-05	6 537E-06

Table D.3 Density and Viscosity of Steam and Compressed Water (*Continued*)

t °C	ρ kg/m³	Z = pv/RT	μ Pa s	ν m²/s	t °C	ρ kg/m³	Z = pv/RT	μ Pa s	ν m²/s
50 Bar (5 MPa)					100 Bar (10 MPa)				
0	1000.31	3.965E-02	1.780E-03	1.779E-06	0	1004.81	7.894E-02	1.769E-03	1.761E-06
25	998.26	3.640E-02	8.898E-04	8.914E-07	25	1001.48	7.257E-02	8.889E-04	8.876E-07
50	990.16	3.386E-02	5.479E-04	5.533E-07	50	992.31	6.757E-02	5.487E-04	5.530E-07
100	960.68	3.022E-02	2.836E-04	2.952E-07	100	962.98	6.030E-02	2.849E-04	2.959E-07
200	867.35	2.640E-02	1.345E-04	1.551E-07	200	871.03	5.257E-02	1.357E-04	1.558E-07
(263.98)	777.52	2.594E-02	9.993E-05	1.285E-07	300	715.58	5.283E-02	8.642E-05	1.208E-07
(263.98)	25.355	0.7955	1.813E-05	7.150E-07	(311.03)	688.63	5.386E-02	8.153E-05	1.184E-07
300	22.073	0.8563	1.986E-05	8.997E-07	(311.03)	55.480	0.6685	2.036E-05	3.670E-07
400	17.299	0.9304	2.438E-05	1.409E-06	400	38.783	0.8300	2.449E-05	6.315E-07
500	14.586	0.9607	2.867E-05	1.966E-06	500	31.172	0.8990	2.898E-05	9.297E-07
600	12.709	0.9763	3.281E-05	2.582E-06	600	26.617	0.9323	3.309E-05	1.243E-06
700	11.299	0.9853	3.679E-05	3.256E-06	700	23.414	0.9509	3.709E-05	1.584E-06
150 Bar (15 MPa)					200 Bar (20 MPa)				
0	1007.28	1.181E-01	1.759E-03	1.746E-06	0	1009.73	1.571E-01	1.749E-03	1.732E-06
25	1003.67	1.086E-01	8.881E-04	8.849E-07	25	1005.84	1.445E-01	8.874E-04	8.822E-07
50	994.43	1.011E-01	5.495E-04	5.526E-07	50	996.53	1.346E-01	5.504E-04	5.523E-07
100	965.25	9.024E-02	2.863E-04	2.966E-07	100	967.48	1.200E-01	2.876E-04	2.973E-07
200	874.60	7.854E-02	1.336E-04	1.528E-07	200	878.10	1.043E-01	1.381E-04	1.573E-07
300	725.90	7.812E-02	8.832E-05	1.217E-07	300	735.00	1.029E-01	9.006E-05	1.225E-07
(342.19)	603.50	8.752E-02	6.930E-05	1.148E-07	(365.80)	491.200	1.381E-01	5.594E-05	1.139E-07
(342.19)	96.72	0.5461	2.276E-05	2.353E-07	(365.80)	170.25	0.3984	2.729E-05	1.603E-07
400	63.89	0.7557	2.491E-05	3.899E-07	400	100.54	0.6403	2.596E-05	2.582E-07
500	48.08	0.8743	2.927E-05	6.088E-07	500	71.93	0.7792	2.982E-05	4.146E-07
600	40.15	0.9271	3.345E-05	8.331E-07	600	58.12	0.8539	3.391E-05	5.834E-07
700	34.94	0.9559	3.746E-05	1.072E-06	700	49.84	0.8935	3.788E-05	7.600E-07
250 Bar (25 MPa)					300 Bar (30 MPa)				
0	1012.14	0.1959	1.739E-03	1.718E-06	0	1014.53	0.1955	1.731E-03	1.706E-06
25	1007.99	0.1802	8.868E-04	8.798E-07	25	1010.11	0.1799	8.864E-04	8.775E-07
50	998.60	0.1679	5.513E-04	5.521E-07	50	1000.66	0.1675	5.523E-04	5.519E-07
100	969.68	0.1497	2.889E-04	2.979E-07	100	971.86	0.1494	2.902E-04	2.986E-07
200	881.40	0.1299	1.393E-04	1.580E-07	200	884.70	0.1294	1.405E-04	1.588E-07
300	743.300	0.1272	9.167E-05	1.233E-07	300	750.90	0.1259	9.318E-05	1.241E-07
400	166.630	0.4829	2.900E-05	1.740E-07	400	358.05	0.2247	4.383E-05	1.224E-07
500	89.900	0.7793	3.061E-05	3.405E-07	500	115.26	0.6079	3.171E-05	2.751E-07
600	70.900	0.8750	3.450E-05	4.866E-07	600	87.48	0.7092	3.520E-05	4.024E-07
700	60.080	0.9265	3.838E-05	6.388E-07	700	73.23	0.7601	3.893E-05	5.316E-07
350 Bar (35 MPa)					400 Bar (40 MPa)				
0	1016.89	0.2730	1.722E-03	1.693E-06	0	1019.23	0.3113	1.714E-03	1.682E-06
25	1012.22	0.2513	8.860E-04	8.753E-07	25	1014.30	0.2866	8.858E-04	8.733E-07
50	1002.69	0.2340	5.533E-04	5.518E-07	50	1004.70	0.2669	5.543E-04	5.517E-07
100	974.00	0.2087	2.915E-04	2.993E-07	100	976.12	0.2379	2.928E-04	3.000E-07
200	887.90	0.1805	1.416E-04	1.595E-07	200	891.00	0.2056	1.428E-04	1.603E-07
300	758.00	0.1746	9.461E-05	1.248E-07	300	764.60	0.1978	9.598E-05	1.255E-07
400	474.90	0.2372	5.578E-05	1.175E-07	400	523.70	0.2459	6.129E-05	1.170E-07
500	144.43	0.6791	3.319E-05	2.298E-07	500	177.97	0.6299	3.516E-05	1.976E-07
600	105.15	0.8260	3.602E-05	3.426E-07	600	123.81	0.8017	3.698E-05	2.987E-07
700	86.78	0.8980	3.955E-05	4.558E-07	700	100.71	0.8843	4.025E-05	3.997E-07

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